ERM-INV Model Solutions
Fall 2016

1. Learning Objectives:
   4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

   5. The candidate will understand the concept of economic capital, risk measures in capital assessment and techniques to allocate the cost of risks within business units.

Learning Outcomes:
(4k) Apply best practices in risk measurement, modeling and management of various financial and non-financial risks faced by an entity.

(5a) Describe the concepts of measures of value and capital requirements (for example, EVA, embedded value, economic capital, regulatory measures, and accounting measures) and demonstrate their uses in the risk management and corporate decision-making processes.

Sources:
ORSA - An International Requirement (Section 3.1 and Section 4.1)

Risk Appetite: Linkage with Strategic Planning Report

SOA 2012 Annual Meeting – Session 53 – Assumption Setting Best Practices (Towers Watson)

Commentary on Question:
This question sought to test the candidate’s knowledge of risk governance, economic capital modeling and ORSA, as well as the application in a hypothetical situation. As such, a simple bullet point list without explanations would not be considered as adequate to demonstrate the knowledge. Partial credit was awarded for correct relevant statements that didn’t address the acquisition (in part b) or the newly grown company (part c).
1. Continued

Solution:
(a) ABC considers adoption of the three lines of defense approach as the risk governance and internal controls system.

(i) Compare XYZ’s risk management structure to the three lines of defense framework.

(ii) Recommend improvements to enhance risk governance.

Commentary on Question:
In order to receive full credit for section one, candidates needed to compare XYZ to the three lines of defense framework from the study note. The answers here are more comprehensive than what is needed for full credit. However, the generic comment, such as “Hire a CRO” or “implement assumption governance”, was not deemed sufficient. In addition, candidates were expected to provide some justification for their part (ii) responses.

(i) Each LOB functions as the first line, pricing and managing their products with little regular oversight.

Second line is a risk management capacity, which should be done by the Control Unit as they conduct periodic reviews of the LOBs and valuation unit for the Chief Actuary. However, it doesn’t look like they challenge LOB pricing or profit projections as would be expected.

Control Unit also performs EC calculation using pricing models but there is no review done on this work. Are pricing models capturing all relevant and material risks in the EC results?

No mention of an internal audit function, which would be the 3rd line of defense.

(ii) The assurance function should be provided by 3rd line, such as internal or external audit, outside of the chief actuary's reporting structure.

No mention of whether or not the control department is responsible for monitoring and reporting on overarching risks (regulatory, compliance, liquidity, etc). If control isn't in charge, this function may not have a home for XYZ.

Control Department, in calculating economic capital, should set their own assumptions and not rely completely on the pricing assumptions -- while both are best estimate, the different purposes for each (real world vs risk neutral) may result in different views of what is considered best estimate.
1. Continued

(b) Assess XYZ’s existing Economic Capital modeling considerations in light of the acquisition.

Commentary on Question:
Many candidates focused on the shortcomings in XYZ’s current EC modeling and failed to address the consideration “in light of the acquisition.” As such, limited partial credit was awarded for generic answers about EC in general or shortcoming’s in XYZ’s current modeling. There were many different correct answers, some of which are mentioned below.

- Company ABC and XYZ may have different risk appetite and business strategy, which would impact the EC model.
- Capital model assumptions need to be updated for best estimate based on Company ABC or combined company’s experience – may leverage on the appraisal.
- Risk identification and assessment may differ (such as operational, catastrophe, liquidity and etc.).
- Company ABC does not have much experience in property and casualty insurance, so ABC may rely on XYZ’s model with adjustment for experience and other best estimate assumptions.
- Shock lapse and other adjustment to behaviors or activated options from merger activity and potential business disruptions.
- May want to use a different metric for Auto/Home vs Life/Annuity blocks

(c) Describe the potential issues, excluding XYZ’s Economic Capital modeling considerations from part (b), to address when preparing the ORSA for the combined company after the acquisition.

Commentary on Question:
Here again, the candidates were expected to explain why or how the list of considerations apply to the newly enlarged company.

- Consider the interaction and diversification benefits resulting from the combined business, given the risk appetite and business plans.
- Stress and scenario testing may need updating to capture the tail risk, thus re-evaluate the capital needs and solvency positions. Simple summation may not be appropriate.
- ORSA should be undertaken at the group level to assess the adequacy of the group's risk management. Certain limitation should be considered:
  - Fungibility of capital
  - Transferability of assets post-acquisition
  - Potential double-counting of capital between LOBs
  - Other business or regulatory restrictions or requirement on capital
1. Continued

- Forward looking assessment of solvency needs by taking into account of the impact of business plans of combined business, such as:
  - Pricing, product development and marketing, especially on acquired LOBs
  - Setting risk appetite / Capital plans / Strategic plans post-acquisition
  - Compensation aligned with risk-adjusted performance
2. **Learning Objectives:**

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.

4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

**Learning Outcomes:**

(1c) Identify and assess the potential impact of risks faced by an entity, including but not limited to market risk, currency risk, credit risk, counterparty risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk, project risk and strategic risk.

(4b) Demonstrate means for transferring risk to a third party, and estimate the costs and benefits of doing so.

(4e) Develop an appropriate choice of a risk mitigation strategy for a given situation (e.g., reinsurance, derivatives, financial contracting), which balances benefits with inherent costs, including exposure to credit risk, basis risk, moral hazard and other risks.

(4j) Demonstrate risk management strategies for other key risks (for example, operational, strategic, legal, and insurance risks).

**Sources:**

ERM-122-16: Chapter 1 of Captives and the Management of Risk, Kate Westover

ERM-114-13: Introduction to Reinsurance, Rudolpho Wehrhahn

Financial Enterprise Risk Management, Sweeting, 2011, Ch. 7 Definitions of Risk

ERM 107-12: Strategic Risk Management Practice, Anderson and Schroder, 2010 Ch. 7 Strategic Risk Analysis

**Commentary on Question:**

This question was to test the candidates’ knowledge of captive insurance, the essential elements of such insurance, risk identification and mitigation and recommending (or not) entering into a retrocession agreement with an offshore affiliate. Overall candidates performed well with this question. Candidates missed points: e.g., not providing what are the essential elements, not justifying the answers, not analyzing the specific case provided in the question stem and sub-stems, and not providing a recommendation.
2. **Continued**

**Solution:**

(a)  

(i) Describe the essential elements of captive insurance.

(ii) Determine whether CP Captive qualifies as a captive insurance company. Justify your response.

(iii) Determine what type of captive CP Captive most resembles. Justify your response.

**Commentary on Question:**

(i) Some candidates did not provide the essential elements but instead provided minor elements. Other candidates only listed elements and did not describe them; these candidates received no points.

1. The insureds in captive insurance choose to put their own capital at risk. They are both willing and able to contribute risk capital.
2. Seek to increase control: the insureds wish to improve control over the way insurance is used to finance their risks. Captive insurance involves financing risks, working outside the commercial regulated marketplace.
3. Captive insurance is used by insureds to achieve their risk financing objectives.

(ii) Some candidates concluded that CP Captive did not qualify as a captive insurance and received full marks if they provided justification. Model solution for CP does qualify as a captive insurance company.

1. It is unclear if CP Captive put up their own capital at risk; however, it is assumed that CP Captive did in order to form a captive insurance company.
2. Each facility (i.e., each insured) is participating in the collective experience of all the facilities (from 1 to N).
3. Each facility is using the captive as a risk tool/technique to be able to apply for stop loss reinsurance coverage with ABC Re. Therefore I conclude that CP Captive qualifies as a captive insurance company.
2. Continued

(iii) Some candidates who provided an alternate answer of Association Captive and also provided justification received full marks.

