

ERM-INV Model Solutions

Spring 2017

1. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

Learning Outcomes:

- (2b) Evaluate how risks are correlated, and give examples of risks that are positively correlated and risks that are negatively correlated.
- (2g) Analyze and evaluate model and parameter risk.
- (2h) Construct approaches to modeling various risks and evaluate how an entity makes decisions about techniques to model, measure and aggregate risks including but not limited to stochastic processes.

Sources:

Financial Enterprise Risk Management, Sweeting, 2011, Ch. 14 Quantifying Particular Risks

ERM-106-12: Economic Capital-Practical Considerations - Milliman

ERM-119-14: Aggregation of risks and Allocation of Capital (Sections 4-7)

ERM 602-12: Investment Management for Insurers, Babbel and Fabozzi, Ch. 11 The Four Faces of an Interest Model

ERM-118-14: Model Validation Principles Applied to Risk and Capital Models in the Insurance Industry

Commentary on Question:

Part (a) was generally answered well by most candidates.

For Part (b)(i), some candidates provided a similar answer to part (a) instead of focusing on the interaction between interest rate and mortality risks for each product as well as across products. Here credit was only given if the candidate explained the interaction between risks and not solely reiterating the source of risk.

Many candidates failed to discuss the risk interaction across products.

1. Continued

Candidates only received full credit when answers given were specific to DML.

For Part (b)(iii), candidates were given substantial credit for choosing any of the three options given as long as the supporting justification was deemed sufficient.

In Part (c)(i), many candidates failed to identify the weaknesses in each of the three activities mentioned, and instead commented on how each item aligned with best practices in model validation.

Solution:

- (a) Describe two important considerations for measuring and modeling each of the following risks for DML:
- Mortality risk
 - Interest rate risk.

Mortality risk

Whole Life - May perform experience studies by geography to assess if mortality level risk differs by country.

SPIA - Reflect future mortality improvement. Mortality trend risk is a major concern for annuity products.

Interest rate risk

Whole Life - Use equilibrium interest rate models under realistic scenarios to stress general account asset/liability cash flows and reflect future dividends arising from interest rate gains/losses.

Deferred Annuity - With rising interest rates, for accounts older than 2 years DML is exposed to surrender risk because customers will be looking to get a better deal/higher credited rate with the competition. Use dynamic policyholder behavior modeling.

1. Continued

(b)

- (i) Explain how mortality and interest rate risks interact within each of DML's three main products as well as across products. Use examples to illustrate your response.
- (ii) Describe the following approaches to aggregate EC:
 - Correlation
 - Copulas
 - Multivariate methods
- (iii) Recommend an appropriate capital aggregation approach for determining the DML's EC. Justify your answer.
- (i) Whole Life - Catastrophic short term mortality and persistent low level of short term rates may create large losses arising from death benefits and cash dividends. This may be partially offset by mortality gains from the SPIA portfolio.

Deferred Annuity - Catastrophic mortality and higher short term rates will create liquidity problems due to higher than expected surrenders.

SPIA - Persistent low interest rates and mortality improvement (trend risk) will adversely affect the DML annuity portfolio. Improvement in trend mortality may allow DML to increase dividends for WL product eventually and pass through future mortality losses to policyholders.

Depending on the level of the mis-estimation of mortality levels, Annuity and Whole Life may offset the impact given their respective liability duration.

- (ii)
 - Correlation - Risks are aggregated using the following formula: Total risk = $(\sum_i \sum_j \rho_{ij} X_i X_j)^{1/2}$. The capital requirements may be aggregated across lines of business and risks using this formula. The capital amounts are first summed for each line and risk. This would be the capital requirement if one did not allow for the diversification effect. A correlation matrix is specified for the correlations between risks, and this is used to calculate new totals for each line of business. The capital requirements are aggregated across the lines using a different correlation matrix.

