SOCIETY OF ACTUARIES
Quantitative Finance and Investment Advanced Exam

Exam QFIADV

AFTERNOON SESSION

Date: Thursday, April 27, 2017
Time: 1:30 p.m. – 3:45 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This afternoon session consists of 6 questions numbered 10 through 15 for a total of 40 points. The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

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Exam QFIADV

****BEGINNING OF EXAMINATION****
Afternoon Session
Beginning with Question 10

10. (*7 points*) ABC Life offers an individual variable annuity (VA) product with two different types of Guaranteed Lifetime Withdrawal Benefit (GLWB) riders:

- Type I is a GLWB that provides an annual look-back ratchet. At issue, the withdrawal benefit base is given by the single premium paid. During the contract term, on each policy anniversary date, the withdrawal benefit base is increased to the account value if the account value exceeds the previous withdrawal benefit base.

- Type II is a GLWB that provides a constant withdrawal benefit base regardless of market movement.

For both rider types, the initial withdrawal benefit base is set to the same premium paid. The fair guaranteed annual withdrawal rate is a percentage, z_{WL}, which when multiplied by the guaranteed withdrawal amount, makes the contract fair at inception. Three Monte-Carlo simulation runs were performed to derive the value of z_{WL} under the same set of capital market and actuarial/behavior assumptions, except that:

Run A: volatility is 18%, GLWB type is Type II
Run B: volatility is 18%, GLWB type is Type I
Run C: volatility is 24%, GLWB type is Type I

(a) (*1 point*) Explain the rationale of using dynamic lapse assumptions in GLWB product pricing.

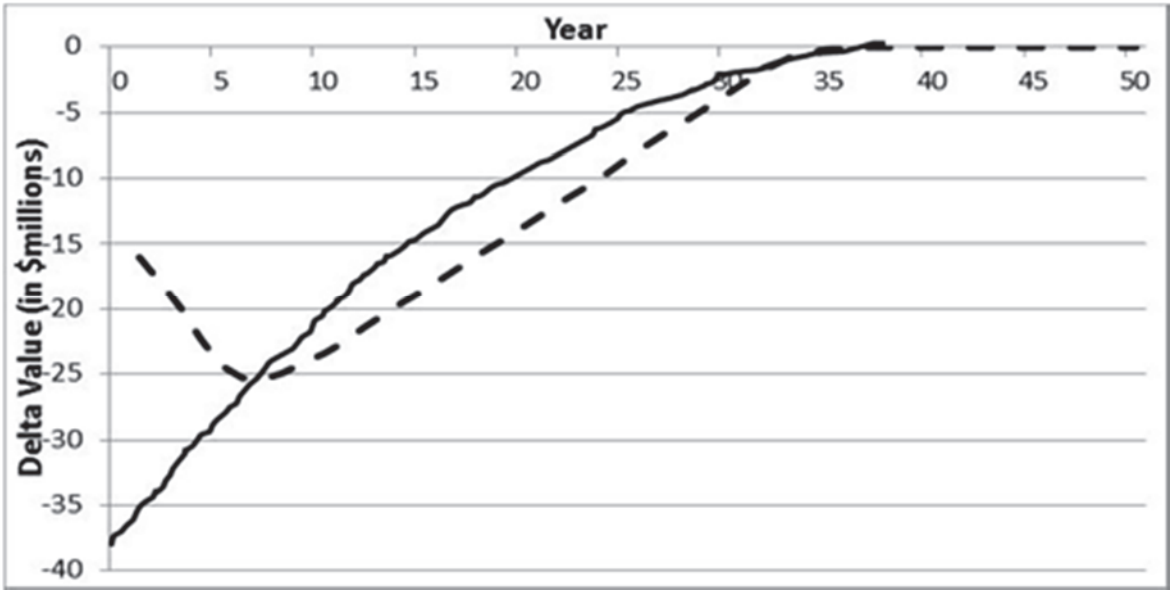
(b) (*1 point*) Outline challenges in managing policyholder behavior risk that may make it difficult to hedge against.

(c) (*2 points*) Determine a ranking of z_{WL} from these three simulations in order of lowest to highest. Justify you answer.

A Monte-Carlo simulation using a real-world equity distribution on a pool of identical policies sold to 60-year-old male with 0% surrender has been performed.

10. Continued

The chart below shows the development over time of the arithmetic average GLWB deltas multiplied by the underlying account value. The solid line and the dotted line each correspond to one of the ABC Life GLWB types (Type I or Type II).



(d) (*1 point*) Justify the GLWB type (type I or II from above) that corresponds to each line on the graph.

