1. **Learning Objectives:**
   1. The candidate will understand the key considerations for general insurance actuarial analysis.

**Learning Outcomes:**
(1k) Estimate written, earned and unearned premiums.
(1l) Adjust historical earned premiums to current rate levels.

**Sources:**

**Commentary on Question:**
*This question tests the candidate’s ability to make correct calculations of earned premium. The candidate also needs to understand earned premiums adjusted to current rate level that are used when projecting expected claim ratios, as well as calculating the weighted average rate level when a new discount is introduced.*

**Solution:**
(a) Calculate the policy year 2015 earned premium evaluated as of December 31, 2016.

2015 policy year earned premium = 12×2,000 = 24,000

(b) Calculate the calendar year 2015 earned premium.

**Commentary on Question:**
*There are two approaches that can be used. Candidates could either assume that the policies written in the first half of the year are renewed during the second half of the year, or candidates could answer the question considering only new policy writings during the year.*

Solution for assuming only new policy writings during the year:

- The policy written on January 1 expires on June 30, and is therefore earned for six months in calendar year (CY) 2015.
- Similarly, the policies written on February 1, March 1, April 1, May 1, June 1, and July 1 are all earned for six months in CY 2015.
- The policy written on August 1 is earned for five months in CY 2015.
1. Continued

- The policy written on September 1 is earned for four months in CY 2015.
- The policy written on October 1 is earned for three months in CY 2015.
- The policy written on November 1 is earned for two months in CY 2015.
- The policy written on December 1 is earned for one month in CY 2015.

CY 2015 earned premium = \(2,000 \times (6/6 + 6/6 + 6/6 + 6/6 + 6/6 + 6/6 + 6/6 + 5/6 + 4/6 + 3/6 + 2/6 + 1/6) = 19,000\)

(c) Calculate the premium on-level factors for 2011 and 2012 used to project expected claim ratios for reserving purposes as of December 31, 2015.

**Commentary on Question:**

*Note that the six-month policy terms cause the height of the triangles to be twice the base.*

<table>
<thead>
<tr>
<th>Rate Level</th>
<th>Rate Level Relative Value</th>
<th>2011</th>
<th>2012</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.0000</td>
<td>93.75%</td>
<td>6.25%</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1.0800</td>
<td>6.25%</td>
<td>68.75%</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1.1232</td>
<td>25.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>1.0446</td>
<td></td>
<td>6.25%</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>1.1699</td>
<td></td>
<td>87.50%</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>1.2401</td>
<td></td>
<td>6.25%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Weighted average rate level</td>
<td>1.0050</td>
<td>1.0858</td>
<td>1.1665</td>
</tr>
</tbody>
</table>

\[\text{e.g., } \% \text{ at rate level C in CY 2012} = \frac{1}{2} \times \frac{1}{2} \times 1 = 25\%\]

Weighted average rate level = sum product of rate level relative value and \% at rate level in CY (e.g., 1.0050 = 1.0000 \times 93.75\% + 1.0800 \times 6.25\%).

Premium on-level factors that are used to project expected claim ratios for reserving purposes as of December 31, 2015 need to use the weighted average rate level for 2015 as the numerator.
1. Continued

Premium on-level factor for 2011 = \( \frac{1.1665}{1.0050} = 1.161 \)

Premium on-level factor for 2012 = \( \frac{1.1665}{1.0858} = 1.074 \)

(d) Calculate the weighted average rate level for 2012.

Average discount is 40% \times 10\% = 4\%.

2012 can be represented using the following graph:

<table>
<thead>
<tr>
<th></th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+8%</td>
<td>+4%</td>
<td></td>
</tr>
</tbody>
</table>

The rate level relative value for A is 1.
The rate level relative value for B is 1.08.
The rate level relative value for C is 1.08 \times 0.96 = 1.0368.
The rate level relative value for D is 1.08 \times 1.04 \times 0.96 = 1.0783.

The areas of each shape are as follows:

A \quad \frac{1}{2} \times \frac{1}{2} \times \frac{1}{4} = 6.25\%
B \quad 0.5 - 0.0625 = 43.75\%
C \quad \frac{1}{2} \times \frac{1}{2} \times 1 = 25\%
D \quad \frac{1}{2} \times \frac{1}{2} \times 1 = 25\%

The weighted average rate level for 2012 = (1 \times 0.0625) + (1.08 \times 0.4375) + (1.0368 \times 0.25) + (1.0783 \times 0.25) = 1.0638.
2. **Learning Objectives:**

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

6. The candidate will understand the need for monitoring results.

**Learning Outcomes:**

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

(2c) Estimate claims-related expenses and recoveries.

(3b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

(3d) Evaluate the estimates of ultimate claims to determine claim liabilities for financial reporting.

(6b) Analyze actual claims experience relative to expectations.

**Sources:**


**Commentary on Question:**

*This question tests the candidate’s understanding of comparing actual vs. expected claims, estimating IBNR reserves using the development method, the Bornhuetter Ferguson method, and the Benktander method, and estimating unpaid ULAE using the Mango and Allen smoothing adjustment.*

**Solution:**

(a) Calculate the difference between actual paid claims and expected paid claims for each accident year.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Actual Paid Claims</th>
<th>Earned Premium</th>
<th>A Priori Expected Claims</th>
<th>Paid Claims CDF</th>
<th>Expected Paid Claims</th>
<th>Actual vs. Expected</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>49,000</td>
<td>90,000</td>
<td>58,500</td>
<td>1.20</td>
<td>48,750</td>
<td>250</td>
</tr>
<tr>
<td>2014</td>
<td>40,500</td>
<td>100,000</td>
<td>65,000</td>
<td>1.60</td>
<td>40,625</td>
<td>−125</td>
</tr>
<tr>
<td>2015</td>
<td>40,000</td>
<td>110,000</td>
<td>71,500</td>
<td>2.00</td>
<td>35,750</td>
<td>4,250</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>4,375</strong></td>
</tr>
</tbody>
</table>
2. Continued

(b) Calculate the accident year 2014 expected paid development from December 31, 2015 to March 31, 2016 using linear interpolation and the a priori expected claims ratio.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Paid CDF</th>
<th>Expected % Paid at Dec. 31, 2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>1.20</td>
<td>83.3%</td>
</tr>
<tr>
<td>2014</td>
<td>1.60</td>
<td>62.5%</td>
</tr>
</tbody>
</table>

Expected % paid at Mar. 31, 2016 = $0.75 \times 62.5\% + 0.25 \times 83.3\% = 67.71\%$

Expected paid claims from December 31, 2015 to March 31, 2016 = \[
\frac{(65,000 - 40,500)}{1 - 0.6250} \times (0.6771 - 0.6250) = 3,404
\]

(c) Explain why linear interpolation might not be appropriate for estimating expected development for accident year 2014.

Development between periods may not be linear (particularly if development factors are large or immature).

(d) Provide one alternative to linear interpolation for estimating expected development between annual evaluations.

Commentary on Question:

Other explanations are possible.

