
SOCIETY OF ACTUARIES
Quantitative Finance and Investment Core

Exam QFICORE

AFTERNOON SESSION

Date: Wednesday, November 1, 2017

Time: 1:30 p.m. – 3:45 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This afternoon session consists of 8 questions numbered 11 through 18 for a total of 40 points. The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFICORE.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

****BEGINNING OF EXAMINATION****

Afternoon Session
Beginning with Question 11

11. (5 points) Your manager asked you to model the monthly log return on ABC stock from January 1960 to December 2015. Since there are more than 600 observations, you are thinking to use ARCH or GARCH to model the stock return.

(a) (2 points) Describe the features of ARCH and GARCH models.

GARCH(1,1) model is described as follows:

$a_t = \sigma_t \varepsilon_t$ where ε_t are independently and identically distributed random variables with mean 0 and variance 1

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \text{ where } \alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0, \alpha_1 + \beta_1 < 1.$$

(b) (1.5 points) Derive the l -step ahead variance forecast $\sigma_t^2(l)$ and find its limit as l goes to infinity.

The current volatility estimate σ_t^2 is 0.00136.

The following parameters of GARCH(1,1) model are given,

$$\alpha_0 = 0.0008$$

$$\alpha_1 = 0.7$$

$$\beta_1 = 0.1$$

(c) (1.5 points) Calculate the forecasted variance in ten days, i.e., the 10-step ahead forecasted variance.

12. (5 points) You are given the following grid. The Treasury bonds in the grid pay coupons once a year.

Term	Par Treasury bond yield	Corporate spread over zero-coupon bonds (basis points)		
		AA	A	BBB
1	1.5%	50	70	86
2	2.0%	55	76	93
3	2.5%	59	84	99
4	3.0%	68	91	104
5	3.5%	77	97	112

You have bought \$20 million of a 3-year, BBB-rated zero-coupon bond.

- (a) (1.5 points) Calculate the yield and the modified duration of the zero-coupon bond.

In order to hedge the interest rate risk of the zero-coupon bond, you decided to short sell the 5-year Treasury bond.

- (b) (2 points) Calculate the amount that you need to short sell in order to hedge the interest rate risk.
- (c) (0.75 points) Describe the simplifications assumed in the hedge of part (b).

Instead of using the 5-year Treasury bond, your manager has suggested two different alternatives to hedge the interest rate risk.

- (d) (0.75 points) Explain the impact on the amount of short-selling needed for each of the alternatives below. No calculation is needed.
- (i) 2-year, annual coupon, A-rated corporate bond
- (ii) 5-year, annual coupon, BBB-rated corporate bond

- 13.** (4 points) You work at ABC Endowment Fund (ABC EF), which invests almost 50% of its \$1 billion fund in fixed income securities.

ABC EF has the following objectives:

Investment Objectives:

- Maintain the endowment's real purchasing power after distributions

Return Objectives:

- Maximize risk adjusted returns
- Average annual return shall support a spending rate of 2.5%, an expected inflation rate of 1.4%, and 0.2% investment expenses

Risk Objectives:

- Maintain diversification levels consistent with prudent investment practices
- Standard deviation of returns shall not exceed 0.1

Currently the portfolio's allocation is as follows.

Asset Class	Current Allocation
1. Government Securities	15%
2. Corporate Bonds	20%
3. High-yield Bonds	5%
4. Equity (domestic & international)	50%
5. Other	10%

- (a) (0.5 points) Explain the role of equity investments in this portfolio.

The investment policy was recently revised to allow leveraged loans as a permissible asset class. There is an internal debate about whether or not to switch from investing in high-yield bonds to leveraged loans.

- (b) (1 point) List two arguments for the switch and two arguments against the switch.