ABC Life is setting a delta and vega hedging program using the Black-Scholes model to determine delta and vega. Hedge positions are established and rebalanced accordingly and you must assume that the actual fund performance follows a Heston model.

Your colleague claims that the following are the specifications and characteristics of the fund performance model:

- The model assumes stochastic volatility;
- Is driven by one stochastic process (S(t));
- $S(t) = S(0)\exp\left(\left(\mu - \frac{\sigma_H^2}{2}\right)t + + \sigma_H W(t)\right)$
- The transformation from the real world probability measure (P) into an equivalent local martingale measure (Q) is not unique.

(e) (*2 points*) Critique each of the claims made by your colleague.

11. (*7 points*) You are given the following for a one-year par bond:

One-year risk free rate = 3.75%
Coupon = 5.47% (entirely received on maturity)
Actual 1 year survival probability = 0.985
Expected present value of the one-year par bond with actual default probability = 101
Recovery rates are the same under both real-world and risk-neutral measures

(a) (*2 points*) Calculate the risk premium, defined as the difference between the risk neutral and actual default probabilities, implied by the one-year par bond.

You are responsible to evaluate the reliability of default frequencies of 4 risky rating classes which have been calculated off of a multi-year experience study of default probabilities. You have made two assumptions regarding your data:

1. the Markov property - the assumption that credit migration is independent of past credit migration given the current credit state and

2. Homogeneity - migration rates are independent of time.

A one year migration matrix M (with each row corresponding to an increasingly risky rating class) has been prepared for you and contains the following properties: M has all non-negative entries, all rows sum to 1, the last column contains 1 year default properties, and the default state is absorbing.

$$M_{data} = \begin{vmatrix} 0.75 & 0.20 & 0.05 & 0.00 & 0.00 \\ 0.08 & 0.69 & 0.06 & 0.10 & 0.07 \\ 0.09 & 0.11 & 0.65 & 0.10 & 0.05 \\ 0.00 & 0.00 & 0.04 & 0.60 & 0.36 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{vmatrix}$$

(b) (*3 points*) Identify in M_{data} all areas which do not pass plausibility constraints.

11. Continued

Starting with single period M_{data} matrix above you wish to develop a generator or Q_{data} matrix for valuation. Your team has proposed 3 different generator matrices with the following $\exp(Q_{data})$ matrices:

$$\exp(Q_{data})_A = \begin{vmatrix} 0.76 & 0.20 & 0.04 & 0.00 & 0.00 \\ 0.10 & 0.70 & 0.10 & 0.06 & 0.04 \\ 0.09 & 0.11 & 0.65 & 0.10 & 0.05 \\ 0.00 & 0.00 & 0.04 & 0.62 & 0.34 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{vmatrix}$$

$$\exp(Q_{data})_B = \begin{vmatrix} 0.75 & 0.15 & 0.08 & 0.02 & 0.00 \\ 0.10 & 0.69 & 0.14 & 0.07 & 0.00 \\ 0.09 & 0.11 & 0.65 & 0.10 & 0.05 \\ 0.00 & 0.00 & 0.04 & 0.60 & 0.36 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{vmatrix}$$

$$\exp(Q_{data})_C = \begin{vmatrix} 0.75 & 0.20 & 0.05 & 0.00 & 0.00 \\ 0.08 & 0.69 & 0.06 & 0.10 & 0.07 \\ 0.09 & 0.11 & 0.65 & 0.10 & 0.05 \\ 0.00 & 0.00 & 0.16 & 0.60 & 0.24 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{vmatrix}$$

(c) (*2 points*) Determine which Q matrix generated would be the best approximation, using an L^1 Norm computation.

12. (*6* points) You are an investment actuary working for XYZ insurance. The investment department is considering three potential investments:

- An S&P 500 index fund.
- An investment in a fund that tracks the National Council of Real Estate Investment Fiduciaries National Property Index (NCREIF NPI), and
- An investment in a fund that tracks the FTSE National Association of Real Estate Investment Trusts U.S. Real Estate Index (NAREIT REIT index).

You are given the following:

Investment (i)	Year 1	Year 2	Year 3	μ_i	σ_i	Sharpe Ratio	Autocorrelation
S&P 500 Index	12%	4%	8%	6.00%	4.00%	150.00%	0
NCREIF NPI	3%	6%	6%	3.00%	1.73%	173.21%	0.5
NAREIT Index	3%	10%	6%	4.33%	3.51%	123.39%	-0.08
Risk Free Rate	2%	2%	2%	0.00%	0.00%		

μ_i= Mean of the excess return of investment i over the risk free rate

σ_i = Standard deviation of the excess return of investment i over the risk free rate

Assume returns in the year before Year 1 were the same as in Year 1.