If data is available, it may be possible to derive development factors at quarterly intervals rather than using interpolation.
2. Continued

(e) Calculate estimated IBNR reserves for each accident year using the following methods applied to paid claim data:

(i) Development method

(ii) Bornhuetter Ferguson method

(iii) Benktander method, one iteration

\[
(1) \quad (3) = (2) \times 0.65 \\
(4) \quad (7) = \frac{1}{(4)} \\
(8) = 1 - (7) \\
\]

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Actual Paid Claims</th>
<th>A Priori Expected Claims</th>
<th>Paid CDF</th>
<th>1/CDF</th>
<th>1 - 1/CDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>49,000</td>
<td>58,500</td>
<td>1.20</td>
<td>0.8333</td>
<td>0.1667</td>
</tr>
<tr>
<td>2014</td>
<td>40,500</td>
<td>65,000</td>
<td>1.60</td>
<td>0.6250</td>
<td>0.3750</td>
</tr>
<tr>
<td>2015</td>
<td>40,000</td>
<td>71,500</td>
<td>2.00</td>
<td>0.5000</td>
<td>0.5000</td>
</tr>
</tbody>
</table>

\[
(9) = (1)(4) \\
(10) = (1) + (8)(3) \\
(11) = (1) + (8)(10) \\
\]

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Development Method</th>
<th>Bornhuetter Ferguson Method</th>
<th>Benktander Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>58,800</td>
<td>58,752</td>
<td>58,794</td>
</tr>
<tr>
<td>2014</td>
<td>64,800</td>
<td>64,875</td>
<td>64,828</td>
</tr>
<tr>
<td>2015</td>
<td>80,000</td>
<td>75,750</td>
<td>77,875</td>
</tr>
</tbody>
</table>

\[
(12) \\
(13) = (9) - (12) \\
(14) = (10) - (12) \\
(15) = (11) - (12) \\
\]

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Actual Reported Claims</th>
<th>Development Method</th>
<th>Bornhuetter Ferguson Method</th>
<th>Benktander Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>54,000</td>
<td>4,800</td>
<td>4,752</td>
<td>4,794</td>
</tr>
<tr>
<td>2014</td>
<td>50,000</td>
<td>14,800</td>
<td>14,875</td>
<td>14,828</td>
</tr>
<tr>
<td>2015</td>
<td>45,000</td>
<td>35,000</td>
<td>30,750</td>
<td>32,875</td>
</tr>
</tbody>
</table>
2. Continued

(f) Describe one situation for which the development method might provide a better estimate for the accident year 2015 IBNR reserves.

Commentary on Question:
Other explanations are possible.

The development method might be more appropriate if accident year 2015 is showing true deterioration.

(g) Describe one situation for which the Bornhuetter Ferguson method might provide a better estimate for the accident year 2015 IBNR reserves.

Commentary on Question:
Other explanations are possible.

The Bornhuetter Ferguson might be more appropriate if accident year 2015 includes unusual large loss(es) at 12 months which are not expected to develop normally.

(h) Estimate accident year 2015 unpaid ULAE as of December 31, 2015 using the classical paid-to-paid method, a multiplier of 50%, estimated IBNR from the Bornhuetter Ferguson method (part (e) above), and the Mango-Allen smoothing adjustment.

<table>
<thead>
<tr>
<th>Calendar Year</th>
<th>(1) Actual Paid ULAE</th>
<th>(2) Expected Paid Claims</th>
<th>(3) Paid-to-Paid Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>5,580</td>
<td>55,800</td>
<td>10.0%</td>
</tr>
<tr>
<td>2014</td>
<td>5,890</td>
<td>60,100</td>
<td>9.8%</td>
</tr>
<tr>
<td>2015</td>
<td>7,100</td>
<td>69,600</td>
<td>10.2%</td>
</tr>
</tbody>
</table>

Selected paid-to-paid ratio = 10%

Bornhuetter Ferguson method IBNR = 30,750
Case estimate = 45,000 – 40,000 = 5,000
Unpaid ULAE = 10%×30,750 + 10%×0.5×5,000 = 3,325
2. Continued

(i) Identify four situations in which the Mango and Allen smoothing adjustment should be considered in the selection of a ULAE ratio.

**Commentary on Question:**
*Other explanations are possible.*

Situations include:
- Sparse data
- Volatile data
- Long-tail lines of business
- Changing exposure volume
3. Learning Objectives:
4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

Learning Outcomes:
(4b) Describe the influences on frequency and severity of changes in deductibles, changes in policy limits, and changes in mix of business.
(4c) Choose trend rates and calculate trend factors for claims.

Sources:

Commentary on Question:
This question tests the candidate’s understanding of analyzing claim trend for long-tail lines when the company has insufficient experience.

Solution:
(a) Identify one distinct consideration, for each of the following options:
   (i) Use industry general insurance data for the applicable line of business and jurisdiction.
   (ii) Combine your company’s experience in one jurisdiction with your company’s regional experience.
   (iii) Combine your company’s experience with that of other affiliated insurers in your group.
   (i) Carefully review the applicability of external data and review the obligations and guidance set out in the Standards.
   (ii) Review the economic, legal, and regulatory environments that influence frequency and severity.
   (iii) Ensure there are similar operational policies, particularly with respect to underwriting, claim management, and reinsurance OR Ensure there are similarities in the types of exposures and products.

(b) Explain the effect this change will have on a claim trend analysis if there is no adjustment in the historical data for the reform-driven policy change.

If the historical data is not adjusted, it will be higher than it should be (relative to the now lower claims level). This will lead to a trend estimate that is lower than it should be.
3. Continued

(c) Calculate the post-reform losses net of deductible using the claim distribution above.

For the 5 claims of amount 50, reinstating the deductible gives a loss of 50 + 500 = 550. The 20% post-reform decrease takes it to 550(0.8) = 440. Reapplying the deductible gives a claim of 0.
For the 10 claims of 500, reinstating the deductible gives a loss of 500 + 500 = 1,000. The 20% post-reform decrease takes it to 1,000(0.8) = 800. Reapplying the deductible gives a claim of 800 – 500 = 300. With 10 claims the total loss is 3,000.

(d) Calculate the accident year 2015 pure premium trend factor.

The pure premium trend rate is \((1 + 3\%) \times (1 - 1.5\%) = 1.01455\). The trending period is 7/1/2015 to 1/1/2018, which is 2.5 years. See the diagram below for a derivation. Thus the AY2015 pure premium trend factor is \(1.01455^{2.5} = 1.0368\).
4. **Learning Objectives:**
   2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**

**Commentary on Question:**
*This question tests the candidate’s understanding of the Bornhuetter Ferguson method as well as estimating ultimate salvage using the Bornhuetter Ferguson method.*

**Solution:**
(a) Explain how the Bornhuetter Ferguson method of estimating ultimate claims combines the development method and the expected method.

The observed experience is based on actual experience through the valuation date; the balance of the ultimate value is based on the a priori estimate of the expected method.