1. Continued

- Copulas - The Copula function can be specified such that the interaction between risks differs at different parts of each of their respective distributions. In practice, marginal risk distributions are transformed to a uniform distribution on the interval $[0,1]$. A uniform random variable is simulated for each risk distribution. The cumulative distribution of each risk is then calculated. The interaction between risks is captured by applying the specified Copula function to the results.
 - Multivariate methods - When using multiple risk factor stresses, the resulting economic capital includes the impact of risk aggregation directly and all risk factors are included. Risk aggregation is implicit in the ESG models and the models are interrelated - either directly or indirectly through risk factor correlations.
- (iii) Use Multivariate Methods to reflect multi-factor stresses. This method will incorporate the various interactions for mortality, interest, and other risks across DML's LOBs. An example of a stress scenario would be a reduction of baseline long-term rate/a percentage increase in mortality trend. ESG might also include policyholder behavior such that the scenario produces higher capital requirements for SPIA but also lower requirement for Whole Life/Deferred Annuity (diversification effect).
- (c) You are reviewing DML's model governance procedures. All you found was evidence of the following three activities:
- Model output should be validated by comparing the projected premiums and benefits from the EC model with those from the cash flow testing model for a set of ten random policies.
 - All individual risk factors should be validated independently and in aggregate.
 - Comprehensive documentation of model output should exist to support model validation.
- (i) Evaluate how each of these activities adhere to core model validation principles.
- (ii) Recommend steps that DML should implement to enhance its model governance procedures.

1. Continued

(i)

- Risk models are designed to evaluate loss and profitability at predetermined level and may not be precise enough at a granular level. Valuation and risk models are not designed for the same purpose and authenticating a risk model for an alternate purpose may lead one to draw inappropriate conclusions about model accuracy.
- Not all risks are material. It may be impractical and time consuming to validate every aspect of the model. The materiality of each risk should be assessed and more effort should be spent validating material risks.
- While model validation needs to be documented in order to provide assurance that it was adequately performed and that there exists an accountable validation owner, prudence should govern when determining documentation standards. Voluminous documentation of model output to support model validation may be at odds with validation's primary purpose.

(ii)

- Establish an owner of model validation. A single individual should be held accountable, be able to escalate related concerns, and resolve issues arising from the validation process.
- Ensure appropriateness of established model governance.
- Ensure that model validation is an independent process.
- Address limitations of model validation. Its objective is to limit the risk of improper use of the model and provide stakeholders a level of comfort as to the credibility of the model results.

2. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

Learning Outcomes:

- (2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas.
- (2e) Evaluate the theory and applications of extreme value theory in the measuring and modeling of risk.
- (2f) Analyze the importance of tails of distributions, tail correlations, and low frequency/high severity events.

Sources:

Value-at-Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion Ch. 5 Computing VaR, Sections 5.1-5.3, including appendices

Article: Modeling Tail Behaviour with Extreme Value Theory, Risk Management, Sept 2009

Financial Enterprise Risk Management, Sweeting, 2011, Ch. 12 Extreme Value Theory

Commentary on Question:

This question aimed to test the candidate's understanding of the GPD model and parameterization techniques. Candidates were asked to compare probabilistic calculations performed using a GPD to calculations based on a normal approximation in order to highlight the fact that the normal approximation tends to understate risk when tails of the actual distribution are considered "heavy".

Overall, this question was considered a fairly straight-forward one. And, especially because there was a similar question offered in previous year's exam, candidates were expected to be able to provide answers that demonstrated a good level of comprehension and ability to communicate through writing effectively.

Solution:

- (a)
 - (i) Explain considerations when selecting a threshold value for parameterizing a GPD.
 - (ii) Recommend an appropriate threshold level. Justify your response.

2. Continued

Commentary on Question:

The majority of candidates explained Part (a)(i) in terms of the appropriateness of data. Additional points were given to those who also considered the stability of parameters in their statements. Candidates who showed clarity in their answers were awarded with higher points than those who were not able to do so.

For Part (a)(ii), considering that there might be some ambiguity of what is meant by the parameters "becoming and remaining stable", partial credits were awarded to other threshold recommendations given that a comparable justification was provided.

(i)

Threshold is too high:

- There is sparse data with which to parameterize the model, though data used to parametrize the model are truly 'extreme'. (Appropriateness of Data)
- Variance of parameter estimates tends to increase as threshold increases. (Stability of Parameters)

Threshold is too low:

- Model will be parameterized based on values that are not truly extreme/not just the tail is being considered. (Appropriateness of Data)
- Variance of parameter estimates decreases as more data become available to parametrize the model. (Stability of Parameters)

(ii) The 95th percentile (\$0.123M) would be an appropriate threshold choice.

Threshold should be selected as the first candidate point where the estimated parameters become, and remain, stable. The stability will begin to wane as the threshold becomes large enough to significantly shrink the count of data values in the associated tail - so there is a sort of "window of stability".