Industrial Insured Group Captive
- If each of the rehabilitation facilities owns a portion of CP Captive and has injected capital into the captive based on a formula, e.g., employee size, then CP Captive is an Industrial Insured Group Captive. This is true for CP Captive.
- If insureds are in the same industry group, or with homogeneous risk, which creates group buying power and other risk management efficiencies CP Captive’s insureds are in the same industry group and their purpose was to participate in their collective experience as a group in order to purchase stop loss coverage.
- If the insureds are sophisticated insureds, meaning that they have sufficient size (based on net worth and number of employees) to qualify under state insurance laws for the purchase of non-admitted insurance. It is assumed that CP Captive is sufficient in size under the state laws. The question stated that CP Captive is "large enough."

(b) You identified the following top four risks for ABC Re in this captive arrangement:

- Reinsurance treaty negotiation
- Insurance
- Legal and regulatory
- Operational

Describe approaches for ABC Re to manage each of these risks.

Commentary on Question:
Some candidates provided general mitigants that could apply to any reinsurance or insurance company, rather than specific mitigants to manage ABC Re’s risks as a reinsurer to a captive insurance company. This was especially prevalent in the topic of Operational Risk. Partial points could be earned in this situation.

Candidates did not receive points if they incorrectly provided answers in the perspective of CP Captive or Retro Life.

Some candidates only listed and did not sufficiently describe the approaches; partial points could be earned in this situation.
2. Continued

Reinsurance Treaty Negotiation

ABC Re should:
- Include treaty provisions for ABC Re’s right to audit CP Captive: claims, underwriting, financial reporting, etc.
- Conduct periodic due diligence review of each of the companies – provided the number of facilities is small enough – or a sample of the facilities to ensure that CP Captive is managing CP Captive’s risks, e.g., good underwriting and claims adjudication.
- Consider a provision for facultative reinsurance versus automatic on new facilities that are added to CP Captive.

Insurance

ABC Re should:
- Request experience data – e.g., detailed first dollar claims - from CP Captive to ensure it can provide the appropriate quote.
- Consider having CP Captive include a coinsurance percentage for claims above the $10 million attachment point/retention level.
- Monitor CP Captive’s facility persistency level.
- Perform stress testing scenario testing and consider holding more capital.

Legal and Regulatory

ABC Re should
- Identify and monitor regulatory changes in the captive insurance space, be a member of regulatory lobby groups or insurance company associations, and be ready to adapt to changes
- Monitor and limit exposure of doing business with captives; set appropriate vetting process on the relationships with new Captives and periodic due diligence review on existing Captives relationships

Operational

ABC Re should:
- Document ahead of the treaty effective date how claims adjudication and payment, as well as premium billing and accounting will be managed under the new treaty, provide training and close supervision to ensure processes will be followed accurately following the treaty’s effective date.
- Conduct testing in the first year and on a periodic basis thereafter.
- Develop Key Risk Indicators or early warnings on claims potentially reaching the attachment point.
2. Continued

(c) As a means to manage capital efficiently, ABC Re proposes ceding a portion of the assumed risks from CP Captive to an offshore affiliate in the Cayman Islands called Retro Life.

(i) Describe key advantages and disadvantages of entering into a reinsurance agreement between ABC Re and Retro Life.

(ii) Recommend a course of action with regard to this proposal. Justify your response.

Commentary on Question:
Some candidates only listed and did not sufficiently describe the advantages and disadvantages. Partial points could be earned in this situation.

Advantages
- Sharing potential profit with an affiliate, versus an external company. If required to post a security, advantage is that it will be with an affiliate.
- Save on reinsurance broker commission since agreement is with an affiliate.
- Often with less demanding regulatory and favorable tax environments.

Disadvantages
- Lack of strict regulation or low capital requirements can lead to failing reinsurers; or require capital injection from parent/holding company
- Potential new regulations/restrictions on claims data leaving domicile state and/or leaving U.S. for offshore.
- May not have the benefit of an external reinsurer expertise in the product development, pricing/modeling, underwriting and claims management, etc. -- especially for XS $10M claims where ABC Re begins its claims exposure.

In general, candidates either recommended or not recommended proceeding with the proposal. Full marks were given if the candidate indicated a need for additional information with sufficient justification. Some candidates did not make any recommendation but provided a course of action that includes mitigating actions that would alleviate the disadvantages outlined in their answer for (c.i); these candidates received partial marks.
2. Continued

I recommend holding off on a retrocession treaty with an offshore affiliate.
- As CRO of ABC Re, I would recommend waiting until ABC’s captive insurance business grows to a significant size – by premium or by number of clients. We ought to wait and monitor the persistency of the facilities that have signed on with CP Captive, i.e., has this one captive's business grown?
- I believe the costs today may outweigh the benefits of such a structure. I assume that this is our first partnership/relationship with a captive. I highly recommend learning from this experience before adding a layer of complexity, such as a retrocession agreement.
- There is a potential stricter regulatory change in the horizon on using captives. I recommend simplifying the arrangement by not retroceding and monitoring the regulatory landscape.

Alternate answer:
I need to have additional information in order to make a recommendation.
- I need to know what the capital saving is compared to the cost of structuring an offshore reinsurance for - at the moment - one captive case.
- I need to know if there are alternative options. Can we go out to bid for an external reinsurance - either onshore or offshore - and reap more benefits than to partner with an affiliate? Can an external reinsurer provide reinsurance expertise that we do not have at ABC Re?
- I need to know what the level of expertise is at the offshore affiliate: Have they been established for some time? How strong are their finance, accounting, legal, systems, governance and actuarial staff. Can we rely on the offshore affiliate to provide the proper support as a retrocessionaire?
3. **Learning Objectives:**

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

**Learning Outcomes:**

(1c) Identify and assess the potential impact of risks faced by an entity, including but not limited to market risk, currency risk, credit risk, counterparty risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk, project risk and strategic risk.

(2d) Apply and analyze scenario and stress testing in the risk measurement process.

(2g) Analyze and evaluate model and parameter risk.

(3c) Analyze quantitative financial data and insurance data (including asset prices, credit spreads and defaults, interest rates, incidence, causes and losses) using modern statistical methods. Construct measures from the data and contrast the methods with respect to scope, coverage and application.

**Sources:**

Financial Enterprise Risk Management, Sweeting, 2011, Ch. 7 Definitions of Risk

ERM-118-14: Model Validation Principles Applied to Risk and Capital Models in the Insurance Industry

ERM-120-14: IAA Note on Stress Testing and Scenario Analysis (pp. 1-6 and 14-17)


ERM 602-12: Investment Management for Insurers, Babbel and Fabozzi, Ch. 11 The Four Faces of an Interest Model

**Commentary on Question:**

*The majority of the candidates attempted to answer this question. Generally speaking, scores were given based on the ability of the candidates to demonstrate the depth of their understanding of the subject in relation to DirectComp’s situation. Partial points were given to candidates who were able to able to describe the concept from the study material but gave no reference to how it can be applied to DirectComp.*
3. Continued

Solution:
(a) Describe the steps to compute DirectComp’s economic reserve requirement using Monte Carlo simulation.

Commentary on Question:
Most of the candidates struggled to describe the steps, especially Steps 2 and 3. To receive full credit, candidates had to demonstrate their understanding of the steps in relation to the given scenario. If only a recital of the steps was given straight out of the textbook, only partial points were awarded.

The economic reserve calculated is CTE70 of the accumulated deficiency

Step 1: Choose a stochastic process to model equity return and a drift equal to the average return of S&P movement. When modeling equity returns, if normal distribution is assumed, it does not capture tail events very well.

Step 2: Simulate S&P prices to the horizon covering the duration of the liability profile.

