INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 40 points.

   This exam consists of 8 questions, numbered 1 through 8.

   The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam GIADV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d’examen pour la version française.
1. (4 points) You are exposure rating a workers compensation treaty that covers the layer 750,000 excess of 250,000 and are given:

- 60% of Standard Premium is in State X and 40% is in State Y.
- The expected loss ratio is 60% in State X and 70% in State Y.
- 50% of Standard Premium in each state is in Hazard Group A and 50% is in Hazard Group B.
- Both states have the following excess loss factors:

<table>
<thead>
<tr>
<th>Loss Size</th>
<th>Hazard Group A</th>
<th>Hazard Group B</th>
</tr>
</thead>
<tbody>
<tr>
<td>250,000</td>
<td>0.0500</td>
<td>0.1000</td>
</tr>
<tr>
<td>500,000</td>
<td>0.0250</td>
<td>0.0500</td>
</tr>
<tr>
<td>750,000</td>
<td>0.0167</td>
<td>0.0333</td>
</tr>
<tr>
<td>1,000,000</td>
<td>0.0125</td>
<td>0.0250</td>
</tr>
</tbody>
</table>

(a) (2 points) Calculate the loss cost rate for the treaty.

You are experience rating the same treaty.

(b) (1 point) Explain how you would handle:

(i) Policy limits

(ii) Discounting

(c) (1 point) Identify an adjustment (unrelated to policy limits or discounting) you would need to make to:

(i) The historical premium

(ii) The historical losses
2. (5 points) You are calculating a risk margin for claim liabilities using the methodology set out in “A Framework for Assessing Risk Margins.”

You have derived the coefficients of variation (CoV) for independent risk, internal systemic risk, and each source of external systemic risk. Your actuarial student suggests using quantitative methods based on historical data to estimate the correlations among these sources of risk.

(a) (1.5 points) Describe three problems with this approach.

The following information is provided:

<table>
<thead>
<tr>
<th>Line of Business</th>
<th>Proportion of Insurance Liabilities</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Outstanding Claim Liabilities</td>
<td>Premium Liabilities</td>
<td></td>
</tr>
<tr>
<td>Motor</td>
<td>15%</td>
<td>35%</td>
<td></td>
</tr>
<tr>
<td>Home</td>
<td>10%</td>
<td>40%</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>25%</td>
<td>75%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Line of Business</th>
<th>Internal Systemic Risk</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Outstanding Claim Liabilities CoV</td>
<td>Premium Liabilities CoV</td>
<td></td>
</tr>
<tr>
<td>Motor</td>
<td>8.5%</td>
<td>6.0%</td>
<td></td>
</tr>
</tbody>
</table>

You assume there is a high correlation between outstanding claim liabilities and premium liabilities within each line of business.

(b) (0.5 points) Propose a numerical value for the correlation between outstanding claim liabilities and premium liabilities within the motor line of business.

(c) (1.5 points) Calculate the internal systemic risk coefficient of variation for the motor line of business.

You have finished your preliminary risk margin analysis of the portfolio and want to internally benchmark your coefficients of variation for independent risk between lines of business. One dimension to consider is the length of claim run-off.

(d) (1.5 points) Describe two implications of differing lengths of claim run-off when performing internal benchmarking of independent risk.
3. (5 points) You model loss sizes using a uniform distribution from 0 to 10.

(a) (1 point) Demonstrate using calculus that the expected loss in the layer 2 to 10 is 3.2.

(b) (1 point) Demonstrate using a graph that the expected loss in the layer 2 to 10 is 3.2.

Over a period of years, loss sizes increase uniformly by 50%.

(c) (1.5 points) Demonstrate using either calculus or a graph that the expected loss in the layer 2 to 10 also increases by 50%.

(d) (1 point) Explain why the leveraged effect of inflation does not cause an increase greater than 50%.

The basic limit is 2.

(e) (0.5 points) Explain, using words, not calculations, why the increased limit factor at 10 is greater than, less than, or the same as it was before the 50% increase in loss sizes.
4. (7 points) You are interested in determining the variability of unpaid claim estimates. The triangle of paid claims data you are working with, by accident year (AY) and development year, is presented below. The shaded cells have been completed using the standard chain ladder method. It is assumed that all claims are fully developed after seven years.

Mack’s method of estimating reserve variability has been applied to this triangle. The key results are provided in the table.

<table>
<thead>
<tr>
<th>Development Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>AY</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>$f_k$</td>
</tr>
<tr>
<td>$\alpha_k^2$</td>
</tr>
</tbody>
</table>

(a) (1.5 points) Demonstrate that the value of $\alpha_k^2$ was correctly calculated. (Your calculation need not match to all four decimal places.)

(b) (1.5 points) Demonstrate that the standard error for accident year 4 was correctly calculated.

Using the normal approximation, a 95% confidence interval for the accident year 6 ultimate claims is $29,187 \pm 1.96(3,782)$ or $(21,774, 36,600)$.

(c) (0.5 points) Explain, referring to this example, why using the normal approximation may not be reasonable.

(d) (1 point) Recommend an approach that may be superior to using the normal approximation. Justify your recommendation.