(b) Calculate the probabilities, for each claim model, that:

- (i) No claim reimbursement will be made in a given month.
- (ii) CMP will receive the experience refund at the end of the upcoming calendar year.

2. Continued

Commentary on Question:

Math results in Part (b) needed to be reasonable and, if not, candidates should have noted that it wasn't a reasonable answer as a way to communicate their understanding and demonstrate their knowledge. For example, many candidates were struggling with Part (b)(i) – They did not recognize the conditional aspect of GPD and, as such, had not arrived at a reasonable answer and mistakenly used $F(0.130)$ as their final answer; candidates were expected to know that 0.1588 is not a reasonable answer for Part (b)(i).

As the answers to Part (b) vary depending on the threshold chosen in Part (a), full credit was given for consistent responses.

- I. The GPD model using the threshold you recommended in part (a)
- II. A normal approximation

(i)

$$\begin{aligned} \text{I) } P\{X \leq 0.130\} &= (1 - P\{X \leq 0.123\}) F(0.130) + P\{X \leq 0.123\} \\ &= (1 - 0.95) * F(0.130) + 0.95 \\ &= 0.05 * (1 - (1 + 0.14(0.130 - 0.123) / 0.04)^{-1 / 0.14}) + 0.95 \\ &= 0.05 * (0.1588) + 0.95 \\ &= 0.9580 \end{aligned}$$

$$\begin{aligned} \text{II) } P\{X \leq 0.130\} &= N[(0.130 - 0.0122)/(0.053)] \\ &= N[2.22] \\ &= 0.9868 \end{aligned}$$

(ii)

$$\begin{aligned} \text{I) } P\{\text{Experience Refund}\} &= P\{X \leq \$130,000 \text{ for at least one month in the} \\ &\text{next 12 months}\} = P\{\text{No loss in excess of } \$130,000 \text{ in next 12 months}\} \\ &= (0.9580)^{12} = 0.5976 \end{aligned}$$

$$\begin{aligned} \text{II) } P\{\text{Experience Refund}\} &= P\{X \leq \$130,000 \text{ for at least one month in the} \\ &\text{next 12 months}\} = P\{\text{No loss in excess of } \$130,000 \text{ in next 12 months}\} \\ &= (0.9868)^{12} = 0.8526 \end{aligned}$$

- (c) Stan is currently preparing cash flow projections for the upcoming calendar year and would like to incorporate the results of your analysis.
- (i) Explain why the GPD model more appropriately reflects the true likelihood of payment of the experience refund.
 - (ii) Describe two shortcomings of the normal approximation in the context of modeling CMP's aggregate claim distribution.

2. Continued

Commentary on Question:

Part (c) is where candidates showed the most differentiation between each other. Candidates who repeated their answers in both Part (c)(i) and Part (c)(ii) were only given grading points for the part where their answers seemed appropriate.

The question in Part (c)(ii) asks for shortcomings of the normal distribution in the context of CMP's loss distribution. Thus, a connection to the problem stem and subsequent analysis is required for full credit. Candidates received grading points for the best 2 shortcoming descriptions, if placed in proper context, since Part (c)(ii) specifically asked for 2 shortcomings only.

Candidate' answers should demonstrate the depth of understanding on the statistical characteristics of the distributions, and therefore purely qualitative statements regarding the distributions might not be given full credit. For example, in Part (c)(ii), only stating that the normal distribution understates the probability of extreme events without commenting on the inherent thinness of the normal distribution's tails cannot receive full credit.

(i)

A loss distribution has a fatter tail than the normal distribution would suggest. The GPD more accurately models the tail of the loss distribution and therefore provides a more accurate view of probability of aggregate losses in excess of the reinsurance trigger.

EVT, in general, focuses explicitly on modeling the behavior of the tails.

- Does not attempt to parametrize data below the selected threshold (makes no assumption regarding true distribution of the data).
- Allows for parameterization of the tail function based on historical data.
- Thicker tails of GPD increases the probability that extreme events will occur.

(ii)

The normal approximation, with its thinner tails, would understate the risk that losses exceed the reinsurance threshold, and thus would overstate the probability that the refund would be paid.

Normal approximation is structured on the central limit theorem which deals with parameterizing the average of i.i.d. variables.