Step 3: Calculate the equity-linked annuity product claims and premiums along each path, and calculate the accumulated deficiency using the claims and premiums.

Step 4: Repeat this process as often as needed. Then take the average of the worst 30% scenario results.

(b) Explain the advantages and disadvantages of using Monte Carlo simulation to determine DirectComp’s economic reserve.

Commentary on Question:
Unlike Part (a) where most of the candidates struggled to give a good answer, some candidates were able to give reasonable answers and score relatively higher than other candidates in Part (b). This part tested candidates' appreciation of using the Monte Carlo method of modeling complicated insurance products rather than relatively simple financial instruments which oftentimes can be valued using closed form formulas.
3. Continued

Pros:
- Simple to explain; since the method is to take some average over a large number of scenarios, it is usually more straight-forward to explain to management
- Does not require a closed-form formula for modeling complex financial derivatives or insurance products; DirectComp’s equity-linked annuity product can be fairly complicated. There rarely is a closed form formula to calculate the value. Monte-Carlo method gives the flexibility to explicitly simulate the payoff along each path.
- Good for modeling path-dependent financial products; a lot of the behavior modeling for complex equity-linked products is path-dependent. Monte Carlo method is superior in handling this.

Cons:
- Intensive computational time requirements; to ensure result convergence, it may require a large number of valuation paths.
- Works relatively well for only one random variable. It’s less straight-forward if multiple risk factors are evaluated together. Computational intensity grows exponentially; if more than one stochastic process needs to be modeled, for example stochastic interest rates in addition to equity returns, the computational intensity will increase dramatically.
- It cannot price options accurately when the holder can exercise early. In the given example, insurance products are subject to policyholder’s early surrender, etc. So it creates complexity.

(c) Determine whether a risk-neutral or real world model is more appropriate when performing DirectComp’s economic reserve calculation. Justify your response.

Commentary on Question:
Similar to Part (b), some candidates were able to give reasonable answers and score relatively higher than other candidates in Part (c). It was important to differentiate between risk neutral and real world in drawing an appropriate conclusion.

When to use each of the model types:

* Risk Neutral
  Mostly used for various types of pricing.

* Real World
  Used for stress testing and reserve and asset adequacy testing.
3. Continued

The economic reserve is calculated to reflect a realistic amount a business believes it needs to meet future liabilities. Unlike pricing exercises, for which the goal is to determine a fair price of some financial instruments or insurance products in a risk neutral framework, the economic reserve calculated here is based on the worst 300 possible cash-flow outcomes generated from a large set of scenarios, usually 1000. These scenarios are meant to represent certain realistic views of the market movements and easier for management to interpret and discuss.

In this case, real world model is most appropriate.

(d)

(i) Analyze the adequacy of the provided stress test.

(ii) Evaluate whether the collateral asset increase is reasonable.

(iii) Identify the risks DirectComp is exposed to from the reinsurance arrangement.

(iv) Recommend actions DirectComp could have taken in advance to mitigate risks from the reinsurance arrangement that you identified in (iii).

Commentary on Question:

Most of the candidates struggled with Part (d-i and d-ii) but did relatively well with Part (d-iii and d-iv). Candidates were expected to be thorough in analyzing and evaluating the answers to Part (d-i and d-ii) while, for Part (d-iii and d-iv), reasonable recommendations for mitigating counter-party risks were considered acceptable.

(i) Analyze the adequacy of the provided stress test.

The analysis focused on market risk. This is appropriate, since the reinsured product had benefits linked to the market (S&P 500 index). Market risk is a major risk factor in determining the value of the product.

However, the shocks considered are too mild. The most severe shock specified was only 30%. For the nature of stress testing, the focus should be on low frequency, but high severity events. Historical market movement also proved that market downturn can be much more severe than 30%. It is recommended to use more severe stresses for this purpose.
3. Continued

For stress testing purposes, one should also consider actuarial risk as well as actuarial and market combo stress. For equity-linked products, even though market risks are important, actuarial risks still play a big role. The dynamic behavior of the actuarial risk resulting from adverse market conditions can significantly impact the results. So it is recommended to add some actuarial and combo stresses.

(ii) Evaluate whether the collateral asset increase is reasonable.

Unlikely but possible:

The 80% increase sounded too high. However, there are two main reasons that the results could possibly be realistic:

1. The sensitivity on CTE70 is not linear to the market performance. Issuing equity-linked insurance products is analogous to the insurance company writing an “option” to the policyholders. The response of the option value to market conditions may be different at different “moneyness” levels.
2. Combo effect from other risks: the sensitivity analysis provided examined only account value shock. Complex insurance products are exposed to many different kinds of risks at the same time. In today’s market, equity market movement is normally coupled with other risk factors; for example, interest rate risk and actuarial risks. The sensitivity analysis can potentially be missing the contribution from movement of these risk factors in reality.
3. Inforce composition change could also contribute to the dramatic reserve increase: e.g. sales volume higher than previously projected, surrender lower than previously priced, etc

(iii) Identify the risks DirectComp is exposed to from the reinsurance arrangement.

Counter-party risk:

Reinsurer may not have enough liquidity and/or can choose to default on the agreement. This will result in DirectComp having to bear the remaining liability and pay off the claims. If in the claim phase, DirectComp may face a liquidity issue which can potentially also lead to insolvency.
3. Continued

(iv) Recommend actions DirectComp could have taken in advance to mitigate risks you identified in (iii) from the reinsurance arrangement.

Require more thorough stress testing analysis to cover more adverse scenarios. This will provide a finer picture for both parties to assess the risk profile of the block of business.

Actions to take in advance include demanding higher collateral assets as a cushion, e.g. a multiple of the DirectComp’s statutory reserve.

Choose reinsurers with a higher rating which indicates better financial strength to overcome severe market conditions.

Involve third party guarantor to ensure DirectComp’s exposure is always covered.

(e) (i) Evaluate the adherence of DirectComp’s model validation process to key model validation principles.

(ii) Recommend any changes or enhancements that DirectComp should make to this process.

Commentary on Question:
Most of the candidates did well in Part (e). Higher scores were given to candidates who were able to be more comprehensive in their answers than others.

(i) Evaluate the adherence of DirectComp's model validation process to key model validation principles.

- Model validation is probably not independent.
- There is not an owner of the model validation; effort is passed around.
- There does not seem to be any model governance.
- Using any reports of testing of the model during the year does not constitute validating what is important for this particular model for this use.

(ii) Recommend any changes or enhancements that DirectComp should make to this process.

- Assign model validation to an independent person, who is not the one who wrote it or maintains it.
- Establish formal model governance.
- The validation report should address what is most important for this model for this use.
3. **Continued**

- Confirm that the model is appropriate for the use being made of it.
- Mandate detailed documentation of the model development process, including assumptions setting, experience study, etc., on a regular basis to facilitate model validation.
- Hire a third party independent consultant, who specializes in the field and has performed similar exercises in the industry, to conduct regular model validation exercises.
4. **Learning Objectives:**

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

**Learning Outcomes:**

(2b) Evaluate how risks are correlated, and give examples of risks that are positively correlated and risks that are negatively correlated.

(2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas.

(3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.

(3b) Analyze and evaluate the properties of risk measures (e.g., Delta, volatility, duration, VaR, TVaR, etc.) and their limitations.

(4a) Demonstrate and analyze applicability of risk optimization techniques and the impact of an ERM strategy on an organization’s value. Analyze the risk and return trade-offs that result from changes in the organization’s risk profile.