(b) Calculate the ultimate salvage for each accident year using the Bornhuetter Ferguson method.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Reported Salvage</th>
<th>Age-to-Age Development Factors</th>
<th>Ultimate Claims</th>
<th>Age-to-Ultimate Development Factors</th>
<th>Projected Ultimate Based on Reported</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>62,000</td>
<td>1.00</td>
<td>230,100</td>
<td>1.00</td>
<td>62,000</td>
</tr>
<tr>
<td>2013</td>
<td>66,000</td>
<td>0.99</td>
<td>229,400</td>
<td>0.990</td>
<td>65,340</td>
</tr>
<tr>
<td>2014</td>
<td>65,000</td>
<td>0.98</td>
<td>232,700</td>
<td>0.970</td>
<td>63,050</td>
</tr>
<tr>
<td>2015</td>
<td>67,000</td>
<td>0.95</td>
<td>239,200</td>
<td>0.922</td>
<td>61,774</td>
</tr>
</tbody>
</table>

Note: Column (4) = Cumulative product of column (2).
\[e.g., \ 1.00 \times 0.99 \times 0.98 \times 0.95 = 0.922\]
4. Continued

\[ (6) = 27\% \times (3) \]
\[ (7) = 1 - 1/(4) \]
\[ (8) = (6)(7) \]
\[ (9) = (1)+(8) \]

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Expected Salvage</th>
<th>Expected % Undeveloped</th>
<th>Expected Salvage Undeveloped</th>
<th>Projected Ultimate Salvage</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>62,127</td>
<td>0.0%</td>
<td>0</td>
<td>62,000</td>
</tr>
<tr>
<td>2013</td>
<td>61,938</td>
<td>-1.0%</td>
<td>-619</td>
<td>65,381</td>
</tr>
<tr>
<td>2014</td>
<td>62,829</td>
<td>-3.1%</td>
<td>-1,948</td>
<td>63,052</td>
</tr>
<tr>
<td>2015</td>
<td>64,584</td>
<td>-8.5%</td>
<td>-5,490</td>
<td>61,510</td>
</tr>
</tbody>
</table>

(c) Compare the actual reported salvage to the expected reported salvage for each accident year.

\[ (10) = 1 - (7) \]
\[ (11) = (6)(10) \]
\[ (12) = (1) - (11) \]
\[ (13) = (12)/(11) \]

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Expected % Developed</th>
<th>Expected Salvage Developed</th>
<th>Difference Actual and Expected</th>
<th>Difference Actual and Expected as a % of Expected</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>100.0%</td>
<td>62,127</td>
<td>-127</td>
<td>-0.2%</td>
</tr>
<tr>
<td>2013</td>
<td>101.0%</td>
<td>62,557</td>
<td>3,443</td>
<td>5.5%</td>
</tr>
<tr>
<td>2014</td>
<td>103.1%</td>
<td>64,777</td>
<td>223</td>
<td>0.3%</td>
</tr>
<tr>
<td>2015</td>
<td>108.5%</td>
<td>70,074</td>
<td>-3,074</td>
<td>-4.4%</td>
</tr>
</tbody>
</table>

(d) Explain whether or not any accident years from part (c) merit further investigation.

Accident year 2013 has much higher actual reported salvage than expected and should be investigated.
Accident year 2015 has much lower actual reported salvage than expected and should be investigated.
5. **Learning Objectives:**
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**
(2a) Use loss development triangles for investigative testing.
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**

**Commentary on Question:**
*This question tests the candidate’s ability to estimate ultimate claims using Berquist-Sherman adjustments when there has been an adjustment in case reserves and a change in settlement rates.*

**Solution:**
(a) Calculate the average case estimate triangle.

\[
\text{Average Case} = \frac{\text{Case}}{\text{Outstanding Counts}}
\]

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>93.2</td>
<td>148.8</td>
<td>100.0</td>
</tr>
<tr>
<td>2014</td>
<td>97.8</td>
<td>373.9</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>400.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{Average Case} = \frac{\text{Reported Claims} - \text{Paid Claims}}{\text{Reported Counts} - \text{Closed Counts}} = \frac{12,800 - 8,700}{200 - 156} = 93.2
\]

(b) Explain whether the average case estimate triangle indicates decreasing, increasing or stable case reserve adequacy.

Average case estimates are increasing in the last calendar year (diagonal). This suggests an increase in case reserve adequacy.

(c) Select ultimate counts for each accident year and justify your selection.

Reported count ultimate needs to be selected, since paid ultimate is distorted by settlement pattern changes.
5. Continued

(d) Calculate the disposal ratio triangle using the selections from part (c).

Disposal ratios = Closed Counts / Ultimate Counts

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>0.520</td>
<td>0.697</td>
<td>0.933</td>
</tr>
<tr>
<td>2014</td>
<td>0.519</td>
<td>0.758</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>0.613</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

e.g., 0.520 = 156 / 300

(e) Explain whether the disposal ratio triangle indicates decreasing, increasing or stable claim settlement rates.

Disposal ratios are higher in last calendar year (diagonal), suggesting increasing settlement rates, consistent with claims manager report of faster settlement of small claims.

(f) Calculate the adjusted paid claims triangle.

Selected Disposal

<table>
<thead>
<tr>
<th>Selected Disposal</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.6132</td>
<td>0.7581</td>
<td>0.9333</td>
</tr>
</tbody>
</table>

Adjusted Closed Counts = Disposal Ratio × Ultimate Reported

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>184</td>
<td>227</td>
<td>280</td>
</tr>
<tr>
<td>2014</td>
<td>190</td>
<td>235</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>195</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Paid to Closed Ratio

| AY13-15 | 55 | 50 | 68 |

Adjusted Paid = Adjusted Closed Counts × Paid to Closed Ratio

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>10,120</td>
<td>11,350</td>
<td>19,040</td>
</tr>
<tr>
<td>2014</td>
<td>10,450</td>
<td>11,750</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>10,725</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Continued

(g) Calculate the adjusted reported claims triangle.

Adjusted Average Case = Selected Case (diagonal) detrended at 3%

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>377.0</td>
<td>363.0</td>
<td>100.0</td>
</tr>
<tr>
<td>2014</td>
<td>388.3</td>
<td>373.9</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>400.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

e.g., 388.3 = 400 / 1.03

Adjusted Open Counts = Reported Counts – Adjusted Closed

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>16</td>
<td>23</td>
<td>20</td>
</tr>
<tr>
<td>2014</td>
<td>16</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>17</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Adjusted Case = Adjusted Average Case × Adjusted Open Counts

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>6,032</td>
<td>8,349</td>
<td>2,000</td>
</tr>
<tr>
<td>2014</td>
<td>6,213</td>
<td>8,600</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>6,800</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Adjusted Reported = Adjusted Paid + Adjusted Case

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>16,152</td>
<td>19,699</td>
<td>21,040</td>
</tr>
<tr>
<td>2014</td>
<td>16,663</td>
<td>20,350</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>17,525</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6. **Learning Objectives:**

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**

(5d) Calculate loadings for catastrophes and large claims.

**Sources:**


**Commentary on Question:**

*This question tests the candidate’s understanding of claim loadings for ratemaking.*

**Solution:**

(a) Describe four considerations for your assessment.

Any four of the following are acceptable:

- The actuary should consider comparing historical insurance data to noninsurance data to determine the extent to which the available historical insurance data are fully representative of the long-term frequency and severity of the perils.
- The actuary should consider the sensitivity of the provision to changes in the historical insurance data relating to the following: (1) the frequency of catastrophes; (2) the severity of catastrophes; and (3) the geographic location of catastrophes.
- The actuary should consider the applicability of historical insurance data for the insured coverage. This includes determining:
  - whether catastrophe losses are likely to differ significantly among elements of the rate structure, such as construction type and location;
  - whether such differences should be reflected in the ratemaking procedures; and
  - how to reflect such differences, taking into account both homogeneity and the volume of data.
- The actuary should consider whether there is a sufficient number of years of comparable, compatible historical insurance data.
6.  Continued

(b) Calculate the hail catastrophe loading as a claim ratio for annual policies starting on April 1, 2016.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Earned House Years</th>
<th>Trended Hail Ultimate Claims (000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>13,929</td>
<td>0</td>
</tr>
<tr>
<td>2011</td>
<td>14,070</td>
<td>0</td>
</tr>
<tr>
<td>2012</td>
<td>14,212</td>
<td>234</td>
</tr>
<tr>
<td>2013</td>
<td>14,356</td>
<td>0</td>
</tr>
<tr>
<td>2014</td>
<td>14,169</td>
<td>358</td>
</tr>
<tr>
<td>Total</td>
<td>70,736</td>
<td>592</td>
</tr>
</tbody>
</table>

(3) Trended Pure Premium for Hail Claims: \( (2) \times 1000 / (1) = 8.37 \)

(4) CY2014 Earned House Years: 14,169

(5) Hail Expected Claims: \( (3)(4) = 118,595 \)

(6) CY2014 Trended Earned Premiums at Current Level: 11,291,000

(7) Catastrophe Claim Hail Loading Expressed as a Claim Ratio: \( (5)/(6) = 1.05\% \)

(c) Describe two concerns you would have in relying on the calculation from part (b) in your rating analysis.