- The normal approximation, with its thinner tails, would likely understate probability of extreme events, understate the risk that losses exceed the reinsurance threshold, and thus would overstate the probability that the refund would be paid.

2. Continued

- The normal distribution does not necessarily fit historical data well because it assumes data is symmetric about the mean with relatively thin tails.
- The tails of the normal distribution decay at an exponential rate.
- The normal distribution exists on the domain of negative infinity to positive infinity, whereas an aggregate claims distribution would, theoretically, exist on the domain of 0 to positive infinity.

3. Learning Objectives:

3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.
4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

- (3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.
- (4g) Demonstrate the use of tools and techniques for analyzing and managing credit and counterparty risk.

Sources:

Counterparty Credit Risk, First Edition, Jon Gregory, Chapter 2: Defining Counterparty Credit Risk
Value-at- Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion Ch. 18 Credit Risk Management]

Commentary on Question:

There was a wide range of credits received by candidates for this question. On average, they understood some of the material like credit derivative price and credit enhancers but they need more work on explaining the reasoning for PFE and EFE.

Solution:

- (a) Determine the price of the ABC Credit Derivative at issue assuming no taxes or profit margin. Show your work.

Commentary on Question:

This is a straight forward calculation but there were a wide variety of approaches by candidates. Some candidates received full credit but most of them missed something like default probability, recovery rates, or timing issues. Most candidates received partial credit depending on what they understood and accomplished.

Alternatively, this could have been priced using risk neutral valuation but very few candidates considered this approach as knowledge of this material (credit derivative pricing) is not presumed for this exam.

3. Continued

Value if default in first year (3 coupons plus derivative provided principal):
 $= 15/1.05 + 15 / 1.05^2 + 15/1.05^3 + (0.5 \times 100 + 0.3 \times 50 + 0.2 \times 25)/1.05^3$
 $= 101.32$

Value if default in second year (2 coupons plus derivative provided principal):
 $= 15 / 1.05^2 + 15/1.05^3 + (.5 \times 100 + .3 \times 50 + .2 \times 25)/1.05^3$
 $= 87.03$

Value if default in third year (1 coupon plus derivative provided principal):
 $= 15/1.05^3 + (.5 \times 100 + .3 \times 50 + .2 \times 25)/1.05^3$
 $= 73.43$

Sum across all years of $P(\text{default in year}) \times (\text{Value of Derivative})$
 $= .05 \times 101.32 + .95 \times .05 \times 87.03 + .95^2 \times .1 \times 73.43$
 $= 15.83$

(b)

(i) Describe how each of the following credit enhancers mitigates credit risk.

- I. Collateral
- II. Netting Agreement
- III. Walkaway features

(ii) Explain whether the credit enhancers listed above are appropriate for the purchaser of the ABC Credit Derivative contract to use with PP.

Commentary on Question:

Most candidates did well although many of them received partial credit only for insufficiently explaining how credit enhancers mitigates credit risk and their appropriateness.

- (i)
- I. Collateral
A party that has a credit risk gives money or another asset to the counterparty to "hold onto" until the contract is settled. If a credit loss occurs, the counterparty has the collateral to offset some of the default.
 - II. Netting Agreement
An agreement where a party that owes on some contracts but overall (combining all the contracts with the same counterparty) is owed can avoid paying on the contracts on which he owes, offsetting part of the money that he is owed (reducing any loss on money that is not received).
 - III. Walkaway features
A clause in a contract that allows a party to walk away from a transaction if the counterparty defaults. Thus, if future amounts were to be owed by the party, it would offset the amounts it was owed by the defaulting counterparty.

3. Continued

- (ii)
 - I. Collateral
Collateral is inappropriate since this is a clearing house and PP posting collateral exposes it to much greater credit risk (it might not get the collateral back if it doesn't owe on the contract), and it has a credit rating at AA, so it shouldn't need to post collateral.
 - II. Netting Agreement
Netting would be appropriate since customers will likely have multiple purchases across different products with PP.
 - III. Walkaway features
Walkaway features are inappropriate: the customer would never want to walk away because they would never be in a position under this contract to owe money.

- (c)
 - (i) Define Potential Future Exposure (PFE) and Expected Positive Exposure (EPE).
 - (ii) Identify which of PFE or EPE is more appropriate for capital purposes.
 - (iii) From PP's perspective:

Determine the PFE to PP from the ABC Credit Derivative assuming a 100% confidence level and a 0% interest rate. Explain your reasoning.
 - (iv) From the ABC Credit Derivative owner's perspective:

Determine the PFE from the ABC Credit Derivative assuming a 100% confidence level and a 0% interest rate. Explain your reasoning.