13. Continued

The standard deviation of the current portfolio is 0.095. The board of trustees decided to revise the risk objective by setting the limit of portfolio standard deviation at 0.07. They performed a mean-variance analysis, which produced the following corner portfolios:

Corner Portfolio	Expected Return	Standard Deviation	Sharpe Ratio	Asset Class (Portfolio Weight)				
				1	2	3	4	5
A	6.9%	0.145	0.379	0%	13%	8%	79%	0%
B	6.0%	0.119	X	1%	22%	10%	63%	4%
C	4.3%	0.074	Y	35%	8%	10%	38%	9%
D	3.5%	0.055	0.381	52%	0%	11%	25%	12%

- (c) (2.5 points) Determine and justify the overall most appropriate strategic asset allocation for ABC EF using the mean-variance analysis.

14. (4 points) ABC Club has a fixed-income portfolio which tracks a benchmark, and their returns in the last four years are given below:

Period	Portfolio Return	Benchmark Return
1	12.90%	12.60%
2	6.80%	6.50%
3	0.80%	1.20%
4	-4.60%	-5.00%

- (a) (0.5 points) List two factors that might affect tracking risk.
- (b) (1 point) Calculate the tracking risk of the portfolio.

ABC Club is re-constructing the portfolio with the following considerations:

- Highly risk-averse
- Either match or exceed the rate of return on a market index
- Income to cover on-going uncertain Club expenses

- (c) (1.5 points)
- (i) Identify three risks associated with selecting a benchmark index.
- (ii) Evaluate each risk in part (c)(i) for ABC Club.
- (d) (1 point) Recommend the most suitable benchmark for ABC Club from the following and justify your recommendation.
- Short-Term Corporate Bond Index
 - Short-Term Corporate High-Yield Bond Index
 - Short-Term Emerging Market Bond Index
 - Long-Term Corporate Bond Index
 - Long-Term Corporate High-Yield Bond Index
 - Long-Term Emerging Market Bond Index

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15. (5 points) HIJ sponsors a defined benefit pension plan. The plan's assets are invested in a portfolio of investment grade corporate bonds. Pension plan participants include both active workers and retirees. HIJ's CFO suggests dividing the pension liabilities into separate portions for active and retired lives to reflect differences in return objectives, risks, liquidity, and time horizon.

- (a) (1 point) Describe the main difference between the active lives and retired lives portion of liabilities for each of the following:
- (i) Inflation sensitivity
 - (ii) Duration

HIJ makes a one-time donation to establish KL, a company-sponsored charitable foundation.

KL's board retains you as the investment advisor to make recommendations for its endowment fund. The following summarizes KL's objectives:

- Making annual distributions of 4% of average asset value.
- Increasing the value of the endowment by seeking a rate of return exceeding the rate needed to maintain the real purchasing power of the portfolio.
- Keeping annual investment management expenses at 0.25% of assets.

KL expects the annual rate of inflation to be 2%.

- (b) (1.5 points) Propose the following investment policy constraints for KL:
- (i) Return objective
 - (ii) Liquidity Requirement
 - (iii) Time Horizon
- (c) (1.5 points) Determine which of the HIJ pension plan and KL:
- (i) Has greater ability to take risks.
 - (ii) Has greater willingness to take risks.

15. Continued

HIJ's investment strategist predicts that over the next two weeks, credit spreads will widen significantly and all interest rates will decline significantly. You are evaluating the following trades. Each trade involves buying and selling an equal value of bonds with identical characteristics, except as noted.

Trade 1:

Buy 7-year AA industrial corporate bonds;
Sell 7-year A industrial corporate bonds.

Trade 2:

Buy 5-year callable corporate bonds;
Sell 5-year non-callable corporate bonds of the same issuer.

- (d) (1 point) Assess the expected effects (positive or negative) on the pension portfolio's value over the next 2 weeks for each potential trade given the strategist's market expectations.

- 16.** (6 points) When the financial director created a new investment policy for Company ABC in 2014, he decided to invest all the funds in equities.

You obtain the following information from your meeting with the investment committee:

- The committee believes that the stock market is efficient.
- The committee fired the previous equity manager because the tracking risk was more than 1%.
- The committee is willing to accept a low information ratio.
- Company ABC pays taxes and wants to minimize investments fees.

