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**SOCIETY OF ACTUARIES**  
**Enterprise Risk Management – Investment Extension**

# Exam ERM-INV

**Date:** Friday, April 28, 2017

**Time:** 8:30 a.m. – 12:45 p.m.

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## INSTRUCTIONS TO CANDIDATES

### General Instructions

1. This examination has a total of 80 points.

This exam consists of 9 questions, numbered 1 through 9.

The points for each question are indicated at the beginning of the question. Questions 8 and 9 pertain to the extension readings and/or the Case Study, which is enclosed inside the front cover of this exam booklet.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

### Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam ERM-INV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.



## **CASE STUDY INSTRUCTIONS**

**The case study will be used as a basis for some examination questions. Be sure to answer the question asked by referring to the case study. For example, when asked for advantages of a particular plan design to a company referenced in the case study, your response should be limited to that company. Other advantages should not be listed, as they are extraneous to the question and will result in no additional credit. Further, if they conflict with the applicable advantages, no credit will be given.**

**\*\*BEGINNING OF EXAMINATION\*\***

- 1.** (10 points) DML is a large life insurance company and is due to report its economic capital (EC) to the regulators. DML currently offers three main products:
- **Single premium immediate annuity (SPIA)**
    - Level payments are guaranteed for life.
    - The current mortality assumption is based on DML's experience, but the experience data are not fully credible.
    - One set of assumptions is used for the entire block.
  - **5-year deferred annuity**
    - The credited rate is guaranteed for the term of the contract.
    - The death benefit is equal to the book value of the account.
    - There are no surrender charges after 2 years.
  - **Participating whole life insurance**
    - This product provides a level death benefit plus accrued dividend.
    - The dividend is based on the difference between actual rates of mortality and interest versus rates assumed in the contract.
    - There is currently no more room to pass adverse experience to policyholders.
    - This product is sold in three different countries but the mortality assumption is based on the DML's experience in its home country.

The assets supporting the DML liabilities are corporate bonds with an average duration of 10 years. The liability duration is 17 years.

- (a) (2 points) Describe two important considerations for measuring and modeling each of the following risks for DML:
- Mortality risk
  - Interest rate risk.
- (b) (5 points)
- (i) Explain how mortality and interest rate risks interact within each of DML's three main products as well as across products. Use examples to illustrate your response.
  - (ii) Describe the following approaches to aggregate EC:
    - Correlation
    - Copulas
    - Multivariate methods
  - (iii) Recommend an appropriate capital aggregation approach for determining the DML's EC. Justify your answer.

## 1. Continued

- (c) (3 points) You are reviewing DML's model governance procedures. All you found was evidence of the following three activities:
- Model output should be validated by comparing the projected premiums and benefits from the EC model with those from the cash flow testing model for a set of ten random policies.
  - All individual risk factors should be validated independently and in aggregate.
  - Comprehensive documentation of model output should exist to support model validation.
- (i) Evaluate how each of these activities adhere to core model validation principles.
- (ii) Recommend steps that DML should implement to enhance its model governance procedures.

2. (8 points) You are an actuarial student at CMP Financial, an insurance company specializing in fire indemnity insurance. CMP Financial has a stop-loss reinsurance agreement with XYZ Re that reimburses aggregate monthly claims in excess of \$130,000. CMP Financial receives an experience refund at the end of any calendar year in which no reimbursements are paid by XYZ Re.

Your boss, Stan, asks for your assistance with modeling aggregate monthly claims activity. Stan would like you to analyze claims using a Generalized Pareto Distribution (GPD).

In addition to providing summary statistics for historical aggregate monthly claim amounts for the block, Stan has also calculated estimates for the GPD parameters at various threshold percentiles.

Average Aggregate Monthly Claim = \$12,200

Standard Deviation of Aggregate Monthly Claim = \$53,000

Percentile	Aggregate Monthly Claim Amount in \$ million	$\xi$	$\beta$
90 <sup>th</sup>	0.095	0.12	0.05
91 <sup>st</sup>	0.100	0.13	0.05
92 <sup>nd</sup>	0.104	0.13	0.05
93 <sup>rd</sup>	0.110	0.15	0.04
94 <sup>th</sup>	0.115	0.15	0.04
95 <sup>th</sup>	0.123	0.14	0.04
96 <sup>th</sup>	0.134	0.14	0.04
97 <sup>th</sup>	0.147	0.14	0.04

GPD's cumulative distribution function is

$$F(x) = 1 - \left( 1 + \xi \frac{x-u}{\beta} \right)^{-\frac{1}{\xi}}, x \geq u$$

- (a) (2 points)
- (i) Explain considerations when selecting a threshold value for parameterizing a GPD.
  - (ii) Recommend an appropriate threshold level. Justify your response.