Commentary on Question:

Most candidates explained (i) and (ii) well. Few candidates received any credit for (iii) and (iv) because they did not do what the question asked for: 100% confidence level and a 0% interest rate.

- (i) PFE is a VaR number (albeit a worst-case gain, instead of a worst-case loss), which gives a credit exposure at a given confidence level, which may be more useful for capital purposes.
EPE is an average future positive exposure (reflecting only the cases where a credit loss would occur), which is more useful for pricing.
- (ii) PFE gives a credit exposure at a given confidence level, which may be more useful for capital purposes.

3. Continued

- (iii) There is no credit exposure from the buyer of the Credit Derivative since they are never in a situation where they would need to pay. However, there is credit risk from XEN bank if it fails to pay on its obligations to PP.

The 100% worst case is that ABC defaults immediately, causing the whole potential payment stream of the ABC Credit Derivative to become payable, which is then defaulted on by XEN Bank.

Thus, the total defaulted amount, and thus the PFE, is the three coupons of \$15 plus the \$100 principal, for a total of \$145.

- (iv) The 100% worst case is that the ABC defaults immediately, causing the whole potential payment stream of the ABC Credit Derivative to become payable, which is then defaulted on by PP.

Thus, the total defaulted amount, and thus the PFE, is the three coupons of \$15 plus the \$100 principal, for a total of \$145.

- (d) “Q: For this product, you list a value for the Potential Future Exposure that is very different from the Expected Positive Exposure. Which value should matter more to an investor?”

Prepare an answer to this question indicating which one is more important.

Commentary on Question:

Most candidates received partial points by proposing either EPE or PFE but did not provide sufficient explanation.

The EPE is the much more realistic value that you are exposed to, as the PFE is meant to capture more of the extreme experience. As an investor, you are likely quite diversified and you are more concerned with “average” performance and risk than extreme performance and risk in a single investment. Thus, you likely should be more concerned with the EPE than the PFE.

- (e) Other than minimizing their credit exposure to ABC, explain how ABC bondholders could benefit from owning the ABC Credit Derivative.

Commentary on Question:

Most candidates received partial credit by explaining some reasonable benefits.

3. Continued

If the bond defaults the bondholder immediately receives the recovered principal, instead of waiting till the maturity of the bond. There is an arbitrage opportunity because the bondholder would receive more than the value of the bond alone if it holds the ABC Credit Derivative and the bond defaults. They could be trying to hedge against an entity similar to ABC, which might be subject to the same systematic risk and default at similar times with similar recovery rates (i.e. an inexact hedge). Owning the derivative may reduce some economic capital that the company would otherwise need to hold.

4. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

Learning Outcomes:

- (2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas.

Sources:

ERM-119-14: Aggregation of risks and Allocation of Capital (Sections4-7)

Commentary on Question:

Candidates were expected to demonstrate the ability to calculate capital at the enterprise level, and then allocate back to LOBs based on the main approaches. They were also expected to be able to recognize how diversification impacts each allocation approach, and recommend the approach that most utilizes the diversification benefit.

Solution:

- (a) Demonstrate that the diversification benefit for XYZ is \$6.35 million. Assume that risks within lines of business are normally distributed.

Commentary on Question:

Candidates generally did well on part (a)

$$\text{Sum of Individual Capital Amounts} = 20 + 6 + 14 = 40$$

$$\begin{aligned}\text{Diversified} &= (20^2 + 6^2 + 14^2 + 2 * 0.8 * 20 * 6 + 2 * 0.4 * 20 * 14 + 2 * 0.5 * 6 * 14)^{1/2} \\ &= 33.65\end{aligned}$$

$$\begin{aligned}\text{Diversification Benefit} &= 40 - 33.65 \\ &= 6.35\end{aligned}$$

- (b) Calculate the capital allocation using the following two methods:

- I. Pro-rata
- II. Discrete marginal contribution

Commentary on Question:

Candidates generally did well on part (b). Common sources of error for candidates were forgetting to scale the capital allocations to the diversified capital amount and calculating the non-scaled amount by subtracting from the undiversified capital amount.