Any two of the following are acceptable:

- Frequency in the area not used
- Severity in the area not used
- No consideration of frequency trending
- No consideration of the number of years used
- No credibility
- No formal cat modeling used
- Change in exposure
6. Continued

(d) Recommend one improvement to address each concern identified in part (c).

Commentary on Question:
The improvement must match the concern from part (c).

- Frequency in the area not used → compare frequency with a larger area
- Severity in the area not used → compare severity with the industry data
- No consideration of frequency trending → frequency seems too low to be credible but should check
- No consideration of the number of years used → more potential data from the industry
- No credibility → results should be credibility weighted
- No formal cat modeling used → use cat model
- Change in exposure → use simulation model or logic tree
Learning Objectives:
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:
(5j) Perform individual risk rating using standard plans.

Sources:

Commentary on Question:
*This question tests individual risk rating.*

Solution:
(a) Identify two items of information to request in order to get a broader perspective on the three companies and their historical experience.

Any two of the following are acceptable (other items are possible):
- What was the premium for each year?
- Explain the claim pattern in the experience period.
- Provide individual claim data.

(b) Evaluate each company for retrospective rating, from the perspective of IRIE.

ABC:
- Claims are relatively stable.
- If claims can be controlled at level of 0.6, there could be an opportunity for savings.

DEF:
- Claims are relatively stable.
- If claims continue at 0.8, there could be additional cost.

GHI:
- Claims are not very stable.
- Could there be a claim limitation if a large claim year like 2013 were to reoccur?
7. Continued

(c) Recommend whether each insured should accept the retrospective premium option or a fixed 1.0 million annual premium. Justify your recommendation.

- All-average claim produces premium of 0.99, slightly less than the fixed premium of 1.0.
- ABC: The premium could range from 0.87 to 1.23 with 0.87 more likely, so this is a reasonable candidate. Recommend retrospective premium option.
- DEF: The premium could range from 0.75 to 1.11 with 1.11 more likely, so this is a reasonable candidate. Recommend retrospective premium option.
- GHI: The premium is likely to be 0.15 but the maximum premium of 1.95 would be reached if there were another year like 2013. Do not recommend retrospective premium option.
8. **Learning Objectives:**
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5h) Calculate deductible factors, increased limits factors, and coinsurance penalties.

**Sources:**

**Commentary on Question:**
This question tests the candidate’s understanding of and application of increased limit factors and deductibles.

**Solution:**
(a) Calculate the observed increased limit factors (ILF) for:

(i) 250,000 limit

(ii) 500,000 limit

(iii) 1,000,000 limit

Limited Average Severity (LAS) @ 100,000 = \( \frac{375,000 + 100 \times (2,000+500)}{15,000+2,000+500} \) = 35.71
LAS @ 250,000 = \( \frac{375,000 + 300,000 + (250 \times 500)}{15,000+2,000+500} \) = 45.71
LAS @ 500,000 = LAS @ 1,000,000 = \( \frac{375,000 + 300,000 + 175,000}{15,000+2,000+500} \) = 48.57

ILF 100,000 to 250,000: 45.71/35.71 = 1.28
ILF 100,000 to 500,000: 48.57/35.71 = 1.36
ILF 100,000 to 1,000,000: 48.57/35.71 = 1.36

(b) Explain whether or not your selected ILF at 1,000,000 should equal your selected ILF at 500,000.

Even though there are no claims in the 500,000 to 1,000,000 layer, this does not mean the ILF should be the same.
8. Continued

(c) Describe two challenges insurers face when determining ILFs for high limits using empirical data.

Answers could include any of the below:

- Absence of complete data – insurers may not maintain ground-up uncapped data on individual claims.
- Claim development – an ultimate loss may reach a high limit, but not before it is fully developed.
- Claim trend – empirical claim data must be adjusted for trends, which will influence the amount of claims at higher limits.
- Empirical data at high limits often lacks credibility due to low frequency of large losses.

(d) Calculate the amount of a covered loss retained by the insured and paid by the insurer for the following covered losses:

<table>
<thead>
<tr>
<th>Covered Loss</th>
<th>Retained by the Insured</th>
<th>Paid by the Insurer</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>900</td>
<td>0</td>
</tr>
<tr>
<td>1,400</td>
<td>600</td>
<td>800</td>
</tr>
<tr>
<td>2,000</td>
<td>0</td>
<td>2,000</td>
</tr>
</tbody>
</table>

e.g., for a covered loss of 1,400:
- Paid by insurer = (1,400 – 1,000)×2 = 800
- Paid by insured = 1,400 – 800 = 600

(e) Explain why increasing deductible amounts will reduce claim frequency, but will not necessarily reduce the insurer’s claim severity.

An insurer is not responsible for paying claims below the deductible. As the deductible increases, the insurer will pay fewer claims, which will reduce frequency.

However, because increasing deductible amounts will lower both claim counts and amounts, the insurer’s severity may increase when small claims are no longer covered. This may lead to an increase in the average amount paid per claim, or a higher severity.
9. **Learning Objectives:**
3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

**Learning Outcomes:**
(3e) Describe the components of premium liabilities in the context of financial reporting.

**Sources:**

**Commentary on Question:**
*This question tests the determination of premium liabilities.*

**Solution:**
(a) Calculate the 2016 pure premium per policy, gross and net of reinsurance.

<table>
<thead>
<tr>
<th>(1) Gross Claim Per Policy</th>
<th>Probability</th>
<th>(2) Gross Pure Premium</th>
<th>(3) = (1)(2)</th>
<th>(4) = (1)[1–0.25]</th>
<th>(5) = min[(4),500]</th>
<th>(6) = (2)(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>56%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>30%</td>
<td>30</td>
<td>75</td>
<td>75</td>
<td>22.50</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>10%</td>
<td>50</td>
<td>375</td>
<td>375</td>
<td>37.50</td>
<td></td>
</tr>
<tr>
<td>2,000</td>
<td>4%</td>
<td>80</td>
<td>1,500</td>
<td>500</td>
<td>20.00</td>
<td></td>
</tr>
</tbody>
</table>

(b) Calculate the premium liabilities as of December 31, 2015, gross and net of reinsurance.

| (1) Average claim cost/year [from part (a)] | 160 | 80 |
| (2) Policies exposed for 2015 [50,000×50%]   | 25,000 | 25,000 |
| (3) Expected Claim Cost [(1)(2)]             | 4,000,000 | 2,000,000 |
| (4) ULAE [4,000,000×10%]                     | 400,000 | 400,000 |
| (5) Additional reinsurance cost [50×25,000]   | 1,250,000 |       |
| (6) General expenses [3,750,000×20%×25%]     | 187,500 | 187,500 |
| (7) Premium liabilities [(3)+(4)+(5)+(6)]    | 4,587,500 | 3,837,500 |

Note: (2) Average written date is July 1, 2015, therefore 50% exposed in 2015.
9. **Continued**

(c) Determine either the premium deficiency reserve or the equity in the unearned premium.