- (a) (1 point) Explain the rationale of using a passive management approach rather than an active management approach, based on the information given above.
- (b) (1 point) Describe biases of each of the following four basic index weighting methods:
- (i) Price-weighted
 - (ii) Value-weighted
 - (iii) Equal-weighted
 - (iv) Float-weighted

You are given the following data:

	Share Price 2014-12-31	Share Price 2015-12-31	Price Change	Market value 2014-12-31 (Millions \$)	Market value 2015-12-31 (Millions \$)	Free Floating Factor
Stock A	100	100	0 %	25,000	25,000	0.6
Stock B	20	30	50 %	60,000	90,000	0.8
Stock C	260	255	-2 %	52,000	51,000	1
Total	380	385		137,000	166,000	

- (c) (1.5 points) Calculate, for each of the four methods in part (b), the return on the index consisting of the above three stocks for the period from Dec 31, 2014 to Dec 31, 2015.
- (d) (1.5 points) Recommend the most appropriate index weighting method for the Company ABC and justify your recommendation.

16. Continued

The investment committee decides to modify the investment strategy by encouraging the fund manager to outperform the market but control the tracking risk within 5%.

- (e) *(1 point)* Recommend an alternative equity investment management approach to the committee.

17. (5 points) An investment committee of a pension fund is seeking equity managers to manage the fund against a benchmark. The expected return of the benchmark is 2%. Four managers are under consideration with uncorrelated alphas:

	Expected Return	Expected Tracking Risk
Manager A	2%	0%
Manager B	3%	2%
Manager C	5%	8%
Manager D	6%	10%

- (a) (1 point) Identify the investment approach of each manager.

The investment committee has the following objectives

- I. Maximizing the utility function: $U_A = r_A - 0.1\sigma_A^2$,
where r_A is the excess return above the benchmark and σ_A^2 is the variance of the active return.
- II. Achieving the tracking risk no more than 4%

The following mixes of managers are proposed to the investment committee.

Manager Mix	Manager A	Manager B	Manager C	Manager D
Option 1	0%	100%	0%	0%
Option 2	60%	0.0%	0.0%	40.0%
Option 3	30%	20%	20.0%	30.0%

- (b) (2.5 points) Identify the best option based on the specified expected utility function and describe the type of portfolio constructed under this option.

The committee introduced a requirement for a minimum information ratio to exceed 60%.

- (c) (1.5 points) Recommend the actions for the committee to take to meet this requirement based on all information described above.

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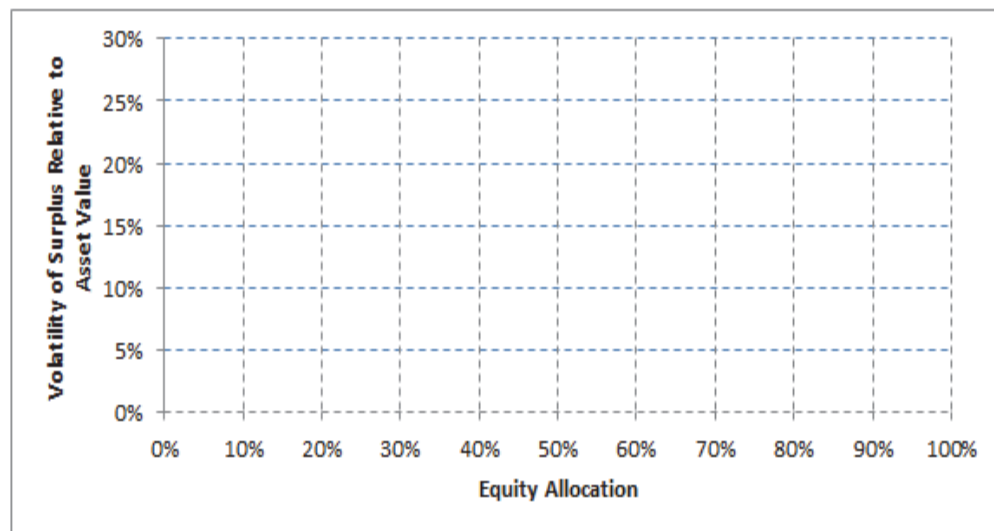
- 18.** (6 points) The duration of ABC pension liabilities is 12. You choose to model the liabilities using a bond index as a proxy. The bond index has a duration of 10.