- I. Annuities = $(20 / 40) * 33.65 = 16.83$
Life = $(6 / 40) * 33.65 = 5.05$
Auto = $(14 / 40) * 33.65 = 11.78$

4. Continued

- II. Without Annuities = $\sqrt{6^2 + 14^2 + 2 * 0.5 * 14 * 6} = 17.78$
Without Life = $\sqrt{20^2 + 14^2 + 2 * 0.4 * 20 * 14} = 28.64$
Without Auto = $\sqrt{20^2 + 6^2 + 2 * 0.8 * 20 * 6} = 25.06$

$$\begin{aligned}\text{Annuities Weight} &= 33.65 - 17.78 = 15.87 \\ \text{Life Weight} &= 33.65 - 28.64 = 5.01 \\ \text{Auto Weight} &= 33.65 - 25.06 = 8.59 \\ \text{Sum of Weights} &= 15.87 + 5.01 + 8.59 = 29.47\end{aligned}$$

$$\begin{aligned}\text{Annuities Allocation} &= (15.87 / 29.47) * 33.65 = 18.12 \\ \text{Life Allocation} &= (5.01 / 29.47) * 33.65 = 5.72 \\ \text{Auto Allocation} &= (8.59 / 29.47) * 33.65 = 9.81\end{aligned}$$

(c)

- (i) Compare the following capital allocation approaches:

- I. Standalone
- II. Pro-rata
- III. Discrete marginal contribution

- (ii) Recommend a capital allocation method for XYZ. Justify your response.

Commentary on Question:

Candidates generally did well on part (c). Candidates were expected to give a recommendation that was related specifically to XYZ's situation. Several candidates chose to recommend Shapley/Game Theory instead of one of the three approaches given. This answer was acceptable provided it was supported with an acceptable justification.

I) Standalone - Simplest to calculate, doesn't take diversification into account

II) Pro rata - Takes diversification into account, but allocated back to business based on standalone risk instead of recognizing each risk's unique contribution to diversification.

III) Discrete Marginal Contribution - Takes diversification into account, arrives at the same total capital as pro rata, allocates back to business unit while taking into account how the risks interact with each other within the company as a whole

4. Continued

Recommendation: Discrete Marginal Contribution

This method will recognize the unique diversification benefit that each LOB brings. Less capital is allocated to Auto under the discrete marginal allocation, and it should get more of the diversification benefit because it's less correlated with the other two lines of business. Pro rata would not reward the Auto line as much.

5. Learning Objectives:

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.
2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.
5. The candidate will understand the concept of economic capital, risk measures in capital assessment and techniques to allocate the cost of risks within business units.

Learning Outcomes:

- (1a) Explain risk concepts and be able to apply risk definitions to different entities.
- (2d) Apply and analyze scenario and stress testing in the risk measurement process.
- (5b) Define the basic elements and explain the uses of economic capital. Explain the challenges and limits of economic capital calculations and explain how economic capital may differ from external requirements of rating agencies and regulators.
- (5e) Demonstrate the ability to develop a capital model for a representative financial firm.

Sources:

Financial Enterprise Risk Management, Sweeting, 2011, Ch. 7 Definitions of Risk

ERM-106-12: Economic Capital-Practical Considerations-Milliman

Commentary on Question:

This question is to test candidates' ability to analyze and identify the risk associated with a product portfolio, to use a deterministic shock approach to quantify the tail risk. This is intended for candidates to apply the risk management technique to support risk management practice and drive business decisions.

In general, candidates did well on this question. However, the relevant comments pertaining to the specifics of the business scenario were expected and awarded more credits than generic answers. The solution below is more complete than what is necessary for full credit.

5. Continued

Solution:

(a) You are evaluating the following risk categories as related to the LTC business:

- I. Insurance Risk
- II. Credit Risk
- III. Market Risk

- (i) Describe the specific risks in each of above categories.
- (ii) Rank the risks. Justify your response.

Commentary on Question:

The ranking below is for illustration of an acceptable answer. Other rankings are acceptable along with reasonable justification of why certain risks are higher than others.

(i)

Insurance risk

- Morbidity: frequency and severity of the claims, claim trend, cost containment measure and etc. The risk of actual experience is worse than originally assumed.
- Lapse rate or policyholder behavior: Claims usually occurs later as liability duration is so long, thus a lapse supported product. The lower the lapse rate, the higher the expected claims.

