1. The probability of being fully functional after two years for a single television is:

\[
\begin{pmatrix}
0.82 \\
0.10 \\
0.08 \\
0.60 \\
0.00
\end{pmatrix}
= 0.82 \times 0.82 + 0.10 \times 0.60 + 0.08 \times 0.00 = 0.7324
\]

The number of the five televisions being fully functional has a binomial distribution with parameters of \( n = 5 \) and \( p = 0.7324 \). The probability that there will be exactly two televisions that are fully functioning is therefore:

\[
\binom{5}{2} 0.7324^2 (1-0.7324)^3 = 10 \times 0.53641 \times 0.019163 = 0.10279
\]

2. 

\[
\mu_x = -\frac{d}{d_x} \ln S_0(x) = -\frac{1}{3} \frac{d}{d_x} \ln \left(1 - \frac{x}{60}\right)
\]

\[
= \frac{1}{180} \left(1 - \frac{x}{60}\right)^{-1} \cdot \frac{1}{3(60-x)}
\]

Therefore, \( \mu_{55} = \frac{1}{3(25)} = \frac{1}{75} = 0.0133 \).

3. Out of 400 lives initially, we expect 

\[
400 \times \frac{l_{45}}{l_{40}} = 400 \times \frac{7,533,964}{9,565,017} = 315.0633
\]

Survivors with standard deviation of 

\[
\sqrt{400 \times \frac{l_{40}^2}{l_{40}} \left(1 - \frac{p_{25}}{l_{40}}\right)} = 8.1793
\]

To ensure 86% funding, using the normal distribution table, we plan for 

\( 315.0633 + 1.08(8.1793) = 323.8969 \).

The initial fund must therefore be 

\[
F = 324 \times 15200 \times \left(\frac{1}{1.06}\right)^{40} = 478,799.80
\]
4. Probability

\[
\begin{align*}
4. & \quad \text{Probability} \\
& = \int_0^5 p_5 \mu_0^{01} p_{5-t} \, dt \\
& = \int_0^5 e^{-0.06t} 0.05e^{-0.08(5-t)} \, dt \\
& = e^{-0.40}(0.05) \int_0^5 e^{0.02t} \, dt \\
& = e^{-0.40} \left( \frac{5}{2} \right)(e^{0.10} - 1) = 0.1762
\end{align*}
\]

5. 

\[
\dot{a}_{x\mid x} = 1 + \nu_p \dot{a}_{x+1\mid x} = 1 + (1 + k) \nu_p \dot{a}_{x+1\mid x} = 1 + (1 + k) \left( \dot{a}_{x\mid x} - 1 \right)
\]

Therefore, we have

\[
k = \frac{\dot{a}_{x\mid x} - 1}{\dot{a}_{x\mid x} - 1} - 1 = \frac{21.167}{20.854} - 1 = 0.015
\]

6. 

\[
100,000 A - \frac{1}{50:60:10} = 100,000 \left[ A_{50:10}^1 + A_{60:10}^1 - A_{50:60:10}^1 \right]
\]

where

\[
A_{50:10}^1 = A_{50:10} - 10 E_{50:60} \times 0.24905 - (0.51081)(0.36913) = 0.060495
\]

\[
A_{60:10}^1 = A_{60:10} - 10 E_{60:70} \times 0.36913 - (0.45120)(0.51495) = 0.136785
\]

\[
A_{50:60:10}^1 = A_{50:60} - (1.06)^{10} \times 10 E_{50:10} E_{60:70} A_{60:70} \times 0.42296 - 1.79085(0.51081)(0.45120)(0.57228) = 0.186751
\]
7. Let $G$ be the annual gross premium. By the equivalence principle, we have

$$G\ddot{a}_{35} = 100,000A_{35} + 0.15G + 0.04G\ddot{a}_{35}$$

so that

$$G = \frac{100,000A_{35}}{0.96\ddot{a}_{35} - 0.15} = \frac{100,000(0.12872)}{0.96(15.3926) - 0.15} = 880.023$$

8. By the equivalence principle,

$$4500\bar{a}_{x:20} = 100,000\bar{A}_{x:20} + R\bar{a}_{x:20}$$

where

$$\bar{A}_{x:20} = \frac{\mu}{\mu + \delta} \left(1 - e^{-20(\mu + \delta)}\right) = \frac{0.04}{0.12} \left(1 - e^{-20(0.12)}\right) = 0.3031$$

$$\bar{a}_{x:20} = \frac{1 - e^{-20(\mu + \delta)}}{\mu + \delta} = \frac{1 - e^{-20(0.12)}}{0.12} = 7.5774$$