**Sources:**


Ch. 7 Portfolio Risk: Analytical Methods


Ch. 9 Forecasting Risk and Correlations


Ch. 13 Liquidity Risk

**Commentary on Question:**

This question was designed to test the candidate’s understanding of parametric risk aggregation techniques and portfolio selection. In addition, the candidate’s understanding of the limitations of the variance/covariance approach to risk aggregation as well as knowledge of alternative methods for assessing risk were tested.
4. Continued

Candidates generally struggled more with the calculation-intensive portions of this question, though partial credit was awarded for proper application of the appropriate concepts despite incorrect final answers. Candidates were not double-penalized if incorrect values calculated in one part were used appropriately in another part.

Solution:
(a) Demonstrate that investing the $3 million in Fund C results in a lower 95% VaR for the aggregate portfolio than investing in Fund B. Show your work.

Commentary on Question:
The purpose of this question was to test the candidate’s understanding of the variance/covariance approach to risk aggregation. The question also implicitly tested the candidate’s understanding of the relationship between correlation and covariance.

Full credit was awarded for correct answers with supporting demonstration of all calculations. Solutions with minor mathematical errors were given substantial credit. The most common mistake was treating the given variances as standard deviations, though candidates committing this error were still entitled to partial credit if the solution was otherwise correct.

First, calculate the covariance of Fund A with each of Fund B and Fund C:

\[ \rho_{AB} = \frac{\sigma_{AB}}{\sigma_A \sigma_B} \]

\[ -0.3 = \frac{\sigma_{AB}}{(0.1)(0.2)} \]

\[ \sigma_{AB} = -0.006 \]

Likewise, \( \sigma_{AC} = -0.003 \)

Now, calculate the 95% VaR for each portfolio:

For AB Portfolio: \( \sigma_{\text{PortAB}}^2 = [5 \ 3] \begin{bmatrix} .01 & -.006 \\ -.006 & .04 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = 0.4300 \)

95% VaR = 1.645 * 0.4300 = $1.079M

For AC Portfolio: \( \sigma_{\text{PortAC}}^2 = [5 \ 3] \begin{bmatrix} .01 & -.006 \\ -.006 & .04 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = 0.4300 \)

95% VaR = 1.645 * 0.4300 = $1.079M
4.  Continued

For AC Portfolio: \[ \sigma_{PortAC}^2 = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} .01 & -.003 \\ -.003 & .0225 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = 0.3625 \]

95% VaR = 1.645 * 0.3625 = $0.990M

As demonstrated, adding fund C produces the lowest 95% VaR.

(b) Calculate the diversification benefit for Mr. Jackson’s aggregate portfolio.

**Commentary on Question:**
In general, candidates who performed well on (a) also performed well on part (b), although a correct answer to part (a) was not required to receive full credit for part (b). Full credit was given for correctly calculating standalone VaR for both funds and using the appropriate value from (a) to determine the diversification benefit.

\[
\text{Diversification Benefit} = (\text{Standalone VaR for Fund A} + \text{Standalone VaR for Fund C}) - \text{VaR for AC Portfolio}
\]

Standalone VaR for A = 1.645*0.1*5 = $0.823M
Standalone VaR for C = 1.645*0.15*3 = $0.740M

\[
\text{Diversification Benefit} = (0.823 + 0.740) - 0.990 = $0.573M
\]

(c) Describe two factors, besides the diversification effect, that an investor should consider when adding funds to a portfolio.

**Commentary on Question:**
The question was designed to emphasize that other important factors, apart from the diversification effect, should be considered when making investment decisions.

*Many candidates provided two valid considerations with explanation on how such factors pertain to Mr. Jackson or other investors. Many candidates lost points for discussing items directly related to the diversification effect (e.g. portfolio variance). Others lost points for simply listing items but failing to describe why such considerations are important for making investment decision.*

An investor should consider the duration of the assets of the fund and any additional liquidity concerns. For example, if the funds contain long duration assets or early withdrawal fees, an investor may be unable to meet future cash obligations without sustaining substantial loss.
4. **Continued**

A prudent investor should also consider the risk/return trade-off associated with a given investment, e.g. the Sharpe ratio. An investor may be willing to take on additional risk if the expected returns are favorable and similarly may avoid “safe” investments if the returns are nominal.

(d) At the end of the following month, Mr. Jackson plans on making an additional contribution in the amount of $100,000 to either Fund A or Fund C.

(i) Calculate $\beta$ for each of these two funds before the additional contribution.

(ii) Estimate, for each of the two funds, the marginal impact to the 95% VaR of investing $100,000.

(iii) Discuss the limitations of using the calculated $\beta$ to estimate marginal VaR.

(iv) Recommend an alternative method to estimate marginal VaR.

**Commentary on Question:**

Candidates, in general, performed poorly on parts (i) and (ii). Substantial partial credit was awarded despite minor mathematical mistakes if the candidate’s work was clear and traceable. Candidates were not penalized if the incorrect results from prior questions were used correctly in their calculations.

Part (iii) asked the candidate to discuss multiple limitations of using $\beta$ to estimate the marginal VaR. Candidates providing only a single response were given partial credit.

Many candidates who responded to part (iv) gave the sought after answer of incremental VaR with a thorough but concise explanation. Other valid responses were given full credit if the candidate provided a similarly thorough explanation.
4. Continued

(i)

\[ \beta = W \times \frac{\sum x}{\sum x'} \]

Where \( W = \) Initial Portfolio Value and \( x = \) vector of dollar exposures

\[ \beta = 8 \times \begin{bmatrix} .01 & -.003 \\ -.003 & .0225 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} \]

\[ \beta = \begin{bmatrix} .0410 \\ .0525 \end{bmatrix} \]

\[ \beta = \begin{bmatrix} .3625 \end{bmatrix} \]

\[ \beta = \begin{bmatrix} .9048 \\ 1.1586 \end{bmatrix} \]

Therefore, \( \beta \) for Fund A is 0.9048 and \( \beta \) for Fund C is 1.1586.

Note: Alternative methods arriving at the same answer were awarded full credit.

(ii)

\[ \Delta VAR = \alpha(\beta_i \times \sigma_p) = \frac{1.645 \left( \begin{bmatrix} .9048 \\ 1.1586 \end{bmatrix} \right. \times \sqrt{.3625}}{8} \]

Marginal VaR from adding to A = $0.1MM * 0.1120 = $0.0112MM = $11,200
Marginal VaR from adding to C = $0.1MM * 0.1434 = $0.0143MM = $14,340

(iii) Estimating the change in portfolio VaR using \( \beta \) results in a linear approximation of a nonlinear function. Such method is only valid for small changes in portfolio size. If the portfolio positions change frequently, it will be necessary to recalculate \( \beta \) each time in order to obtain accurate approximations for future changes to VaR. In addition, calculating \( \beta \) for large portfolios consisting of multiple funds can be computationally intensive, perhaps to the point of impracticality.
4. Continued

(iv) A valid alternative for calculating marginal change in portfolio VaR is to determine the incremental VaR associated with a $100,000 addition to each Fund.

First, calculate the aggregate VaR for the combined portfolio with $5.1M invested in Fund A and $3.0M in Fund C, then subtract the VaR calculated in part (a) to determine the marginal change in VaR associated with an additional $100,000 in Fund A.

Likewise, calculate the aggregate VaR for the combined portfolio with $5.0M invested in Fund A and $3.1M in Fund C and subtract the VaR calculated in part (a) to determine the marginal change in VaR associated with an additional $100,000 in Fund C.
5. **Learning Objectives:**
2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

**Learning Outcomes:**
(2e) Evaluate the theory and applications of extreme value theory in the measuring and modeling of risk.

(2f) Analyze the importance of tails of distributions, tail correlations, and low frequency/high severity events.

(2g) Analyze and evaluate model and parameter risk.