Credit Risk

Default or downgrade risk of high-yield corporate bond portfolio that will result in the loss of asset value, especially under stressed market condition. <Note: Market-tradable derivatives are subject to collateral, netting and frequent mark-to-market. The inherent credit risk is low. >

Market Risk

Interest rate risk, yield curve and duration mismatch. Due to the long duration and regular premium payment, the reinvestment and new money rates have significant impact on investment income. A high-yield corporate bond portfolio is especially subject to market volatility.

(ii) Ranking:

Insurance risk is highest, as the morbidity and lapse may present significant risks if the emerging experience deviates from assumed rates, especially in the long term. XYZ has limited ability to adjust the pricing to mitigate the risk.

5. Continued

Market risk is second due to the long duration of the liability and level premium structure. This risk is partially mitigated by interest derivatives, but mismatch may still present a risk, and may subject to the short term market volatility.

Credit risk is ranked last in absence of quantitative information. However it could become significant if the high-yield bond is under stress.

- (b) XYZ uses the deterministic stress tests to calculate EC.

Describe how you would select shocks for a deterministic stress test.

Commentary on Question:

Mere listing of suggested shocks is not considered as adequate for full credit.

- The shocks are used to determine the capital strain when applied to the economic balance sheet. The shocks are calibrated to capture the tail event or tail risks.
- Economic capital represents the amount sufficient to cover losses that can occur with a specified confidence probability. This requires the calibration of the shocks to capture the tail risk to a specific risk measure (VAR or CTE).
- The design of the shocks is determined with actuarial judgment, and the results can differ if such a subjective assumption changes.
- The shock design should incorporate all the major risks associated with the LTC line of business – both asset and liability, ideally to capture the interaction among different risks.
- The company’s own experience supplemented with industry experience and other expert opinion could help determine the magnitude of the shocks.

- (c)

- (i) Explain the CRO’s concerns.
- (ii) Provide recommendations to address the CRO’s concerns.

Commentary on Question:

Some candidates proposed a listing of shocks with some arbitrary numbers, which were not sufficient for full credit.

5. Continued

(i) CRO's concerns:

I. Due to long duration of the liability and supporting assets, parallel shift of interest rate shock does not represent the inherent risks. Yield curve shape change, such as flattening or steeping, also have significant impact on the EC. This impact is exacerbated by the non-perfect hedging, which is extremely difficult for such long duration liability. Almost no available marketable instruments, including swaps or swaptions can provide perfect delta matching at such a long duration.

II. Shock design only represents one stress scenario, which is difficult to determine if this truly reflects the stress situation that can cause large amount of losses to the insurer.

The shock design may be stale and needs to be refreshed to reflect the current experience, including product portfolio/mix, investment strategy and hedging activities. In addition, the market condition and emerging experience may warrant additional examination on this assumption.

- The shocks need to incorporate all material risks.
- The aggregate EC results should identify the major risk drivers for LTC – interest, consumer behavior, hedging effectiveness and etc.
- Interactions and correlations between risks need to be evaluated.

(ii) Recommendations – possible answers include:

- Evaluate each major risk factor separately by performing sensitivity on major assumptions.
- For interest risk, add more stress scenarios, such as yield curve change, pop-up then flat, pop-down or other scenario to test which produce larger EC. In addition, not only are the LTC products subject to the interest risk, the hedging instruments are highly sensitive as well.
- Shock bond portfolio values and other actuarial assumptions
- Taking interaction of the risk categories into consideration when aggregating the risks to examine the capital shortfall.
- Calibrate the stress scenario for EC calculation using economic scenario generator or historical stress events.
- Supplement the EC amount with sensitivity result by risk factor for more complete illustration of the product risk profile.

6. Learning Objectives:

4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

- (4d) Demonstrate how derivatives, synthetic securities, and financial contracting may be used to reduce risk or to assign it to the party most able to bear it.

Sources:

ERM-115-13 Creating an Understanding of Special Purpose Vehicles, PWC

Commentary on Question:

The question was to test the candidates' knowledge of Special Purpose Vehicles (SPV), and how an SPV is used to raise capital. Overall candidates performed well with this question, and many performed better in parts d & e (recommending go or no-go and describing other strategic considerations), than in parts a, b & c (defining, illustrating and identifying).

Solution:

- (a) Define an SPV.

