1. \[ 2.5q_{50} = 2q_{50} + 2p_{50} \cdot 0.5q_{52} \]
   \[ = 0.02 + (0.98) \left( \frac{0.5}{2} \right)(0.04) \]
   \[ = 0.0298 \]
   ANSWER: B

2. \[ 20q_{30} = \frac{S_0(30) - S_0(50)}{S_0(30)} = \frac{\frac{220}{250} - \frac{3}{4}}{\frac{220}{250}} = \frac{\frac{440 - 375}{440}}{\frac{220}{250}} = \frac{65}{440} = \frac{13}{88} = 0.1477 \]
   ANSWER: B

3. The 20-year female survival probability = \( e^{-20\mu} \)
   The 20-year male survival probability = \( e^{-30\mu} \)

   We want 1-year female survival = \( e^{-\mu} \)

   Suppose that there were \( M \) males and \( 3M \) females initially. After 20 years, there are expected to be \( Me^{-30\mu} \) and \( 3Me^{-20\mu} \) survivors, respectively. At that time we have:

   \[ \frac{3Me^{-20\mu}}{Me^{-30\mu}} = \frac{85}{15} \Rightarrow e^{10\mu} = \frac{85}{45} = \frac{17}{9} \Rightarrow e^{-\mu} = \left( \frac{9}{17} \right)^{\frac{10}{10}} = 0.938 \]

   ANSWER: C
4. \[ E[Z] = \int_0^\infty 0.04 e^{-0.04t} \left( e^{0.02t} \right) e^{-0.06t} \, dt \]

\[ = 0.04 \int_0^\infty e^{-0.08t} \, dt = \frac{0.04}{0.08} = \frac{1}{2} \]

\[ E[Z^2] = \int_0^\infty (0.04 e^{-0.04t}) (e^{0.04t}) (e^{-0.12t}) \, dt = \frac{0.04}{0.12} = \frac{1}{3} \]

\[ Var[Z] = \frac{1}{3} - \left( \frac{1}{2} \right)^2 = \frac{1}{12} = 0.0833 \]

ANSWER: E

5. \[ A_1^{[50]:3} = v \left( q_{[50]} + p_{[50]} v \left( q_{[50]+1} + p_{[50]+1} v q_{52} \right) \right) \]

where: \[ v = \frac{1}{1.04} \]

\[ q_{[50]} = 0.7(0.045) = 0.0315 \]

\[ p_{[50]} = 1 - q_{[50]} = 0.9685 \]

\[ q_{[50]+1} = 0.8(0.050) = 0.040 \]

\[ p_{[50]+1} = 1 - q_{[50]+1} = 0.960 \]

\[ q_{52} = 0.055 \]

So: \[ A_1^{[50]:3} = 0.1116 \]

ANSWER: D
The median of \( K_{48} \) is the integer \( m \) for which
\[
P(K_{48} < m) \leq 0.5 \text{ and } P(K_{48} > m) \leq 0.5.
\]
This is equivalent to finding \( m \) for which
\[
\frac{l_{48+m}}{l_{48}} \geq 0.5 \text{ and } \frac{l_{48+m+1}}{l_{48}} \leq 0.5.
\]
Based on the ILT, we have \( m = 30 \) since
\[
l_{78} \geq 4,522,840 \text{ and } l_{79} \leq 4,522,840.
\]
So:
\[
APV = 5000A_{48} + 5000E_{48}A_{78}
= 5000(0.22892) + 5000(0.24193)(0.36044)
= 1422.14
\]
ANSWER: A
7. The minimum premium to prevent lapse will be the premium such that \( AV_5 = 0. \) 

Let \( P \) be this premium.

\[
AV_5 = 0 = \left( AV_4 + 0.95P - 500 - \frac{20,000}{1.045} \right)(1.045) \Rightarrow P = 20,146.06
\]

ANSWER: A

8. \( EPV(\text{NET PREMIUM}) + EPV(\text{EXPENSE LOADING}) = \text{GROSS PREMIUM} \)

\[
\uparrow p^e
\]

So: \( 1,000,000 \frac{A_{50}}{a_{50}} + p^e = 19,526 \)

So: \( p^e = 753.76 \)

ANSWER: C

9. Let \( P \) be the net premium for year 1.

Then:

\[
P \left[ 1 + \frac{1.01}{1.05} \cdot 0.99 \right] = \frac{10^5}{1.05} \left( 0.01 + \frac{0.99}{1.05} (1.01)(0.02) \right) \Rightarrow P = 1416.93
\]

ANSWER: B
10. The policy is fully discrete, so all cash flows occur at the start or end of a year. There is a loss if death occurs in year 1 or year 2, otherwise the policy was profitable.

\[
\Pr(\text{death in year 1 or 2}) = 1 - e^{-2\mu} = 0.113
\]