**Sources:**
Financial Enterprise Risk Management, Sweeting, 2011, Ch. 12 Extreme Value Theory (pages 276 – 278)

ERM-104-12: Study Note on Parameter Risk, Venter and Sahasrabuddhe (pages 2, 5-10)


Article: Modeling Tail Behaviour with Extreme Value Theory, Risk Management, Sept 2009 (pages 15-17)

SOA Monograph- A New Approach to Managing Operational Risk -Chapter 8 (pages 46-49)

**Commentary on Question:**
This question was looking to test on the areas of model and parameter risk, as well as Extreme Value Theory. While cyberliability was the specific application of these concepts, it was not expected that the candidates would have specific knowledge about this relatively new risk class.

**Solution:**
(a) 
(i) Describe challenges that you would encounter in developing a cyberliability risk model for Bank XYZ.

(ii) Explain how you would address these challenges.
5. Continued

Commentary on Question:
We were looking for answers covering three broad areas: Data, Parameters, and Model Choice. Most candidates pointed out appropriate aspects of data challenges, with fewer touching on the other two. The answer for part a(ii) was supposed to relate to the specific challenges mentioned in part a(i), and most candidates did this.

(i) Data challenges:
Data are limited – experience is from last five years only; data only from media reports so smaller losses are unlikely to be captured

Parameterization challenges:
There’s a risk of missing important relationships in model being developed; some of the companies reporting losses may have different vulnerabilities to cyberliability or different base exposures

Model choice challenges:
If we pick wrong model structure, such as normal distribution vs. Generalized Pareto, we may have insufficient risk measures;

(ii) To address data challenges:
Collect more data, get consortium data, try to capture older information. Normalize or make adjustment on the existing data according to the relative scale of the company size or exposure.

To address parameter challenges: investigate specific data points more closely to learn exposure (how many accounts/policies), type of hack attempted, company own hacking protections – allow margins on parameters to reflect uncertainty

To address model choice challenges: attempt to fit multiple models, do sensitivity tests/goodness of fit tests, develop scenarios to capture dependencies

(b) Explain why this distribution may be appropriate for fitting these data.

Commentary on Question:
We were looking for two aspects here to explain why a GPD would be a good model structure choice: data aspects and aspects about the fundamental qualities of cyberliability as a risk. Most candidates received at least some credit on this part.
5. **Continued**

Our data are likely only tail data – we see only the most extreme losses in media; this is a tail distribution for tail data

Cyberliability is a relatively new, highly uncertain risk – much of the trouble is in low frequency/high severity events, which this model captures as a long-tailed risk

(c) Explain the considerations in setting the threshold parameter.

**Commentary on Question:**
Most candidates got full points on this part – we were looking for answers that talked about the balance between setting the threshold too low and too high. Some candidates explained the mathematical approach of looking at the behavior of the mean excess loss function to set the threshold, but that was not necessary for full credit.

If the threshold $u$ is set too high, there will be too little data to do a good model fit

If the threshold $u$ is set too low, then one may be capturing data that are not extreme and not representing tail risks

(d)

(i) Calculate the range for beta under these assumptions. Show your work.

(ii) Calculate the range for the 99th percentile of the loss distribution under these assumptions. Show your work.

**Commentary on Question:**
The most important part of this problem was recognizing the nature of the conditional cdf and setting it up appropriately. The bulk of the credit for this problem was weighted towards setting up the cdf formula correctly. Many candidates struggled with this part. However, candidates were not double-penalized for carrying the same error from part d(i) into part d(ii).

In addition, candidates were expected to recognize the unreasonable parameters/results, such as negative beta, the 99th percentile losses lower than $100 million, and etc.
5. Continued

(i) $100 million is at 95\textsuperscript{th} percentile, so we adjust the cdf

If \( x > 100 \),

\[
F(x) = 0.95 + 0.05 \cdot \left[ 1 - \left( 1 + \frac{x - 100}{\beta} \right)^{-1} \right]
\]

\[
F(x) = 1 - 0.05 \cdot \frac{\beta}{\beta + x - 100}
\]

Largest loss = $700 million

\[
F(700) = 1 - 0.05 \cdot \frac{\beta}{\beta + 600}
\]

\[
\frac{\beta}{\beta + 600} = 20[1 - F(700)]
\]

One end of interval: \( F(700) = 0.98 \)

\[
\frac{\beta}{\beta + 600} = 20[0.02] = 0.4
\]

\[0.6\beta = 240, \beta = \$400 \text{ million}\]

Other end of interval: \( F(700) = 0.995 \)

\[
\frac{\beta}{\beta + 600} = 20[0.005] = 0.1
\]

\[0.9\beta = 60, \beta = \$66.7 \text{ million}\]

Beta in the range of [\$66.7 million, \$400 million]

(ii) Looking for 99\textsuperscript{th} percentile:

\[0.99 = 1 - 0.05 \cdot \frac{\beta}{\beta + x - 100}\]

\[0.2 = \frac{\beta}{\beta + x - 100}\]

\[x = 100 + 4\beta\]

using the results from d(i),

the range for the 99\textsuperscript{th} percentile loss is [\$366.7 million, \$1.7 billion]
5. Continued

(e) Recommend two methods to validate inputs for the cyberliability model.

**Commentary on Question:**
*This was looking for model validation techniques involving inputs, but they had to make sense for cyberliability risk in this context. Some of the methods recommended by candidates could not apply to this type of model. In addition, we were looking for short explanations of what these validation methods involved; no credit was given for just the name of the technique.*

Back-test distribution: look at new cyberliability loss data as it comes in; are newly-reported cyber events in line with our fitted model?

Benchmarking: compare model result against appropriate industry benchmarks for cyberliability risk.

Expert judgment – consider emerging developments, esp. given limited data; seek independent experts to review assumptions and opinions.
6. **Learning Objectives:**

3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

**Learning Outcomes:**

(3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.

(4c) Demonstrate means for reducing risk without transferring it.

(4d) Demonstrate how derivatives, synthetic securities, and financial contracting may be used to reduce risk or to assign it to the party most able to bear it.

(4g) Demonstrate the use of tools and techniques for analyzing and managing credit and counterparty risk.

**Sources:**

Jorion, Chapter 18: Credit risk management, in Value at Risk: The new benchmark for managing financial risk, 3rd edition

PWC, Creating an understanding of Special Purpose Vehicule, ERM-115-13

Sweeting, 2011. Chapter 16: Responses to risk, in Financial enterprise risk management

**Commentary on Question:**

This question pertains to credit risk, its definition and credit/accounting/rating agency issues related to the use of a Special Purpose Vehicle (SPV) for the purpose of managing credit risk.

**Solution:**

(a) Describe three components of credit risk.

**Commentary on question:**

In order to receive maximum points, a candidate had to list and briefly describe the three major components that constitute the basic building blocks of credit risk. These can be found in any credit risk model or assessment of the credit worthiness of any counterparty, asset, investment, or derivative. Some candidates listed instead, different types of credit risk definitions – ex. Spread risk - or models – ex. Credit scoring, actuarial model, credit migration, KMV - which was not the purpose of the question.
6. Continued

The three components of credit risk are:

- Default risk: The probability that a counterparty will default on its obligations, which can take on many definitions depending on legal context of a situation.

- Exposure-at-default risk: When in default, this component measures the exposure to a counterparty. If the counterparty is a traded security, this can be measured by the market value, actual or future – mark-to-market/mark-to-model – of that security/investment/asset, whether real or derivative.

- Recovery risk: After default, the potential and variable rates of recovery of a claim against a counterparty, an asset, an investment or, in the context of a derivative instrument, the replacement value of that security. This is based on the legal context of a situation.

(b) Your manager states that because VaR and CTE are used extensively in ABC’s risk management activities, these are appropriate risk measures for credit risk analysis.