## 2. Continued

- (b) (3 points) Assume that aggregate monthly claim amounts are independent of prior months' claim activity. Consider the following two claim models.
- I. The GPD model using the threshold you recommended in part (a)
  - II. A normal approximation

Calculate the probabilities, for each claim model, that:

- (i) No claim reimbursement will be made in a given month.
  - (ii) CMP will receive the experience refund at the end of the upcoming calendar year.
- (c) (3 points) Stan is currently preparing cash flow projections for the upcoming calendar year and would like to incorporate the results of your analysis.
- (i) Explain why the GPD model more appropriately reflects the true likelihood of payment of the experience refund.
  - (ii) Describe two shortcomings of the normal approximation in the context of modeling CMP's aggregate claim distribution.

- 3.** (12 points) You are a new CERA hired by the finance department of Protection Partners (PP), a clearinghouse that trades derivatives and other financial instruments. PP has maintained an AA rating ever since its inception. You are analyzing a new credit derivative issued by XEN Bank that PP wishes to offer.

You are given:

- The annual risk free rate is 5%

ABC Bond

- Issued in \$100 notional amount increments
- 3 year term
- Pays 15% annual coupons
- If the bond defaults, no coupon is paid, and the bondholder immediately receives whatever principal is recovered, terminating the bond contract.
- The probabilities of default are:
  - 5% in each of years 1 and 2
  - 10% in year 3
- The recovery rates given default are:
  - 0% with probability 50%
  - 50% with probability 30%
  - 75% with probability 20%

ABC Credit Derivative

- The derivative is on ABC bonds
- Each \$100 ABC Credit Derivative backs \$100 in ABC bonds
- When an ABC bond defaults, the ABC Credit Derivative will
  - Immediately pay the defaulted coupon
  - Pay any future coupons at their normal due date
  - Pay the unrecovered principal (with no adjustment for interest) at the planned ABC bond maturity.

- (a) (2.5 points) Determine the price of the ABC Credit Derivative at issue assuming no taxes or profit margin. Show your work.



### 3. Continued

- (b) (3.5 points)
- (i) Describe how each of the following credit enhancers mitigates credit risk.
    - I. Collateral
    - II. Netting Agreement
    - III. Walkaway features
  - (ii) Explain whether the credit enhancers listed above are appropriate for the purchaser of the ABC Credit Derivative contract to use with PP.
- (c) (3.5 points)
- (i) Define Potential Future Exposure (PFE) and Expected Positive Exposure (EPE).
  - (ii) Identify which of PFE or EPE is more appropriate for capital purposes.
  - (iii) From PP's perspective:

Determine the PFE to PP from the ABC Credit Derivative assuming a 100% confidence level and a 0% interest rate. Explain your reasoning.
  - (iv) From the ABC Credit Derivative owner's perspective:

Determine the PFE from the ABC Credit Derivative assuming a 100% confidence level and a 0% interest rate. Explain your reasoning.
- (d) (1.5 points) PP is putting together a brochure to help with understanding the ABC Credit Derivative. You have been asked to draft the answer to the following question in the brochure:
- “Q: For this product, you list a value for the Potential Future Exposure that is very different from the Expected Positive Exposure. Which value should matter more to an investor?”
- Prepare an answer to this question indicating which one is more important.
- (e) (1 point) Other than minimizing their credit exposure to ABC, explain how ABC bondholders could benefit from owning the ABC Credit Derivative.

4. (9 points) You are given the following capital information on company XYZ for its three lines of business.

Line of Business	Stand-Alone Capital (\$ million)
Annuities	20
Life	6
Auto	14

A matrix for the risk correlation between lines of business is provided below.

	Annuities	Life	Auto
Annuities	1	0.8	0.4
Life	0.8	1	0.5
Auto	0.4	0.5	1

- (a) (2 points) Demonstrate that the diversification benefit for XYZ is \$6.35 million. Assume that risks within lines of business are normally distributed.
- (b) (4 points) Calculate the capital allocation using the following two methods:
- I. Pro-rata
  - II. Discrete marginal contribution
- (c) (3 points)
- (i) Compare the following capital allocation approaches:
    - I. Standalone
    - II. Pro-rata
    - III. Discrete marginal contribution
  - (ii) Recommend a capital allocation method for XYZ. Justify your response.