The investment department is relying on the Ex-Post Sharpe ratio relative to the risk free rate to select which investment to make.

(a) (*5 points*) Recommend one of the three investments and justify your recommendation.

(b) (*1 point*) Explain why price smoothing can be a problem for risk and portfolio management.

13. (*7 points*) Rebonato's Volatility and Correlation described the features and methods of smile-modeling approaches. Rebonato gives four categories of models for stock prices that give rise to a volatility smile:

1. Fully-stochastic-volatility models
2. Local-volatility (restricted-stochastic-volatility) models
3. Jump-diffusion models
4. Variance-gamma (pure jump) models

(a) (*2 points*) For a fully-stochastic-volatility model category:

(i) Describe this model category.

(ii) Explain why this model category gives rise to a smile.

(b) (*2 points*) For a local-volatility (restricted-stochastic-volatility) model category:

(i) Describe this model category.

(ii) Explain why the market is complete under this model.

One of the most salient empirical features of equity volatility surfaces is that short maturities display pronounced smiles, while distant maturities display shallow smiles.

(c) (*1 point*) Explain two processes that are capable of producing this empirical behavior.

Volatility smile models must be calibrated (fitted) to market inputs.

(d) (*1 point*) Contrast the advantages and disadvantages of calibrating to pure market prices as opposed to applying pre-processing to the inputs before calibrating the model.

An actuary is attempting to calibrate a complex volatility smile model with many parameters, for real estate prices. The model will be calibrated using historic asset prices and option prices from the small real estate derivatives market.

(e) (*1 point*) Recommend whether pure market prices or pre-processed market inputs should be used for calibration.

14. (*5 points*) You have gathered information on prices of 1 year European call options at different strike prices over a range of underlying stock prices. The information was recorded over a short period of time assuming that the underlying volatility function did not change.

Current stock Price	99
Interest rate	3%
Value of at the money option	9.32

Implied Volatilities

Stock Price	Strike Price		
	99	**100**	**101**
99	20.0%	19.8%	19.5%
100	20.3%	20.0%	19.8%
101	20.5%	20.2%	20.0%

Delta (based on implied volatilities)

Stock Price	Strike Price		
	99	**100**	**101**
99	0.5987	0.5792	0.5591
100	0.6175	0.5987	0.5794
101	0.6355	0.6174	0.5987

Vega (based on implied volatilities)

Stock Price	**Strike Price**		
	99	**100**	**101**
99	38.28	38.72	39.06
100	38.15	38.67	39.10
101	37.95	38.53	39.05

(a) (*1 point*) Describe the characteristics of the observed implied volatility.

The stock price has increased to 100.

(b) (*1 point*) Calculate the value of the at-the-money option.

You intend to delta-hedge the at-the-money option based on the Black formula and the at-the-money implied volatility.

(c) (*1 point*) Assess the appropriateness of this approach.

(d) (*1 point*) Propose an alternative delta hedging strategy.

(e) (*1 point*) Calculate the delta according to the strategy in (d).

15. (*8 points*) Given the current low interest rate environment, the traditionally conservative investment company you work for is looking to take on more risk than the current strategy of investing in high rated corporate bonds. You have been asked to advise on alternative asset classes.

(a) (*1 point*) Identify four alternative assets classes (other than infrastructure) available to investors.

(b) (*1 point*) Describe the investment role that these asset classes typically perform for an institutional investor. Note: Relevant issues may include diversification, return enhancement, and inflation.

Your company has two clients:

Client 1
A large educational endowment which has a well-staffed investment research department. This client wants to grow the endowment, and is not concerned about short term liquidity.

Client 2
A wealthy individual who currently uses a single trusted investment advisor. The individual has a desire to protect their wealth from inflation and 20% of their wealth is tied up in the family business.

For each client, you have been asked to recommend two alternative asset classes to add to their basic stock and bond portfolios. Your recommendation must discuss appropriate investment vehicles within each asset class, approximate allocations that may be appropriate, and any advantages and disadvantages that each asset class would present.

(c) (*3 points*) Outline your recommendations.

You have now been asked to review the risk-return profile for investing in infrastructure investments.

(d) (*1 point*) Explain emerging issues related to the risk-return profile of infrastructure investments.

(e) (*2 points*) Identify five risks relating to infrastructure investments, noting how applicable they are to each client.

****END OF EXAMINATION****
Afternoon Session

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