Critique your manager’s statement.

Commentary on question (b):
The purpose of this question was to test the understanding of both VAR and CTE, their usefulness and limitations, as valid risk measures in the specific context of credit risk. While used extensively and historically in the context of market risk activities, mostly for traded activities, many claim that these can also be used in the context of credit risk.

Thus, in order to receive maximum points, a candidate had to outline valid arguments in favor and against the use of these two potential measures for credit risk analysis. Some candidates simply listed and compared the underlying mathematical properties of each measure – ex. subadditivity, coherence, better tail assessment. Although valid per se, this was not the purpose of the question. One had to relate these two measures back to the context of credit risk measurement in order to be considered valid answers.

Although the use of either VAR or CTE could bring coherence to the overall risk reporting process at any company when risks are consolidated – ERM and economic capital evaluation - and although individuals, company employees, board members, regulators, auditors, rating agencies, company analysts are already familiar with these measures, these are not usually appropriate to measure credit risk for the following reasons:
6. Continued

-Time horizon:
VAR/CTE are usually calculated over a short-time horizon – ex. 10-day VAR – using historical data, while credit risk evolves over a longer-time period which, for example, a credit-migration model tries to capture.

-Incomplete models:
Mean-reversion is very often an important assumption of any credit risk assessment, which both CTE and VAR underlying models do not explicitly capture.

While VAR and CTE are focused on the evaluation of a counterparty/asset/investment/derivative – the equivalent of credit exposure – they do not integrate the other two components of credit risk, both the probability of default and the recovery rates, which impact any credit risk assessment.

-Sources of credit information:
Credit information is not always publicly available compared to market risk activities. Ex. An asset/derivative may be publicly traded - although liquidity might affect its credit exposure value – while a bank loan is not unless it is securitized.

While CTE is an improvement as it captures tail events, which credit events are, data is usually not available and if available, it would not be appropriate as credit risk is heavily dependent on the legal context affecting a counterparty – affecting particularly recovery rates - and historical data is usually inappropriate as the economic context will have changed.

(c) The CFO recommends that ABC establish a special purpose vehicle (SPV) into which many of the below investment grade bonds would be transferred. The CFO claims that this arrangement would move these risks off ABC’s balance sheet.

(i) Explain bankruptcy remoteness and off-balance sheet accounting as they relate to the CFO’s proposal.

(ii) Recommend alternatives for managing ABC’s credit risk.
6. Continued

Commentary on question:

(i) In order to receive maximum points, a candidate had to explain these two concepts in the context of a SPV. In general, this question was answered well by most candidates familiar with these two concepts.

(ii) In order to receive maximum points, a candidate had to outline and briefly describe potential alternatives to using a SPV to manage credit risk.

(i) Bankruptcy remoteness describes the reality that if an SPV, that is operated as a distinct legal entity from the sponsor firm files for bankruptcy, the sponsor firm is not liable for the SPV's debts. Thus, the financial risk to the sponsor firm in the event of bankruptcy or default is mitigated. This could allow ABC to minimize losses in the event of default on the riskier bonds of the portfolio, while at the same time retain potential gains.

Off-balance sheet accounting refers to not including the assets and liabilities of the SPV within the sponsoring firm’s balance-sheet. However, this could be used to restrict the flow of information about the risky bond portfolio to regulators, rating agencies, and the public, allowing ABC to appear more financially stable than it may be.

(ii)

-Credit Default Swaps: could be expensive depending on the type of assets held by ABC, and would expose ABC to the credit risk of another party.
- Collateralized debt obligations(CDO): sell credit risk to external investors in tranches.
- Credit Linked Notes: similar as a CDO, but classified as a bond for investment purposes, which allow investors to bypass rules concerning exposure to credit derivatives.
- Change the distribution of assets in the portfolio: this will impact the return of the portfolio, but could also reduce credit risk. However, it could also have an impact on other risks like liquidity.
- Implement other more operational measures: netting of derivative agreements, imposing and monitoring credit limits by counterparty, defining minimum credit ratings for investment purposes – ex. Above AA -, imposing collateral postings.
6. Continued

(d) 

(i) Describe the term “controlled” in the context of SPV accounting under IFRS.

(ii) Explain why the SPV is considered “controlled” under IFRS.

(iii) Propose changes to the SPV so that it will not be considered “controlled” under IFRS.

Commentary on Question d:

In order to receive maximum points to this question, a candidate had to clearly describe what “control” implies in the context of IFRS without referring back to the stem of the question, - question (i) – followed by a clear explanation that linked his/her previous answer to the characteristics of this particular SPV - question (ii) – and finally clearly laying out some proposals – question (iii) – that would modify the fundamental characteristics of this SPV, so that it would no longer be considered “controlled” in the context of IFRS as answered in question (i).

(i) In general, the new IFRS standard classifies any company – a SPVs is a separate legal entity - as “controlled” if the sponsor firm effectively “controls” the entity. It lists specific criteria in order to make this determination. The sponsoring firm must be subject to IFRS rules, it must determine a SPV's strategic goals, exercising a majority control over board decisions, can provide an explicit/implicit guarantee if losses occur, can obtain the majority of revenues net of risks, directly/indirectly benefiting its sponsor and solely functioning for its benefit. Thus, in this case, the assets and liabilities of the “controlled” SPV must be included/consolidated in the sponsoring firm’s balance sheet.

(ii) This particular SPV would more than likely be considered controlled because:
-ABC falls under IFRS jurisdiction.
-This SPV would undertake activities on ABC's behalf and ABC would be the main beneficiary from this.
-ABC would be the sole investor in the SPV: ABC would effectively control the SPV and its board decisions.
-ABC, as the sole investor in the SPV, would retain the majority of risks of the SPV.
-ABC is the sole investor in the SPV, would receive the majority of the benefits – better performance – while diminishing its own risk capital investment as risks would now reside in the SPV.
6. Continued

-Legally speaking, this SPV, being set up as a limited-liability corporation, doesn't imply a full bankruptcy remoteness as envisioned by the CFO and probably would have to be consolidated for IFRS purposes as subsidiaries are.

(iii) In order to reduce the likelihood that the implementation of this IFRS standard might apply in the case of this particular SPV, the following changes might have to be envisioned in order to effectively eliminate the assumed “control” that ABC exercises over the SPV, enhancing bankruptcy remoteness and potentially eliminating the consolidation of assets and liabilities of the SPV with ABC:

- Elect an independent board to govern the SPV where ABC no longer has the majority.
- Have the SPV manage investment portfolios for other companies.
- Open up the capital of the SPV to outside investors, who would then share profits/losses/risks/equity capital resulting from the SPV’s operations.
- Establish internal rules/regulations that govern the SPV: narrowly define its actions, restrict the SPV’s ability to act solely on ABC’s behalf.
- Envision setting up the SPV in a territory outside the jurisdiction of IFRS.

(e) The CFO states that an SPV will address the concerns expressed by the rating agency.

Critique this statement.