Solving for $R$, we have

$$R = 4500 - 100,000 \left(\frac{0.3031}{7.5774}\right) = 500$$

9. By the equivalence principle, we have

$$G\ddot{a}_{35:10} = 50,000A_{35} + 100a_{35} + 100A_{35}$$

so that

$$G = \frac{50,100A_{35} + 100(\ddot{a}_{35} - 1)}{\ddot{a}_{35} - 10E_{35}\ddot{a}_{45}}$$

$$= \frac{50,100(0.12872) + 100(14.3926)}{15.3926 - 0.54318(14.1121)}$$

$$= 1020.828$$
10. Let \( P \) be the annual net premium

\[
P = \frac{1000 \bar{A}_{x:n}}{\bar{a}_{x:n}} = \frac{1000(0.192)}{\bar{a}_{x:n}}
\]

where

\[
\bar{a}_{x:n} = \frac{1 - A_{x:n}}{d} = \frac{(1.05)}{(0.05)} \left(1 - A_{x:n}^{1} - A_{x:n}^{\frac{1}{2}} \right)
\]

\[
\bar{A}_{x:n} = \frac{i}{\delta} \left( A_{x:n}^{1} \right) + _aE_x
\]

\[
\Rightarrow 0.192 = \frac{0.05}{0.0488} \left( A_{x:n}^{1} \right) + 0.172
\]

\[
\Rightarrow A_{x:n}^{1} = 0.01952
\]

\[
\Rightarrow \bar{a}_{x:n} = \frac{1.05}{0.05} \left(1 - 0.01952 - 0.172 \right) = 16.97808
\]

Therefore, we have

\[
P = \frac{1000(0.192)}{16.97808} = 11.31
\]

11. Premium at issue for (20): \( \frac{65.28}{16.5133} = 3.9531 \)

Premium at issue for (50): \( \frac{249.05}{13.2668} = 18.7724 \)

Lives in force after ten years:

Issued at age 20: \( 10,000 \times \frac{9,501,381}{9,617,802} = 10,000 \times 0.9878953 = 9878.953 \)

Issued at age 50: \( 10,000 \times \frac{8,188,074}{8,950,901} = 10,000 \times 0.9147765 = 9147.765 \)

The total number of lives after ten years is therefore: \( 9878.953 + 9147.765 = 19,026.718 \)

The average premium after ten years is therefore:

\[
\frac{(3.9531 \times 9878.953) + (18.7724 \times 9147.765)}{19,026.718} = 11.078
\]
12.  
\[ V[L_0 \#1] = \left( B_1 + \frac{P_1}{d} \right)^2 \left( \frac{A_x - A_x^2}{w} \right) = 20.55 \]  
\[ = \left( 8 + \frac{1.25(1.06)}{0.06} \right)^2 \times w = 20.55 \]  
\[ V[L_0 \#2] = \left( 12 + \frac{1.875}{0.06}(1.06) \right)^2 \times w \]  
\[ \frac{V[L\#2]}{V[L\#1]} = \left( \frac{12 + \frac{1.875}{0.06}(1.06)}{8 + \frac{1.25}{0.06}(1.06)} \right)^2 = 2.25 \]  
\[ \Rightarrow V[L\#2] = 2.25 \times 20.55 = 46.24 \]  
Or:  
\[ W = \left( \frac{12}{8} \right)^2 \times V[L_0 \#1] = (1.5)^2 \times (20.55) = 46.24 \]  
(because both premium and benefit are scaled by 1.5)

13. Calculating the reserve,  
\[ _{15}V = A_{50:15} - \frac{A_{35:30}}{\hat{a}_{50:15}} \]  
Where  
\[ \hat{a}_{35:30} = \frac{1 - A_{35:30}}{d} = \frac{1 - 0.255}{0.05} = 15.645 \]  
And  
\[ \hat{a}_{50:15} = \frac{1 - A_{50:15}}{d} = \frac{1 - 0.506}{0.05} = 10.374 \]  
So that  
\[ _{15}V = 0.506 - \frac{0.255}{15.645} \times 10.374 = 0.3369128 \]  
SC = surrender charge  
\[ _{15}V - SC = 0.40A_{50:15} \Rightarrow SC = _{15}V - 0.40A_{50:15} \]  
\[ = 0.3369128 - 0.40(0.506) \]  
\[ = 0.1345128 \]  
For insurance of 2000, SC = 269.0256
14.