ANSWER: D

11. \(APV(\text{expenses}) = 0.35G + 8 + 0.15G\dot{a}_{30:31} + 4a_{30:31}\)

\[= 0.20G + 4 + 0.15G\dot{a}_{30:31} + 4\ddot{a}_{30:31} + 200,000\dot{a}_{30:31}\]

\[G = \frac{200,000\dot{a}_{30:31} + 4 + 4\ddot{a}_{30:31}}{0.85\dot{a}_{30:31} - 0.20}\]

\[200,000\dot{a}_{30:31} = 200,000\left[\dot{a}_{30} - 10\dot{E}_{30}\dot{A}_{40}\right]\]

\[= 200(102.48) - (200)(0.54733)(161.32)\]

\[= 2836.94\]

\[G = \frac{2836.94 + 4 + 4(7.7465)}{0.85(4.4516) - 0.20} = \frac{2871.926}{3.58386} = 801.35\]

ANSWER: C

12. \(J^{FPT} = 100,000A_{[55] 3} - 100,000 P_{[55] 3} \cdot \ddot{a}_{[55] 3}\)

\[= 100,000A_{58} - 100,000 \cdot \frac{A_{[55] 1} \cdot \ddot{a}_{58}}{\ddot{a}_{[55] 1}}\]

\[= 100,000 \left( 0.27 - \frac{0.24}{1 - 0.24} \cdot \frac{1 - 0.27}{d} \right)\]

\[= 3947.37\]

ANSWER: B
13. \[ V = (V + P)(1 + i) - (25,000 + V - V)q_x \]
\[ V = (V + P)(1 + i) - (50,000 + V - V)q_{x+1} = 50,000 \]
\[ ((P(1 + i) - 25,000q_x) + P)(1 + i) - 50,000q_{x+1} = 50,000 \]
\[ ((P(1.05) - 25,000(0.15)) + P)(1.05) - 50,000(0.15) = 50,000 \]

Solving for \( P \), we get

\[ P = \frac{61,437.50}{2.1525} = 28,542.39 \]

ANSWER: D

14. \[ AV_2 = (AV_1 + 3000(1 - 0.07) - 10)(1.05) - \frac{3.0}{1000}(50,000 - AV_2) \]
\[ = 5113.211 + 0.003AV_2 \]
\[ \Rightarrow AV_2 = \frac{5113.211}{0.997} = 5128.60 \]

ANSWER: D

15. \[ DPP = \min \{ t : NPV(t) \geq 0 \} \]
\[ NPV(0) = \pi_0 = -550 \]
\[ NPV(1) = \pi_0 + \pi_1v = -550 + \frac{300}{1.12} = -282 \]
\[ NPV(2) = NPV(1) + \pi_2v = -282 + \frac{275}{1.12^2} = -62.91 \]
\[ NPV(3) = NPV(2) + \pi_3v^3 = -62.91 + \frac{75}{1.12^3} = -9.53 \]
\[ NPV(4) = NPV(3) + \pi_4v^4 = -9.53 + \frac{150}{1.12^4} = 85.80 \]
\[ NPV(4) \geq 0 \Rightarrow DPP = 4 \]

ANSWER: D
16. 

\[ IV = \frac{(505 + 220 - 30)(1.05) - 10,000q_{s3}}{1 - q_{s3}} = 666.2807 \]

The profit for policy year 4 is

\[ 4885 \left[ (505 + 220 - 30)(0.01) + (30 - 34)(1.06) + (10,000 - 666.2807)(0.0068 \frac{42}{4885}) \right] \]

\[ = -68,730.37 \]

ANSWER: C

17. 

\[ V^e = \left( \frac{p^e}{G - 187 - 0.25G - 10} \right)(1.03) \]

\[ = -38.7 \]

\[ \Rightarrow 0.75G = \frac{-38.7(0.992)}{1.03} + 187 + 10 = 159.72 \]

\[ \Rightarrow G = 212.97 \]

ANSWER: B

18. At 66, the total retirement fund is 1,500,000(1.08) and is to be used to purchase a quarterly annuity equal to \( X \bar{a}_{66}^{(4)} \) so that:

\[ X = \frac{1,500,000(1.08)}{\frac{3}{8} - \frac{3}{8}} = 1,620,000 \]

\[ \bar{a}_{66} = \frac{9.6362 - \frac{3}{8}}{\frac{3}{8}} = 174,923.30 \]

Replacement Ratio

\[ \frac{X}{250,000} = \frac{174,923.30}{250,000} = 0.70 \]

ANSWER: E
19. No projected salary for traditional unit credit method:

\[ V' = (0.02)(10)(150,000)v^{20}p_{45}\bar{a}_{65}^{(12)} = 127,157.50 \]

\[ V = (0.02)(11)(150,000)v^{19}p_{46}\bar{a}_{65}^{(12)} \]

Let \( C \) = normal contribution

\[ dV + C = vp_{45}V' \Rightarrow C = \frac{1}{10} dV = 12,715.75 \]

ANSWER: A

20. Kaitlyn’s annual retirement benefit is

\[
\frac{50,000}{5} \left( 1.025^{26} + 1.025^{27} + 1.025^{28} + 1.025^{29} + 1.025^{30} \right) \times 31 \times (1 - 0.07(3)) \times (0.02) = 10,000(31)(0.02)(0.79)(9.988563) = 48,923.98
\]

ANSWER: D