**Commentary on Question (e):**

*In order to obtain maximum points for this question, a candidate had to clearly lay out how an outside party, namely a rating agency, might assess the credit impact of this SPV on ABC's ultimate credit rating, outlining arguments that relate to the criteria used by rating agencies for this purpose, which are separate but complementary to the ones used by IFRS and tested previously.*

Rating agencies look beyond purely accounting criteria - ex. Standard and Poors' ERM rating criteria - in order to truly assess if this SPV would affect ABC's overall risk profile, which could then affect negatively its creditworthiness and overall credit rating. They take into account the legal environment in which ABC operates, which would affect its “bankruptcy remoteness” determination and its potential impact on its credit rating. They are concerned with some of these aspects:
6. Continued

-Lack of transparency related to the possibility of “hiding risks” from investors and even the rating agency itself.
-Operational risk considerations: the impact of setting up a SPV, which would then make ABC less capable to monitor/track risk. Rating agencies might view the SPV arrangement as purely window dressing, the fundamental management of credit risk was not performed by ABC. Ex. Lax underwriting.
-Legal/SOX compliance: a SPV increases legal risk, because of additional operational complexity, affecting accounting controls and financial reporting processes - ex. SOX compliance – which could impact ABC’s overall credit rating. Ex. fines
-Equity/Capital Risk: ABC will hold a large equity position in the SPV. If the SPV runs into financial problems, ABC runs the risk of losing its equity investment, resulting in a negative impact to ABC's capital position, which is an important aspect to any rating agency’s credit assessment of a company.
-Reputation Risk: even if the SPV were to be considered “not controlled” by ABC according to IFRS and even considered “bankrupt remote” according to local law, ABC’s reputation and ultimate financial value and capital could be affected negatively if the SPV runs into problems/underperforms since ABC might be considered to be the original sponsor of this vehicle. Its overall credit rating would be affected.
-Signaling Effect: poor performance of the SPV may attract additional attention to ABC and thus signal that the sponsoring firm is not doing well either.
-Regulation: lax regulation poses an indirect risk to the originating firm. ABC may be doing things that are “legal”, but not appropriate in the eyes of a credit rating agency. Ex. Enron. It might lead to negative political reactions, which would then force ABC to rescue its SPV, in order to reduce the potential for systemic risk, thus affecting ABC's credit rating and even the rating agency’s own reputation.
7. **Learning Objectives:**

3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

**Learning Outcomes:**

(3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.

(4d) Demonstrate how derivatives, synthetic securities, and financial contracting may be used to reduce risk or to assign it to the party most able to bear it.

(4h) Analyze funding and portfolio management strategies to control equity and interest rate risk, including key rate risks. Contrast the various risk measures and be able to apply these risk measures to various entities. Explain the concepts of immunization including modern refinements and practical limitations.

(4k) Apply best practices in risk measurement, modeling and management of various financial and non-financial risks faced by an entity.

**Sources:**

ERM-111-12: Key Rate Durations: Measures of Interest Rate Risks

ERM-304-12: Mind the Gap Using Derivative Overlays to Hedge Pension Duration

**Commentary on Question:**

The goal of this question was to test candidates’ understanding of surplus interest rate risk exposure and possible approaches to managing this exposure including hedging. Candidates were expected to apply their knowledge of key rate duration analysis to identify exposures to non-parallel yield curve shifts. Candidates generally did relatively well on the calculations, but many struggled to apply their knowledge and results to develop a risk management strategy.
7. Continued

Solution:
(a) Calculate the overall duration and key rate durations of the asset portfolio.

**Commentary on Question:**
Most candidates received full credit on this portion. Full credit was given for reasonable rounding differences from answers below.

Overall Asset Duration = \( \frac{(461600\times0 + 302100\times17.46 + 10900\times3.63)}{461600 + 302100 + 10900} = \frac{5274666 + 395670}{774600} = \frac{5314233}{774600} = 6.86 \)

KRD Short = \( \frac{(461600 \times 0 + 302100 \times 0.14 + 10900 \times 0.44)}{461600 + 302100 + 10900} = \frac{42294 + 4796}{774600} = \frac{47090}{774600} = 0.0608 \)

KRD Mid = \( \frac{(461600 \times 0 + 302100 \times 6.47 + 10900 \times 2.05)}{774600} = \frac{1954587 + 22345}{774600} = 2.55 \)

KRD Long = \( \frac{(461600 \times 0 + 302100 \times 10.85 + 10900 \times 1.14)}{774600} = \frac{3277785 + 12426}{774600} = 4.248 \)

(b) Calculate the dollar durations for each key rate and the overall dollar-duration exposure of the DB Plan surplus. Show your work.

**Commentary on Question:**
To receive full credit, candidates needed to calculate the correct dollar durations in magnitude and direction. That is, they needed to either show the values as negative numbers or indicate that an increase in rates would result in a decrease to surplus and vice-versa. Values were accepted as $KR D's or $KR D100's. Many candidates lost credit for not indicating the correct direction of the impact. Some candidates misunderstood the question and gave dollar durations for the asset portfolio rather than the surplus.

Surplus is total assets less total liabilities. The total asset KRD's are derived in part (a). Dollar durations are MV * duration * 1% shift.

Surplus dollar duration is the difference between asset and liability dollar duration.

Short $KR D = ($774,600 * 0.06) - ($736,325 * 0.07) = $46,476 - $41,970.5 = -$373,229.25 or -$3,732 per 1% shift ($KR D100)

Mid $KR D = ($774,600 * 2.55) - ($736,325 * 4.41) = $1,975,230 - $3,247,193.25 = -$1,271,963.25 or -$12,720 per 1% shift
7. Continued

Long $KRD = ($774,600 * 4.25) - ($736,325 * 5.02) = $3,292,050 - $3,696,351.5
= -$404,301.5 or -$4043 per 1% shift

Overall $KRD = ($774,600 * 6.86) - ($736,325 * 10) = $5,313,756 - $7,363,250 =
= -$2,049,494 or -$20,495 per 1% shift (also the sum of Short/Mid/Long $KRD)

(c) The pension plan decides to enter into an interest rate swap to hedge against the
duration mismatch. Consider a 30-year interest rate swap with a duration of 25.

(i) Determine whether the Plan should be the receive-fixed or the receive-
floating party and state why.

(ii) Calculate the notional principal of the swap that would hedge the Plan’s
interest rate risk. Show your work.

Commentary on Question:
To receive full credit on part (i), candidates needed to identify which party the
Plan should be in the swap and explain why. Partial credit was given for a
correct answer with no explanation. The explanation below is more thorough than
was required for full credit.

Most students did well on part (ii). A common mistake was to target matching
asset and liability durations rather than dollar durations. Reasonable rounding
differences were accepted for either of the calculation methods shown below.

(i) The Plan should be the receive fixed party in the swap.

From (b) we see that the Plan’s overall surplus dollar duration is negative,
so the Plan is short dollar duration. Duration on an interest rate swap
comes from the fixed leg, so to increase the asset duration, the Plan needs
to receive fixed. The pay-floating leg has very low duration.

(ii) The overall surplus dollar duration from (b) is -$2,049,494.
The notional required to fully hedge = ($2,049,494/25) = $81,980

Alternatively, target asset duration = (Liability MV * Liability
Duration)/Asset MV = ($736,325 * 10)/$774,600 = 9.50587
Notional required = Asset MV * (Target duration – Asset duration)/Swap
Duration = $774,600 * (9.50587 – 6.86)/25 = $81,980
7. Continued

(d) 

(i) Determine the impact on the Plan’s surplus. Show your work.

(ii) Explain whether this hedge meets the Plan’s objective of minimizing interest rate risk.

**Commentary on Question:**

The key observation expected of candidates here was that the Plan is exposed to non-parallel shifts in the yield curve because the key rate durations are not well-matched between assets and liabilities.

Due to calculation errors in previous parts of the question, some candidates got an incorrect numerical answer that indicated that the swap had reduced the adverse impact of this particular change in rates. Credit was not taken off again for calculation errors on prior parts, but candidates were still expected to observe that the Plan would not be immunized against non-parallel rate changes in general.