\[ AV_0 = 0 \]
\[ P_1 = 4,450 \]
\[ EC_1 = 56 + 2\% \times 4,450 = 145.00 \]
\[ \text{COI rate} = q_{36} = 1.2 \times 0.00214 = 0.002568 \]
\[ \text{COI}_1 = 200,000 \times 0.002568 \times (1/1.06) = 484.53 \]
\[ \text{Credited Interest:} \ 6\% \times (4,450 - 145 - 484.53) = 229.23 \]
\[ AV_1 = 4,450 - 145 - 484.53 + 229.23 = 4,049.70 \]

15. We have

\[ vq_x + \beta (\bar{a}_{25:20} - 1) + P \bar{a}_{20} E_{20} \bar{a}_{45:20} = P\bar{a}_{25:40} \]
\[ \Rightarrow \beta = \frac{P(\bar{a}_{25:20}) + P \bar{a}_{20} E_{20} \bar{a}_{45:20} - vq_x}{\bar{a}_{25:20} - 1} \]

Where \( P = \frac{A_{25:40}}{\bar{a}_{25:40}} = \frac{1}{\bar{a}_{25:40}} - d = 0.02161656 \)

\[ \Rightarrow \beta = \frac{0.02161656(11.087) - \frac{1}{1.04}(0.005)}{11.087 - 1} = 0.02328295 \]

For insurance of 10,000, \( \beta = 233 \).

16.

\[ q_{50} = 0.00592, \quad q_{51} = 0.00642 \]

\[ \Rightarrow AV_1 = 1369.895 \]

\[ AV_2 = \left( AV_1 + 5000(1 - 0.035) - 75 - (500,000 - AV_2) \left( \frac{1.20q_{51}}{1.03} \right) \right) \times 1.045 \]

\[ \Rightarrow AV_2 = 2506.787 \]
17. Let $P$ be the annual net premium at $x+1$.

$$P\ddot{a}_{x+1} = 1000 \sum_{k=0}^{\infty} (1.03)^k q_{x+1} = 1000A_{x+1}'$$

We are given

$$110\ddot{a}_{x+1} = 1000 \sum_{k=0}^{\infty} (1.03)^k v^{k+1} q_{x+1} = 1000A_x'$$

Which implies that

$$110(1 + v p_x\ddot{a}_{x+1}) = 1000(1.03v q_x + 1.03v p_x A_{x+1}')$$

Solving for $A_{x+1}'$, we get

$$A_{x+1}' = \frac{110}{1000} \left[ 1 + v (0.95)(7) \right] - 1.03v (0.05) \left( \frac{1}{1.03v (0.95)} \right) = 0.8141032$$

Thus, we have

$$P = \frac{1000(0.8141032)}{7} = 116.3005$$

18. Under PUC:

$V = \text{accrual rate} \times \text{years of past service} \times \text{survival to retirement} \times \text{discount to retirement benefit}$

$$36V = \frac{\text{years of service} + 1}{\text{years of service}} \times 35V$$

$$35V + C = 36V = \frac{36}{35} 35V$$

$$C = \frac{36}{35} 35V - 35V \Rightarrow C = \frac{35V}{35}$$
19. By age 65, member would have served total of 35 years in which case, benefit would be $35 \times 0.02 = 70\%$. Thus set it at 60%.

$$\text{EPV(benefits)} = 0.60 \times 50,000 \times (1.03)^{19} \times \frac{1}{1.05^{20}} \frac{l_{65}^{(z)}}{l_{45}^{(z)}} \ddot{a}_{65}^{(12)}$$

$$= 0.60 \times 50,000 \times \left(\frac{1}{1.05}\right)^{20} \left(\frac{3}{5}\right)(7.8)(1.03)^{19}$$

$$= 92,787.29$$

20. Replacement ratios

Plan 1: $R = \frac{1250 \times 25}{S_0 \times (1.04)^{24}}$

Plan 2: $R = \frac{S_0 \times 0.02 \times 25 \times \frac{1.04^{25}-1}{0.04} \times \frac{1}{25}}{S_0 \times (1.04)^{24}}$

The two are equal, so that

$$S_0 = \frac{1250 \times 25}{0.02 \left(\frac{1.04^{25}-1}{0.04}\right)} = 37,518.69$$