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**5.** (7 points) You are asked to calculate economical capital (EC) for the Long Term Care (LTC) business of the XYZ Insurance Company. This block of business has the following characteristics:

- Level premium for life
- Policy pays certain qualified expenses incurred to assist with activities of daily living
- Liability duration is approximately 40 years at issue
- Supporting assets are invested in government securities and high-yield corporate bonds
- Interest rate swaps and options are used to manage interest rate risk
- Economic Liability is based on best estimate assumptions.

(a) (2 points) You are evaluating the following risk categories as related to the LTC business:

- I. Insurance Risk
- II. Credit Risk
- III. Market Risk

- (i) Describe the specific risks in each of above categories.
- (ii) Rank the risks. Justify your response.

(b) (2 points) XYZ uses the deterministic stress tests to calculate EC.

Describe how you would select shocks for a deterministic stress test.

## 5. Continued

(c) (3 points) XYZ uses the following deterministic shocks:

- +/- 1% parallel shift in interest rates
- +/- 20% in lapse rates in all durations
- 10% increase in claims

XYZ's Chief Risk Officer (CRO) commented that:

- I. Interest rate shock may not capture the inherent interest risk in this line of business.
- II. The shock design seems overly simplistic to reflect all the material risks.

- (i) Explain the CRO's concerns.
- (ii) Provide recommendations to address the CRO's concerns.

- 6.** (8 points) LMN Insurance Company, a BBB-rated insurer, plans to expand into a new market. The CFO of LMN is exploring the possibility of securing additional capital using Special Purpose Vehicles (SPV) to fund the expansion. The CFO thinks that the current financing rates are too high for what he thinks is a very low risk venture.

Major rating agencies recently changed LMN's credit rating outlook to negative.

LMN holds high-quality commercial real estate investments in a fast-growing city. This accounts for more than half of its asset portfolio.

The CFO suggests using the real estate portfolio to collateralize the bonds issued by an SPV to finance the expansion.

- (a) (0.5 point) Define an SPV.
- (b) (3 points) Illustrate how LMN can structure this SPV to obtain financing.
- (c) (2 points) Identify four of the key risks and four of the key benefits of the SPV transaction for LMN.
- (d) (1 point) Recommend whether LMN should proceed with the SPV transaction based on your response to (c). Justify your response.
- (e) (1.5 points) Describe three other strategic considerations that LMN would need to address prior to initiating the SPV transaction.

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7. (6 points) XYZ is an insurance company that sells life insurance and hurricane insurance. In order to grow, XYZ is investigating selling a new tornado insurance product. For hurricane and tornado insurance combined, XYZ is willing to absorb a loss of \$100 million, which is 10% of expected company-wide profits.

The CFO has asked you to quantify the risk of offering the tornado insurance. An outside consultant provided you with the sales data from several other companies that sell tornado insurance. The data for one company is shown below.

Date	Sales in Millions
Jan. 2010	15.34
Feb. 2011	18.23
Mar. 2012	18.23
Apr. 2013	18.23
May 2014	99.99
Jun. 2015	19.74

- (a) (3 points) You are reviewing ASOP 23 prior to performing your work.
- (i) Describe concerns you have with the data you have been provided.
  - (ii) Identify two key data-related disclosures that you would include as part of your actuarial report on the profitability of the tornado insurance.



## 7. Continued

Your colleague has estimated the 90% VaR for hurricane product's losses over 5 years to be \$60 million; and the corresponding CTE to be \$90 million.

Your calculations for the tornado product's losses show a 90% VaR over 5 years of \$75 million, with a corresponding CTE of \$120 million.

- (b) (1.5 points) Explain whether VaR or CTE is the more appropriate risk measure to use in this situation.
- (c) (1.5 points) The CFO has directed you to use the 90% VaR measure with complete independence between hurricane and the tornado products.

Recommend whether or not XYZ should offer the tornado product. Justify your response.