To manage the pension surplus, you choose to use a portfolio of U.S. equity and the bond index. You are given the following information:

	Annual Expected Return	Annualized Volatility σ	Correlation Matrix	U.S. Equity	Bond Index	Liability Index
U.S. Equity	8.0%	15%	U.S. Equity	1.0		
Bond Index	5.5%	10%	Bond Index	0.2	1.0	
Liability	4.0%	8%	Liability Index	0.15	0.9	1.0

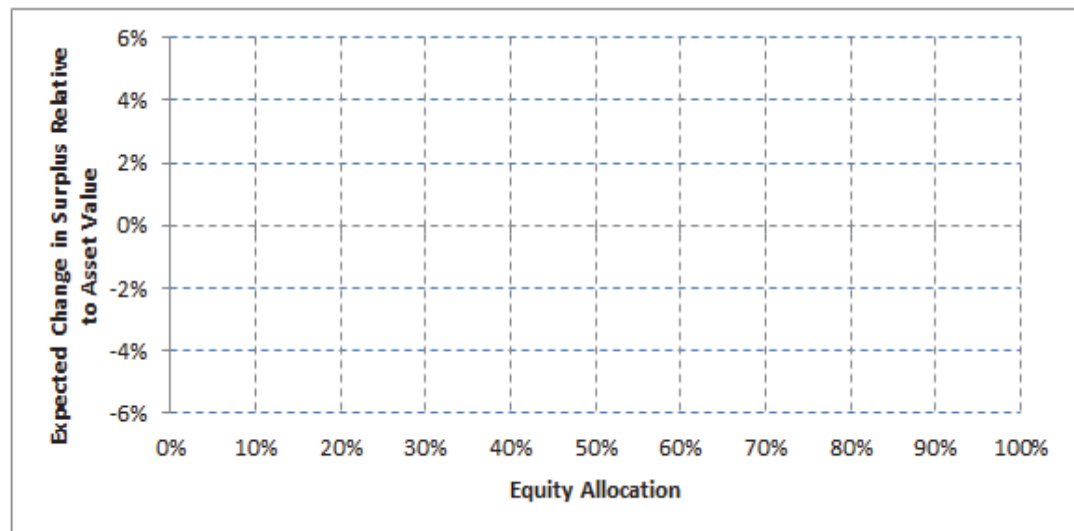
The risk-free rate is 1% per annum, compounded continuously.

- (a) (2 points) Assume that the pension funding ratio $\left(\frac{\text{Asset}}{\text{Liability}}\right)$ is 2.0.
- Sketch a curve in the following chart to show the sensitivity of the surplus risk relative to the equity allocation.
 - Determine the equity allocation that minimizes the surplus risk on your curve.
 - Determine the equity allocation that maximizes the surplus risk on your curve.



18. Continued

- (b) (2 points) Now assume that the pension funding ratio is 1.0.
- Sketch a curve in the following chart to show the expected change in surplus relative to asset value.
 - Determine the equity allocation level at which the expected future surplus starts to decrease.



You choose a 40/60 asset allocation between U.S. Equity and the Bond index. The expected excess return of assets over liabilities, $R_{x,t}$, is given by:

$$E[R_{x,t}] = E\left[\frac{1+R_{A,t}}{1+R_{L,t}} - 1\right] = 2.45\%$$

where $R_{A,t}$ and $R_{L,t}$ are the returns of assets and liabilities at time t , respectively.

Now assume that the pension funding ratio is 0.9 and the plan pays out 7.5% of its liability value per year.

- (c) (2 points) Evaluate whether the pension plan is expected to be underfunded, exactly funded, or overfunded 10 years from now.

****END OF EXAMINATION****
Afternoon Session

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