Premium deficiency reserve = $3,837,500 - $3,750,000 = $87,500.

(d) State the maximum deferred policy acquisition expense (DPAE) ABC Insurance could record as an asset.

Maximum DPAE = 0 (lower of the gross equity in the unearned premium and the net equity in the unearned premium plus the ceded unearned commissions).
10. **Learning Objectives:**

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**


**Commentary on Question:**

This question tests the estimation of Allocated Loss Adjustment Expenses (ALAE) using the development method and the expected method.

**Solution:**

(a) Identify one practical consideration in your decision.

Either of the following are acceptable:

- The reporting and payment patterns are similar for indemnity and ALAE.
- Differences in the reporting and payments of indemnity and ALAE are consistent from year to year and consistent in the relationship of one to another.

(b) Identify one situation where you should project indemnity and ALAE separately.

Either of the following are acceptable:

- Where the insurer's practices for setting case estimates differ.
- IT systems considerations, i.e., the insurer separates case estimates between indemnity and ALAE.
10. Continued

(c) Calculate the ratio of ALAE to claims for each report year.

<table>
<thead>
<tr>
<th>Report Year</th>
<th>Projected Ultimate ALAE Based on Reported Development</th>
<th>Selected Ultimate Claims</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>720</td>
<td>15,000</td>
<td>0.048</td>
</tr>
<tr>
<td>2013</td>
<td>740</td>
<td>16,100</td>
<td>0.046</td>
</tr>
<tr>
<td>2014</td>
<td>930</td>
<td>16,000</td>
<td>0.058</td>
</tr>
<tr>
<td>2015</td>
<td>820</td>
<td>16,400</td>
<td>0.050</td>
</tr>
<tr>
<td>Total</td>
<td>3,210</td>
<td>63,500</td>
<td>0.051</td>
</tr>
</tbody>
</table>

(d) Select an expected ratio of ALAE to claims. Justify your selection.

- 2014 seems to be an outlier.
- Average of 2012, 2013, 2015 = 4.8%.
- Select 4.8% as a reasonable estimate.

(e) Calculate the projected ultimate ALAE by report year using the expected ratio from part (d).

<table>
<thead>
<tr>
<th>Report Year</th>
<th>Projected Ultimate Claims Based on Reported</th>
<th>Expected ALAE Based on Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>15,000</td>
<td>720</td>
</tr>
<tr>
<td>2013</td>
<td>16,100</td>
<td>773</td>
</tr>
<tr>
<td>2014</td>
<td>16,000</td>
<td>768</td>
</tr>
<tr>
<td>2015</td>
<td>16,400</td>
<td>787</td>
</tr>
<tr>
<td>Total</td>
<td>63,500</td>
<td>3,048</td>
</tr>
</tbody>
</table>
10. Continued

(f) Calculate the indicated ALAE IBNR by report year using the projected ultimate ALAE from part (e).

<table>
<thead>
<tr>
<th>Report Year</th>
<th>Expected ALAE Based on Ratio</th>
<th>Reported ALAE at Dec. 31, 2015</th>
<th>Expected Method Indicated IBNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>720</td>
<td>655</td>
<td>65</td>
</tr>
<tr>
<td>2013</td>
<td>773</td>
<td>630</td>
<td>143</td>
</tr>
<tr>
<td>2014</td>
<td>768</td>
<td>735</td>
<td>33</td>
</tr>
<tr>
<td>2015</td>
<td>787</td>
<td>570</td>
<td>217</td>
</tr>
<tr>
<td>Total</td>
<td>3,048</td>
<td>2,590</td>
<td>458</td>
</tr>
</tbody>
</table>

(g) Select the ALAE IBNR by comparing the indicated ALAE IBNR calculated in part (f) with the indicated ALAE IBNR from the reported development method. Justify your selection.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>720</td>
<td>655</td>
<td>65</td>
<td>65</td>
</tr>
<tr>
<td>2013</td>
<td>740</td>
<td>630</td>
<td>143</td>
<td>110</td>
</tr>
<tr>
<td>2014</td>
<td>930</td>
<td>735</td>
<td>33</td>
<td>195</td>
</tr>
<tr>
<td>2015</td>
<td>820</td>
<td>570</td>
<td>217</td>
<td>250</td>
</tr>
<tr>
<td>Total</td>
<td>3,210</td>
<td>2,590</td>
<td>458</td>
<td>620</td>
</tr>
</tbody>
</table>

Candidates could recommend either of two options (the key is the justification for the selection):

Option 1: IBNR for 2014 is too low using the expected method so select development method IBNR of 195.
10. Continued

Option 2: The IBNR for 2014 using the expected method is reasonable, even if it is a little low. The question says that there is uncertainty in the development method so it is reasonable to use the expected method.

<table>
<thead>
<tr>
<th>Report Year</th>
<th>Selected IBNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>65</td>
</tr>
<tr>
<td>2013</td>
<td>143</td>
</tr>
<tr>
<td>2014</td>
<td>33</td>
</tr>
<tr>
<td>2015</td>
<td>217</td>
</tr>
<tr>
<td>Total</td>
<td>458</td>
</tr>
</tbody>
</table>
11. **Learning Objectives:**

   5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

   (5g) Calculate risk classification changes and territorial changes.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 32.

**Commentary on Question:**
*This question tests the candidate's understanding of risk classification.*

**Solution:**

(a) Describe how a more refined risk classification system might lead to a competitive advantage for a company.

An insurer with more refined pricing can charge a rate that is more closely related to expected claims, attracting customers with a low risk of claims. Conversely, an insurer with less refined pricing will encourage adverse selection; higher risk customers will be attracted to the company.

(b) Calculate age one-way relativities and gender one-way relativities.

Overall average pure premium:
\[
\frac{(90 \times 200 + 100 \times 125 + 80 \times 150 + 110 \times 120)}{(90+100+80+110)} = 146.58
\]

One-way relativities for age:
Young: \[
\frac{(90 \times 200 + 80 \times 150)}{(90+80)}/146.58 = 1.20
\]
Old: \[
\frac{(100 \times 125 + 110 \times 120)}{(100+110)}/146.58 = 0.83
\]

One-way relativities for gender:
Male: \[
\frac{(90 \times 200 + 100 \times 125)}{(90+100)}/146.58 = 1.10
\]
Female: \[
\frac{(80 \times 150 + 110 \times 120)}{(80+110)}/146.58 = 0.90
\]

(c) Calculate indicated pure premiums for each age and gender combination without rebalancing.

Young Male = 146.58 \times 1.20 \times 1.10 = 193.49
Old Male = 146.58 \times 0.83 \times 1.10 = 133.83
Young Female = 146.58 \times 1.20 \times 0.90 = 158.31
Old Male = 146.58 \times 0.83 \times 0.90 = 109.50
11. Continued

(d) Explain why one-way analysis fails to replicate the observed pure premiums in this scenario.

Distributional bias and dependence cause the one-way relativities to produce different pure premiums than observed.