**Questions 8-9 pertain to the Case Study and/or extension readings.  
Each question should be answered independently**

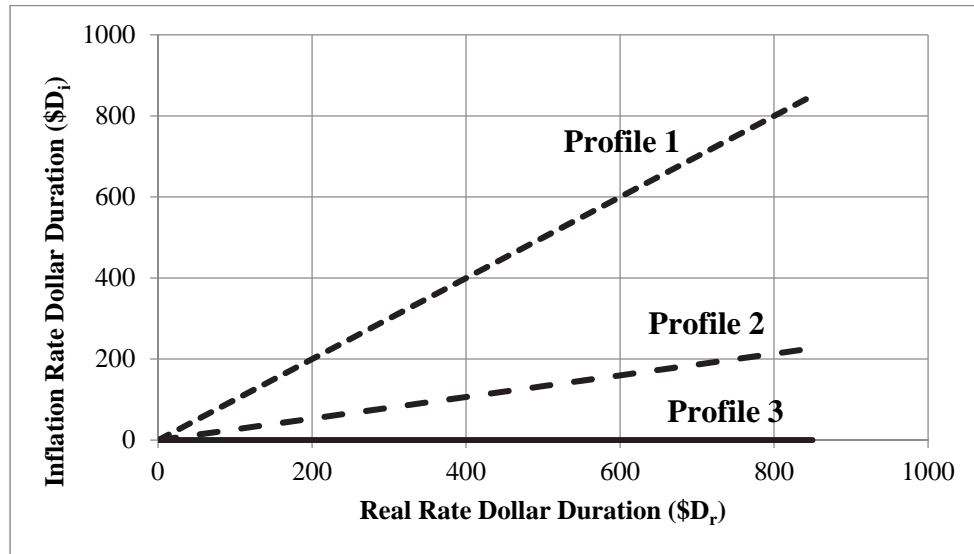
**8.** (11 points) Denise Henning, VP, HR at SLIC has asked you to explain aspects of a liability-relative investing presentation she received from a consultant. She would like your opinion on whether this is something SLIC should consider for its DB Pension Plan.

The presentation starts with an illustration of a simplified DB plan seeking to immunize interest rate risk, with assets allocated to 60% equity and 40% nominal bonds. Liabilities total \$100 million and have a nominal duration of 9.8. Assets total \$80 million with equity having an implied nominal duration of 3 using statistical properties, and nominal bonds having a nominal duration of 20.

- (a) (2 points)
- (i) Demonstrate whether the illustrated plan is duration matched.
  - (ii) Demonstrate whether the plan is dollar-duration matched.
  - (iii) Explain why dollar-duration matching should be the preferred objective.

The presentation shows that nominal interest rate risk is composed of real interest rate ( $r$ ) risk and inflation ( $i$ ) risk, resulting in “dual duration” components  $D_r$  and  $D_i$ . The presentation also introduces inflation-indexed bonds called Treasury Inflation-Protected Securities (TIPS) as an additional asset class.

The presentation includes the following graph of the three different dual dollar-duration profiles ( $\$D_i$ ,  $\$D_r$ ) for equity, nominal bonds, and TIPS:



**8. Continued**

- (b) (1.5 points) Match each of the three assets classes: equity, nominal bonds and TIPS, with one of the dual dollar-duration profiles in the graph. Justify your response.

The presentation next demonstrates dual dollar-duration matching for two different types of plan liabilities using the asset classes: equity, nominal bonds, and TIPS.

- I. Plan X has a dual-duration pair of (15, 1)
- II. Plan Y has a dual-duration pair of (15, 4)

- (c) (1.5 points) Identify which dual-duration pair best matches SLIC’s DB Pension Plan liabilities. Justify your response.

For the Plan Y example, liabilities are assumed to be \$100 million, with plan assets totaling \$80 million. It is now assumed that 60% of assets are allocated to equity with dual durations of (20, 4), and the remaining assets are allocated between nominal bonds and TIPS to simultaneously immunize plan surplus.

Part of the solution “family” for a 40% fixed income (nominal bonds, TIPS) allocation is as follows:

Nominal Bond Duration	Nominal Bond %	TIPS Real Rate Duration	TIPS %
	43.3%		
	20.0%		20.0%
		16.8	
			33.5%

- (d) (3.5 points) Calculate the missing values in the table. Show your work.
- (e) (1 point) Select one of the four Nominal Bond/TIPS allocations as being the most practical to implement. Justify your choice.
- (f) (1.5 points) Explain whether you think SLIC should pursue dual duration analysis to assist in setting its DB Plan asset allocation strategy.

**Questions 8-9 pertain to the Case Study and/or extension readings.  
Each question should be answered independently**

**9.** (9 points) Two alternative bond portfolio strategies are being considered to back SLIC's SPIA product.

- Portfolio A is a diversified portfolio of Corporate A bonds.
- Portfolio B is a diversified portfolio of Corporate BBB bonds.