One of Mack’s assumptions is $E(C_{i,k+1} | C_i, \ldots, C_{ik}) = C_{ik} f_k$. Mack observes that this is consistent with a regression model with a slope of $f_k$ and an intercept of 0. Mack states that a weighted regression should be used to estimate the slope.

(e) (1 point) Explain why it is necessary to perform a weighted regression.
4. Continued

Venter proposes several approaches for comparing or evaluating different models. One proposal is to calculate the sum of squared errors for each model and then adjust them to reflect the number of observations and the number of estimated parameters.

(f) (1.5 points) Describe two other approaches that Venter proposes for comparing or evaluating different models.
5. (5 points) You are given the following triangle of cumulative paid losses:

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Months of Development</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>2013</td>
<td>4,000</td>
</tr>
<tr>
<td>2014</td>
<td>5,000</td>
</tr>
<tr>
<td>2015</td>
<td>6,000</td>
</tr>
</tbody>
</table>

On level premium for each year is 12,000.

You apply Clark’s stochastic reserving model using the Cape Cod method and an exponential distribution with cumulative distribution function $G(x) = 1 - e^{-\frac{x}{\theta}}$.

(a) (0.5 points) State two of the key assumptions of Clark’s method.

The maximum likelihood estimates of $ELR$ and $\theta$ are 71.15% and 7.293, respectively.

(b) (1 point) Estimate ultimate losses for accident year 2015.

The estimate of $\sigma^2$ is 273.

(c) (0.5 points) Estimate the process standard deviation of the accident year 2015 reserve.

The estimated parameter standard deviation of the accident year 2015 reserve is 852.

(d) (0.5 points) Estimate the total coefficient of variation of the accident year 2015 reserve.

(e) (1 point) Estimate the expected payments in 2016 for accident year 2015.

The estimated parameter standard deviation of the payments in 2016 for accident year 2015 is 512.

(f) (1 point) Estimate the total standard deviation of payments in 2016 for accident year 2015.

Actual payments in 2016 for accident year 2015 were 3,800.

(g) (0.5 points) Indicate whether this suggests that the reserving model is invalid. Justify your answer.
6. (5 points) Your company is renewing two accounts, X and Y, each of which is exposed
to two possible independent claim events, 1 and 2. You are given the following
information:

<table>
<thead>
<tr>
<th>Event (i)</th>
<th>Loss for Account (L)</th>
<th>Variance = $L^2 p (1 – p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>$i$</td>
<td>$p(i)$</td>
<td>X</td>
</tr>
<tr>
<td>1</td>
<td>2%</td>
<td>30,000</td>
</tr>
<tr>
<td>2</td>
<td>1%</td>
<td>15,000</td>
</tr>
</tbody>
</table>

- $p(i)$ represents the probability of Event $i$.
- The risk load multiplier, $\lambda$, is 0.000025.

(a) (2.5 points) Calculate the risk load for each account using the Shapley method.

You decide to recalculate risk loads using the Covariance Share method, using a
weighting scheme that assigns the shared covariance by event to each account in
proportion to its loss for that event.

(b) (2 points) Calculate the risk load for each account using the Covariance Share
method.

(c) (0.5 points) Evaluate which method is more likely to produce appropriate risk
loads to be used in pricing.
7. (4 points) You are using the Capital Asset Pricing Model to determine the underwriting profit margin to factor into the upcoming year’s premiums for homeowners insurance. The following information is provided:

- Homeowners coverage consists of three distinct payment pattern groups.
  - Group 1, which represents 40% of homeowners premiums, has loss payments expected to occur three months after receipt of premiums.
  - Group 2, which represents 40% of homeowners premiums, has loss payments expected to occur six months after receipt of premiums.
  - Group 3, which represents 20% of homeowners premiums, has loss payments expected to occur nine months after receipt of premiums.
- The liability beta is –0.2.
- The risk-free rate is 1%.
- The market risk premium is 5%.
- The effect of taxation is ignored.

(a) (0.5 points) Calculate the funds generating coefficient estimate, \( k \).

(b) (0.5 points) Calculate the underwriting beta.

(c) (1.5 points) Calculate the underwriting profit margin.

(d) (0.5 points) Indicate whether the underwriting profit margin would be higher, lower, or the same if taxes were not ignored.

(e) (1 point) Provide two criticisms of models that apply the Capital Asset Pricing Model to insurance.
8. (5 points) Property R Us Reinsurance Company has been presented with an opportunity to write a quota share treaty with two different adjustable feature options:

Option 1: Minimum commission of 20% at a 70% loss ratio sliding 0.5:1 to a maximum commission of 30% at a 50% loss ratio

Option 2: Flat commission of 27.5% with the ceding company reassuming 50% of the loss between a 60% and 70% loss ratio

The loss ratio is uniformly distributed in the range from 30% to 80%.

(a) (1.5 points) Calculate the expected technical ratio (loss ratio plus commission ratio) for Option 1.

(b) (1.5 points) Calculate the expected technical ratio for Option 2.

(c) (1 point) Recommend which option Property R Us should choose. Justify your recommendation.

Suppose a surplus share treaty were used instead of a quota share treaty.

(d) (1 point) Explain how the loss ratio distribution on the surplus share treaty would qualitatively differ from the loss ratio distribution on the quota share treaty.

**END OF EXAMINATION**