(e) Calculate the revised pure premiums.

Young = 146.58 × 1.20 = 175.90
Old = 146.58 × 0.83 = 121.66

(f) Describe the potential rating effects on male and female policyholders.

Restrictions on using gender as a rating variable would result in females being charged a rate higher than their relative risk and males being charged a rate lower than their relative risk.
12. **Learning Objectives:**

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**

(2a) Use loss development triangles for investigative testing.

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**


**Commentary on Question:**

*This question tests the investigation of reported count triangles as well as estimating ultimate counts using the development method.*

**Solution:**

(a) Calculate the ultimate claim counts for each accident year using the development method with a simple all-year average and a tail factor of 1.05.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12-24</th>
<th>24-36</th>
<th>36-48</th>
<th>48-Ult</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>3.700</td>
<td>1.369</td>
<td>1.135</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>3.724</td>
<td>1.403</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>4.398</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simple average</td>
<td>3.941</td>
<td>1.386</td>
<td>1.135</td>
<td>1.050</td>
</tr>
<tr>
<td>Age-to-Ultimate</td>
<td>6.510</td>
<td>1.652</td>
<td>1.192</td>
<td>1.050</td>
</tr>
</tbody>
</table>

(e.g., 3.700 = 1,850 / 500)

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>(1) Reported Counts at Dec. 31, 2015</th>
<th>(2) Age-to-Ultimate Development Factors</th>
<th>(3) = (1)(2) Ultimate Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>2,875</td>
<td>1.050</td>
<td>3,019</td>
</tr>
<tr>
<td>2013</td>
<td>3,030</td>
<td>1.192</td>
<td>3,612</td>
</tr>
<tr>
<td>2014</td>
<td>2,243</td>
<td>1.652</td>
<td>3,705</td>
</tr>
<tr>
<td>2015</td>
<td>515</td>
<td>6.510</td>
<td>3,353</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td><strong>13,689</strong></td>
</tr>
</tbody>
</table>

(b) Identify one item to investigate based on the reported count triangle and the development factors calculated in part (a).

2014 12-24 development factor is higher than 2012 and 2013.
12. **Continued**

(c) Describe how you would investigate the item identified in part (b).

One could approach the claims department to see if there has been a process change in the speed of settling claims.

(d) Identify three reasons a reinsurer’s experience may be more variable than the primary insurer’s experience.

Any three of the following are acceptable (other answers are possible):

- Reinsurer typically covers higher layer which has more uncertainty.
- Reinsurer will have less data so lower credibility.
- There are often lengthy lags in reporting experienced by reinsurers.
- Random deviations in reported claims will have a magnified effect because the projected ultimate values are highly dependent on reported claims.
13. **Learning Objectives:**

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**


**Sources:**

**Commentary on Question:**
*This question tests the frequency-severity closure method of estimating ultimate claims.*

**Solution:**

(a) Describe a data adjustment to use with the closure method if the line of business has a significant number of partial payments.

The closed count triangle should ideally be matched with a triangle of paid claims on closed counts.

(b) Calculate the proportion of closed counts at each maturity age for accident year 2012.

First calculate the incremental closed counts at each maturity age for 2012:

12: 9,670
24: 12,120 – 9,670 = 2,450
36: 12,980 – 12,120 = 860
48: 13,380 – 12,980 = 400

Next, at each maturity age, the proportion of closed counts is equal to the incremental counts closed divided by the counts not yet closed.

12: 9,670 / 13,380 = 0.723
24: 2,450 / (13,380 – 9,670) = 0.660
36: 860 / (13,380 – 12,120) = 0.683
48: 400 / (13,380 – 12,980) = 1.000
13. Continued

(c) Calculate the incremental closed counts for accident year 2014 at all maturities 12 through 48 months.

The incremental closed count for age 12 is given as 5,960. The incremental closed count for age 24 is 7,420 – 5,960 = 1,460.

For the remaining ages, it is equal to (the ultimate count minus the cumulative closed count) × selected proportion of closed counts:

36: (8,500 – 7,420) × 0.70 = 756
48: (8,500 – 7,420 – 756) × 1.000 = 324

(d) Calculate the incremental paid severity for accident year 2014 at all maturities 12 through 48 months.

The incremental paid severity for ages 12 and 24 are as shown in the table provided: 1,175 for age 12 and 4,200 for age 24.

For the remaining ages, it is simply the selected severity adjusted for the cumulative trend of 1.035 from 2015 to 2014:

36: 12,800 / 1.035 = 12,367
48: 14,500 / 1.035 = 14,010

(e) Calculate the accident year 2014 projected ultimate claims.

Incremental paid claims are equal to the incremental closed counts from part (c) multiplied by the corresponding incremental severities from part (d). Thus:

12: 5,960 × 1,175 = 7,003,000
24: 1,460 × 4,200 = 6,132,000
36: 756 × 12,367 = 9,349,452
48: 324 × 14,010 = 4,539,240

Total = 7,003,000 + 6,132,000 + 9,349,452 + 4,539,240 = 27,023,692
14. **Learning Objectives:**
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**

**Commentary on Question:**
*This question tests the candidate’s understanding (calculation and purpose) of the Cape Cod method and the Generalized Cape Cod method.*

**Solution:**
(a) Calculate the used-up on-level earned premiums for each accident year.

<table>
<thead>
<tr>
<th></th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>B</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
</tr>
</tbody>
</table>

6% rate decrease July 1, 2014

On-level Calculation:

<table>
<thead>
<tr>
<th>Rate Level</th>
<th>Rate Level Relative Value</th>
<th>% Earned in CY</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>100.0%</td>
<td>87.5%</td>
<td>12.5%</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.94</td>
<td>0.0%</td>
<td>12.5%</td>
<td>87.5%</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>1.0000</td>
<td>0.9925</td>
<td>0.9475</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

On-level factor: 0.948 0.955 1.000 (relative to 2015 average rate level)
14. Continued

\[
\begin{array}{cccccccc}
(1) & (2) & (3) = (1)(2) & (4) & (5) = 1/(4) & (6) = (3)(5) \\
\hline
\text{Accident Year} & \text{Earned Premium} & \text{Premium On-Level Factor} & \text{On-Level Earned Premium} & \text{Reported CDF} & \text{Expected % Reported} & \text{Used-Up On-Level Earned Premium} \\
2013 & 40,000 & 0.948 & 37,900 & 1.250 & 0.80 & 30,320 \\
2014 & 41,000 & 0.955 & 39,141 & 2.000 & 0.50 & 19,571 \\
2015 & 40,000 & 1.000 & 40,000 & 5.000 & 0.20 & 8,000 \\
\hline
\text{Total} & & & & & & 57,891 \\
\end{array}
\]

(b) Calculate the expected claims for each accident year.

\[
\begin{array}{cccccc}
(7) & (8) & (9) & (10) = (7)(8)(9) & (11) = 0.694 \times (3)/[(8)(9)] \\
\end{array}
\]

\[
\begin{array}{cccccc}
\text{Accident Year} & \text{Actual Reported Claims} & \text{Trend Factor} & \text{Tort Reform} & \text{Adjusted Claims} & \text{Expected Claims} \\
2013 & 22,000 & 1.040 & 0.900 & 20,600 & 28,083 \\
2014 & 14,000 & 1.020 & 0.950 & 13,566 & 28,026 \\
2015 & 6,000 & 1.000 & 1.000 & 6,000 & 27,753 \\
\hline
\text{Total} & 40,166 & & & & 83,862 \\
\end{array}
\]

Notes: Trend factor = 1.02^{(2015-AY)}
Adjusted expected claim ratio = \frac{40,166}{57,891} = 0.694

(c) Calculate the estimated ultimate claims for each accident year.

\[
\begin{array}{cccc}
(7) & (12) = 1 - (5) & (13) = (11)(12) & (14) = (7) + (13) \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{Accident Year} & \text{Actual Reported Claims} & \text{Expected % Unreported} & \text{Expected Unreported} & \text{Projected Ultimate} \\
2013 & 22,000 & 0.20 & 5,617 & 27,617 \\
2014 & 14,000 & 0.50 & 14,013 & 28,013 \\
2015 & 6,000 & 0.80 & 22,202 & 28,202 \\
\hline
\text{Total} & 41,832 & & 83,832 \\
\end{array}
\]
14.  Continued

(d)  Explain why the Cape Cod method may not be appropriate for coverages such as property or collision.