You have been asked to analyze the two bond portfolios from a market risk/reward perspective. To simplify your analysis, you focus on a small set of three interest rate risk factors: Treasury zero-coupon bond yields at maturities of 3 years, 7 years and 20 years. You compile the following information:

Maturity Term (Years)	Treasury Zero Yield	Yield Volatility	Modified Duration D*	Correlation			Variance-Covariance		
				3	7	20	3	7	20
3	1.5%	1.06%	2.96	1.00	0.97	0.83	0.0112%	0.0101%	0.0077%
7	2.3%	0.98%	6.84	0.97	1.00	0.92	0.0101%	0.0096%	0.0079%
20	2.8%	0.88%	19.46	0.83	0.92	1.00	0.0077%	0.0079%	0.0077%

You note the linear relationship between returns volatility and yield volatility:

$$\sigma\left(\frac{dP}{P}\right) = |D^*| \times \sigma(dy)$$

- (a) (1.5 points) For each of the three treasury zero-coupon maturity terms:
- (i) Calculate the 1-year Yield VaR at the 95% confidence level.
  - (ii) Calculate the 1-year Returns VaR at the 95% confidence level.

Show your work.

## 9. Continued

You use a cash-flow mapping process that groups the SPIA liability cash flows to the three Treasury zero-coupon maturity points. You produce the following table of values for a market value of \$100 million of the SPIA liability:

Liabilities (\$millions)			
MaturityTerm (Years)	MV	Cash Flow	Cash Flow Map PV
3	\$40		\$40.00
7	\$33		\$33.00
20	\$27		\$27.00
Total	\$100		\$100.00

(b) (0.5 points)

- (i) Derive the liability cash flows that were mapped to each treasury zero-coupon maturity term.
- (ii) Calculate the SPIA liability modified duration.

Show your work

Similarly, you cash-flow map the asset cash flows for a \$100 million market value portfolio for each of Portfolio A and Portfolio B as follows:

MaturityTerm (Years)	Portfolio A (\$millions)		Portfolio B (\$millions)	
	MV	Cash Flow Map PV	MV	Cash Flow Map PV
3	\$49	\$50.46	\$19	\$20.14
7	\$25	\$26.76	\$65	\$74.44
20	\$26	\$31.55	\$16	\$23.52
Total	\$100	\$108.78	\$100	\$118.10

*Question 9 is continued on next page.*

## 9. Continued

You generate partial results for 1-year Yield VaR and 1-year Returns VaR at the 95% confidence level as follows:

	Liabilities		Portfolio A		Portfolio B	
	1-yr Yield VaR(95%)	1-yr Returns VaR(95%)	1-yr Yield VaR(95%)	1-yr Returns VaR(95%)	1-yr Yield VaR(95%)	1-yr Returns VaR(95%)
Total (millions)	\$1.574	\$13.686	\$1.713	\$14.902		

(c) (2 points)

- (i) Calculate the modified duration of Portfolio A and Portfolio B.
- (ii) Calculate the 1-year Yield VaR for Portfolio B at the 95% confidence level.
- (iii) Calculate the 1-year Returns VaR for Portfolio B at the 95% confidence level.

Show your work.

You determine that the effective convexity, spread duration and spread volatility for the liabilities and each of the alternative bond portfolios are as follows:

	Effective Convexity	Spread Duration	Spread Volatility
Liabilities	65.03	0.00	0%
Portfolio A	67.16	9.08	35%
Portfolio B	57.60	7.99	30%

You refine the  $VaR(dP)$  measure to incorporate the following convexity (C) adjustment:

$$-\left(\frac{1}{2}CP\right)VaR(dy)^2$$

The convexity-adjusted 1-year  $VaR(dP)$  at the 95% confidence level for the SPIA liabilities is \$12.880 million, and for Portfolio B is \$14.965 million.

## 9. Continued

Assume that:

- Portfolio A has a static spread of 1% over the Treasury zero-coupon bonds.
- Portfolio B has a static spread of 2% over the Treasury zero-coupon bonds.
- The required return on the liabilities equals the expected return under the Treasury zero-coupon bond cash-flow map.

(d) (3 points)

- (i) Calculate the 1-year 95%  $VaR(dP)$  for Portfolio A incorporating the convexity adjustment.
- (ii) Calculate the excess return of the bond portfolio over the liabilities for each of Portfolio A and Portfolio B.
- (iii) Calculate the 1-year 95% VaR due to credit spreads for each of Portfolio A and Portfolio B.

Show your work.

- (e) (2 points) Recommend one of Portfolio A or Portfolio B as providing better market risk/reward merits relative to the SPIA liabilities portfolio. Justify your choice on the basis of your analysis.

**\*\*END OF EXAMINATION\*\***

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