(i) The change in value is -1 * surplus $KR D * yield curve shift in %
OR -1 * surplus $KR D100 * (yield curve shift in %/1%)

Before the IR swap:
Short $KR D = -1 * -$373,229 * 0.0010 = +$373.23
Mid $KR D = -1 * -$1,271,963 * -0.0070 = -$8903.74
Long $KR D = -1 * -$404,302 * 0.0050 = +$2021.51
Total surplus change before the IR swap = -$6509.00

IR Swap impact:
Short $KR D = -1 * -$81,980 * 3.50 * 0.0010 = -$286.93
Mid $KR D = -1 * -$81,980 * 5.60 * -0.0070 = +$3,213.62
Long $KR D = -1 * -$81,980 * 15.90 * 0.0050 = -$6,517.41
Total IR swap impact = -$3591

Total change in surplus with IR swap = -$6509 + -$3591 = -$10,100

(ii) This hedge does not meet the objective of minimizing interest rate risk.

The swap hedge actually increased the loss. The Plan is protected against parallel shifts in the yield curve but is still exposed to non-parallel shifts using this approach since the dollar duration is matched in total but not for the key rates.
7. Continued

(e) Recommend and justify an approach to more effectively mitigate the interest rate risk of the DB Plan.

**Commentary on Question:**
Many candidates skipped this section. Those who articulated a recommendation with clear justification mostly received full credit. Several potential recommendations are listed below, but candidates only needed to provide one to receive full credit.

Possible recommendations:
A basket of swaps of different tenors to offset the dollar duration mismatch for each of the key rate durations.

A custom hedge with a bank or other counterparty designed to offset the dollar duration mismatch for each of the key rates.

Adjusting the Plan’s asset portfolio to better match the liability IR sensitivity for each of the key rates.

Justification: The recommended approach would better match the key rate dollar durations. When key rate durations are matched, the Plan is protected against changes in the shape of the yield curve.
8. Learning Objectives:

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.

3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

(1a) Explain risk concepts and be able to apply risk definitions to different entities.

(3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.

(4a) Demonstrate and analyze applicability of risk optimization techniques and the impact of an ERM strategy on an organization’s value. Analyze the risk and return trade-offs that result from changes in the organization’s risk profile.

(4c) Demonstrate means for reducing risk without transferring it.

(4h) Analyze funding and portfolio management strategies to control equity and interest rate risk, including key rate risks. Contrast the various risk measures and be able to apply these risk measures to various entities. Explain the concepts of immunization including modern refinements and practical limitations.

(4i) Analyze the application of Asset Liability Management and Liability Driven Investment principles to Investment Policy and Asset Allocation.

(4k) Apply best practices in risk measurement, modeling and management of various financial and non-financial risks faced by an entity.

Sources:
ERM-605-12: Modern Investment Management, Litterman, Chapter 10, Strategic Asset Allocation in the Presence of Uncertain Liabilities

Commentary on Question:
This question required candidates to demonstrate knowledge of how to use strategic asset allocation analysis to assess the main drivers of surplus volatility risk in DB Plans and to recommend LDI strategies to control surplus risk appropriate to the Plan’s funding status. Many candidates were able to reach the correct conclusions, but this often seemed to be by intuition. To receive full credit, candidates needed to demonstrate their understanding by providing clear explanation/justification for their conclusions.
8. Continued

Solution:
(a) 
(i) Classify each hypothetical Plan A, B, and C as exactly funded, underfunded, or overfunded. Show your work.

(ii) Identify and explain which of the three hypothetical Plans most resembles SLIC’s DB Plan.

Commentary on Question:
To receive full credit for (i), candidates needed to solve for the funding ratio for each Plan or otherwise show how they got to their classifications. Many candidates recognized that Plans with higher funding ratios should have higher equity allocations and correctly guessed the classifications without showing significant work.

On part (ii), most candidates correctly explained that SLIC’s Plan most resembled the underfunded Plan identified in part (i). Full credit was given for that response even if the candidate had mis-identified the underfunded Plan in part (i).

(i) We know the equity allocation to minimize surplus volatility is:

\[ \alpha = \frac{(1 - \beta \frac{L_t}{A_t})(\sigma_B^2 - \rho \sigma_B \sigma_E)}{\sigma_E^2 + \sigma_B^2 - 2 \rho \sigma_B \sigma_E} \]

To identify each Plan’s funding status, we need to solve for L/A. For all Plans, \( \beta \) is the leverage factor of DB liability index duration to bond index duration, or \( 10/9 = 1.11 \)

For all Plans, we get:

\[ \frac{(\sigma_B^2 - \rho \sigma_B \sigma_E)}{\sigma_E^2 + \sigma_B^2 - 2 \rho \sigma_B \sigma_E} = 0.12069 \]

Re-arranging terms, we get:

\[ \frac{L_t}{A_t} = 1 - \frac{\alpha}{0.12069} \]

So for Plan A, \( \frac{L_t}{A_t} = 66.75\% \) \( \rightarrow \) Plan A is overfunded

For Plan B, \( \frac{L_t}{A_t} = 100\% \) \( \rightarrow \) Plan B is exactly funded

For Plan C, \( \frac{L_t}{A_t} = 117.75\% \) \( \rightarrow \) Plan C is underfunded

(ii) SLIC’s Plan has a funding ratio < 1, so SLIC’s Plan is underfunded and most resembles Plan C.
8.  Continued

(b)  

(i)  Explain the 1-year static RACS measure to the Committee.

(ii) Explain the reasons leading to the relationship between RACS and the equity allocations for Plan A and Plan C.

Commentary on Question:
Candidates did relatively well on part (i). Many candidates described the measure or just gave the formula, but to receive full credit, candidates needed to explain why it was important or how it might be used.

On part (ii), many candidates described the shape of the graph, but to receive substantial credit, candidates needed to explain why an over/under-funded Plan would produce the graph provided. Candidates struggled with explaining this clearly and thoroughly.

(i) The RACS is the expected surplus return in excess of the risk-free return divided by the volatility of surplus. It is a measure of the risk-return tradeoff similar to the Sharpe Ratio except it considers both assets and liabilities. A higher value corresponds to a more favorable risk-return profile.

(ii) Since Plan A is overfunded, increasing the allocation to equities introduces unnecessary risk and reduces RACS.

Since Plan C is underfunded, increasing the allocation to equities increases the likelihood that the Plan can become fully funded and as a result increases RACS.

(c) Explain the reasons leading to the relationship between the change in RACS in moving to Global Equity and the equity allocation levels for Plan A and Plan C.

Commentary on Question:
Candidates struggled with this. Similar to part (b)(ii), many described the shape of the graph but did not explain the underlying drivers.

For Plan A, at lower equity allocations, liability duration can be matched with bonds and increasing the allocation to global equity increases RACS because global equity has lower volatility. At higher equity allocations, the benefit diminishes because the liability duration is not matched with bonds and global equity is less correlated with the liabilities relative to domestic equity.
8. Continued

For Plan C, there are not enough assets to match the liability duration with bonds, so RACS decreases at any equity allocation since global equity is less correlated with the liabilities.

(d) Conclude whether or not SLIC should modify its asset allocation strategy with respect to each of the two proposed changes in investment strategy that you have considered, using your RACS analysis. Justify your response.

Commentary on Question:
The question asked for a recommendation and justification for each of the two proposed changes. Most candidates who correctly identified SLIC’s Plan as most resembling Plan C were able to interpret the graphs to give the correct conclusions. However, to receive full credit, candidates needed to provide justification. Those who struggled with parts (b) and (c) generally could not give a sufficient justification here and lost substantial credit on part (d).

1. A significant reduction in overall allocation to equity is not desirable for SLIC. SLIC is underfunded and needs to increase equity allocation as shown by the line for Plan C in the graph from part (b).

2. A significant increase in the sub-allocation to global equity within the equity allocation is not desirable for SLIC. As shown by the line for Plan C in the graph from part (c), underfunded plans do not benefit from allocating to global equity.