Development factor may be less than 1.0 which will result in used-up exposures that are greater than the original exposures.

(e)  Explain the purpose of a decay factor.

The decay factor allows different weighting of the years in the experience period with the greatest weight being applied to the year under consideration (origin year) and then decreasing weights to the years preceding and subsequent to the origin year. The decay factor is judgmentally selected between 0% and 100%.

(f)  Identify the methods that the Generalized Cape Cod method approaches when the decay factor approaches zero and approaches one.

When decay factor = 0, the Generalized Cape Cod method returns the development method result.

When decay factor = 1, the Generalized Cape Cod method returns the traditional Cape Cod method result.
14. Continued

(g) Calculate the expected claims for accident year 2015 using the Generalized Cape Cod method with a decay factor of 70%.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Earned Premium</th>
<th>Used-Up On-Level Earned Premium</th>
<th>(10)</th>
<th>(15)</th>
<th>(16)</th>
<th>(17) = (1)(16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>40,000</td>
<td>30,320</td>
<td>20,600</td>
<td>49%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>41,000</td>
<td>19,571</td>
<td>13,566</td>
<td>70%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>40,000</td>
<td>8,000</td>
<td>6,000</td>
<td>100%</td>
<td>0.700</td>
<td>28,000</td>
</tr>
<tr>
<td></td>
<td>57,891</td>
<td>40,166</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Expected claim ratio (16) = sumproduct[(10),(15)] / sumproduct[(6),(15)]
Expected claims (17) = (1)(16) = 40,000×0.700
Learning Objectives:
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

Learning Outcomes:
(2d) Explain the effect of changing conditions on the projection methods cited in (2b).

Sources:

Commentary on Question:
This question tests the candidate’s understanding of changing conditions on data, assumptions, and methods.

Solution:
(a) Explain the likely row, column, or diagonal effects each event had on the data.

Commentary on Question:
Other explanations are possible.

(i) Expect higher claim ratios in AY 2012 and subsequent (multiple rows).

(ii) Expect a temporary slowdown (decrease) in reported claim activity on accident year (AY) 2013 which means the reported claims (frequency) evaluated at 12 months could be low. However, cumulative patterns will self-correct by 24 months.

There could also be a temporary slowdown (decrease) in paid or closed activity on other AYS in the calendar year (CY) 2013 diagonal. This slowdown (decrease) should be offset by an increase in the next CY diagonal (2014).

(iii) Expect claims to increase for CYs 2010 and subsequent (CY effect) (i.e., all open claims are affected).

(b) Explain how you would handle each event through a data adjustment, assumption, or method selection.

Commentary on Question:
Other explanations are possible.

(i) Development on AYS 2012 and subsequent should be given higher weight in making age-to-age development factor selections on AYS 2012 and subsequent.
15. Continued

(ii) Since we are now evaluating experience as of Dec 31, 2015, this anomaly should not be an issue in estimating AY 2013 claims. AY 2013 12-24 month development factor (or the entire row) should be excluded from future analyses.

(iii) Development factors from CYs 2010 and subsequent should be given higher weight in making age-to-age development factor selections on all AYs.

(c) Describe one situation (different from those above) that might lead you to use a Berquist-Sherman adjustment in estimating ultimate claim ratios for auto liability business.

Commentary on Question:
Other explanations are possible.

A change in claim department settlement patterns.
16. **Learning Objectives:**
7. The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

**Learning Outcomes:**
(7b) Apply catastrophe models to insurance ratemaking, portfolio management, and risk financing.

**Sources:**

**Solution:**
(a) Rank the following portfolios from least to most catastrophe risk from GIC’s perspective, with the possibility that some may be roughly equal in risk. Justify your ranking.

I. ABC only
II. FGH only
III. XYZ only
IV. ABC and FGH
V. ABC and XYZ
VI. FGH and XYZ
VII. ABC, FGH, and XYZ

**Commentary on Question:**
Candidates generally did poorly, failing to understand that combining portfolios brings diversification and thus reduces risk. Alternative answers could earn full credit. For example, ABC and XYZ could be ranked with one riskier than the other. This is acceptable provided a reasonable explanation was provided. Another alternative is to have so little risk associated with FGH that any portfolio that includes it will be viewed as less risky than those that do not.

For the individual portfolios, FGH is less risky than ABC and XYZ, which are about equal. That is because tornados cause less widespread damage (and hence are not discussed in the text). The other consideration is that combining any of these portfolios provides diversification and thus a reduction in overall riskiness. This leads to a ranking of

VII < IV = VI < V < II < I = III.

(b) Describe an action GIC may take to improve its underwriting to account for ABC’s earthquake risk.

GIC could obtain detailed underwriting information about each property, perhaps including inspection by an engineer.
16. Continued

(c) Describe coverage modifications GIC may use to reduce its earthquake risk.

GIC should consider coverage limits both for each location and for aggregate losses.

(d) Describe how a catastrophe model could be used to set the coverage modifications from part (c).

The model can be run incorporating various trial limits. Each run produces an exceedance curve. This continues until limits are found that produce a curve with the desired risk profile.

(e) Describe an action other than using coverage modifications that GIC may use to reduce its earthquake risk.

 Commentary on Question:
*Any one of the actions listed is sufficient to receive credit.*

- Purchase reinsurance
- Securitization
- Insure additional risks to add diversification
- Improve the quality of the insured structures
17. **Learning Objectives:**

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**

(5j) Perform individual risk rating using standard plans.
(5k) Calculate rates for claims-made coverage.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 34 and 35.

**Commentary on Question:**

*This question tests individual risk rating and claims-made ratemaking.*

**Solution:**

(a) Provide two advantages of claims-made and two advantages of occurrence coverage for ERR.

Two advantages of claims-made:
- less uncertainty in pricing
- less effect due to sudden changes in either the trend or the reporting pattern

Two advantages of occurrence coverage:
- greater opportunity for investment income
- less risk of coverage gaps

(b) Identify three risk characteristics that can be used in schedule rating for the group.

Any three of the following are acceptable (other characteristics are possible):
- risk management program
- geographical scope of practice
- range of assignments
- experience
- peer review

(c) Provide a reason why the total allowable schedule rating credits or debits for all risk characteristics combined is generally limited.

Schedule credits and debits generally have a limit due to either regulatory rules and regulations or internal company policies.
17. Continued

(d) Assess the credibility of the historical data from ERR.

For a long experience period of ten years, there are only 100 claims. Even assuming a credibility standard of 400 claims and the square root rule, ERR’s experience is only partially credible \((\sqrt{100/400} = 0.5)\).

(e) Calculate the step factors from first-year through maturity for the following cases:

(i) Claims-made

(ii) Claims-paid

**Commentary on Question:**
An amount of 10 per year is used as an illustrative example.

<table>
<thead>
<tr>
<th>Claims-Made Reporting Pattern</th>
<th>Report Year</th>
<th>AY Lag</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td></td>
<td>0</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>10</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Claims-Paid Payment Pattern</th>
<th>Report Year</th>
<th>AY Lag</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td></td>
<td>0</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>10</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

(i) Claims-made step factors:
\[
\begin{align*}
10/40 &= 0.25 \\
20/40 &= 0.50 \\
30/40 &= 0.75 \\
40/40 &= 1.00 \\
\end{align*}
\]

(ii) Claims-paid step factors:
\[
\begin{align*}
5/40 &= 0.125 \\
15/40 &= 0.375 \\
25/40 &= 0.625 \\
35/40 &= 0.875 \\
40/40 &= 1.000 \\
\end{align*}
\]
17. Continued

(f) *(1 point)* Calculate the tail factor applicable to a mature policy for the following cases:

(i) Claims-made

(ii) Claims-paid

(i) Claims-made tail factor = $\frac{60}{40} = 1.5$

(ii) Claims-paid tail factor = $\frac{80}{40} = 2.0$
18. **Learning Objectives:**

4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

**Learning Outcomes:**

(4d) Describe the influences on exposures and premiums of changes in deductibles, changes in policy limits, and changes in mix of business.

(4e) Choose trend rates and calculate trend factors for exposures.

**Sources:**

**Commentary on Question:**

*This question tests the candidate’s understanding of premium trend analysis, particularly when the trend rate changes.*

**Solution:**

(a) Explain one reason not to use actual premium when analyzing premium trend.

Using actual (unadjusted) premiums could result in trend estimates that reflect rate changes and not an underlying trend.

(b) Calculate the 2010 premium trend factor.

**Commentary on Question:**

*Candidates had difficulty setting the dates to which the two trend factors are applied.*

Policies written between January 1, 2009 and December 31, 2010 contribute toward 2010 earned premiums. The average written date is thus January 1, 2010. The first trending period is from January 1, 2010 to January 1, 2012, which is 2 years. The total trend is $1.02^2 = 1.0404$.

The rates will be in effect for one year starting September 1, 2016. The average written date in the forecast period is six months later, or March 1, 2017. Thus, the second trending period is from January 1, 2012 to March 1, 2017, which is 5 1/6 years. The trend for this period is $1.0075^{5.167} = 1.0394$.

The 2010 premium trend factor is $1.0404 \times 1.0394 = 1.0814$. 

18. Continued

(c) Describe what would have been different in the calculation if the work done in part (b) was for a self-insurer.

The difference is that a self-insurer is essentially a single policy, not a series of policies written over the period. Therefore, the average written dates would reflect the actual date the policy is written.

(d) Explain how the premium trend factors would be affected by the following:

(i) An increasing proportion of insureds choosing a higher policy limit at the beginning of 2014

(ii) An increasing proportion of insureds choosing a higher deductible at the beginning of 2014

(i) The increased policy limit would increase the premiums and thus the premium trend factor would increase.

(ii) The higher deductible would decrease the premiums and thus the premium trend factor would decrease.
19. **Learning Objectives:**

4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**

(4c) Choose trend rates and calculate trend factors for claims.

(5b) Calculate expenses used in ratemaking analyses including expense trending procedures.

(5f) Calculate overall rate change indications under the claims ratio and pure premium methods.

**Sources:**

**Commentary on Question:**
This question tests expense loadings and basic ratemaking.

**Solution:**

(a) Calculate the weighted average trended pure premium.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Earned Exposures</th>
<th>Ultimate Claims</th>
<th>Average Earned Date</th>
<th>Trending Period</th>
<th>Trended Ultimate PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>5,100</td>
<td>2,082,000</td>
<td>7/1/2013</td>
<td>10/1/2017</td>
<td>4.25</td>
</tr>
<tr>
<td>2014</td>
<td>5,250</td>
<td>2,250,000</td>
<td>7/1/2014</td>
<td>10/1/2017</td>
<td>3.25</td>
</tr>
<tr>
<td>2015</td>
<td>5,200</td>
<td>2,178,000</td>
<td>7/1/2015</td>
<td>10/1/2017</td>
<td>2.25</td>
</tr>
</tbody>
</table>

Weighted average trended pure premium  

444.90

Notes:  
(6) = (2)(1.02)^5/(1)  
(6)_{\text{Total}} = 0.2 \times 444.08 + 0.3 \times 457.06 + 0.5 \times 437.93 = 444.90
19. Continued

(b) Recommend how you would include a provision for the health levy in your ratemaking analysis. Justify your recommendation.

**Commentary on Question:**
*Either option is acceptable. The key is the justification for the approach.*

Option 1: Recommend using a percentage of premium approach. The justification is that higher premium drivers are higher risks and therefore should pay more toward the health levy.

Option 2: Recommend using a flat fee per policy (exposure). The justification is that every risk should pay an equal amount toward the levy.

(c) Calculate the provision for the health levy to include in your ratemaking analysis.

Either method is acceptable.

Option 1 (percentage): \( \frac{119,000}{2,761,000} = 4.31\% \) (assumes the levy in rating period will continue to be the same ratio to premium so no trending required).

Option 2 (fixed expense):
- Fixed expense = \( \frac{119,000}{5,200} = 22.88 \)
- Need to trend to future rating period (from 2015) = 2.25 years
- Trended fixed expense for levy = \( 22.88 \times 1.01^{2.25} = 23.40 \)

(d) Calculate the indicated rate.

Option 1 (levy as variable):
Total variable expense = \( 0.12 + 0.0431 = 0.1631 \)

Indicated rate = \( \frac{PP \times (1 + ULAE) + F}{1 - V - Q} = \frac{(445 \times 1.05 + 20)}{1 - 0.1631 - 0.03} = 603.85 \)

Option 2 (levy as additional fixed expense):
Total F = \( 20 + 23.40 = 43.40 \)

Indicated rate = \( \frac{PP \times (1 + ULAE) + F}{1 - V - Q} = \frac{(445 \times 1.05 + 43.40)}{1 - 0.12 - 0.03} = 600.77 \)
(e) Determine whether or not the target for profit and contingencies will be met based on your indicated rate from part (d).

Use formula 31.11: \( P = C + (F \times E) + (V \times P) + (Q \times P) \)

Solve for \( Q = 1 - \frac{C \times (1 + ULAE) + (F \times E) - V}{P} \)

Option 1:
\[ F \times E = 20 \times 5,400 = 108,000 \]
\[ V = 16.31\% \]
\[ Q = 1 - \frac{2,450,000 \times (1.05) + 108,000}{603.85 \times 5,400} - 16.31\% = 1.5\% \]

Therefore, since \( Q < 3\% \) target, target is not met.

Option 2:
\[ F \times E = 43.40 \times 5,400 = 234,360 \]
\[ V = 12\% \]
\[ Q = 1 - \frac{2,450,000 \times (1.05) + 234,360}{600.77 \times 5,400} - 12\% = 1.5\% \]

Therefore, since \( Q < 3\% \) target, target is not met.