Commentary on Question:

Many candidates did not receive full marks when they did not indicate the relationship of the SPV to the sponsoring firm and/or the off-balance sheet characteristic of the relationship.

A Special Purpose Vehicle (SPV) sometimes referred to as a Special Purpose Entity (SPE) is an off-balance sheet vehicle (OBSV) comprised of a legal entity created by the sponsor or originator, typically a major investment bank or insurance company, to fulfil a temporary objective of the sponsoring firm.

- (b) Illustrate how LMN can structure this SPV to obtain financing.

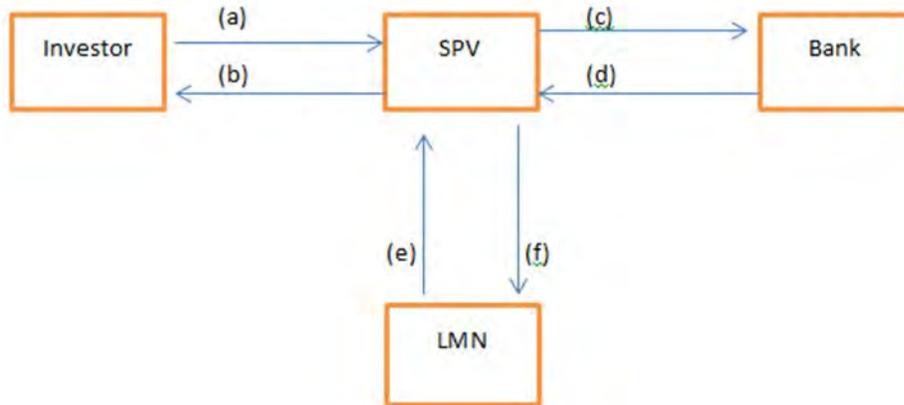
Commentary on Question:

- *Many candidates illustrated and/or described only the interactions/transactions between the SPV and investors, and not the other transactions. At times, the candidates wrote too much detail about the different tranches that could be set up.*
- *Some candidates failed to recognize that the real estate assets were being securitized and instead attempted to securitize the new venture or something generic.*
- *Many candidates did not mention the investment bank and/or did not describe the transactions between the bank and the SPV.*
- *Several candidates did not describe the transaction from the SPV to the LMN well.*

6. Continued

- *Some merely illustrated or relied on the illustration as the only answer - with minimal words to describe the transactions.*
- Create an SPV using the high quality real estate investment as the starting asset.
- Use these assets to back a bond issue by the SPV.
- Regular payouts will be paid to the investors using the cash flows coming (i.e. rent) from the real estate asset.
- The assets can be used as collateral in obtaining financing from the bank.
- Since the block of business is very profitable, LMN can expect a more favorable borrowing rate in this SPV compared to if LMN issued the bonds directly.

An alternate solution is an illustration/diagram:



- (a) Investor purchases bond from SPV
 - (b) SPV pays investor profits on real estate
 - (c) SPV gives real estate as collateral to bank
 - (d) SPV receives financing from bank at a better rate than LMN would have gotten
 - (e) LMN gives SPV the real estate
 - (f) SPV gives profits from Investor and funds received from bank to LMN in order to fund the new venture
- (c) Identify four of the key risks and four of the key benefits of the SPV transaction for LMN.

Commentary on Question:

Many candidates only listed and did not describe the risks & benefits. Some candidates wrote similar definition for reputational and signaling risks.

6. Continued

Key Risks

- Franchise Risk – some investors in the SPV can be invested in LMN as well. If these investors are not satisfied with the SPV performance, this can affect their relationship with LMN as well.
- Equity risk – depending on how the SPV is structured, LMN might be holding a huge equity tranche on the investment. If the underlying real estate portfolio fails to perform, LMN will be left to absorb the losses from the equity tranche.
- Regulatory risk – Since the SPV will be less regulated than LMN, the SPV can pose as an indirect risk to LMN.
- Liquidity & funding risk – If the SPV fails to perform, it can exacerbate LMN's liquidity situation and make it harder for them to access the capital markets.

Key Benefits

- LMN can access much needed capital for a lot cheaper.
- Minimal red tape - It is relatively cheap and easy to set up an SPV for some jurisdictions (see next bullet).
- Freedom of jurisdiction – LMN is free to incorporate the SPV in the most attractive jurisdiction.
- Tax benefits – LMN can incorporate the SPV in tax shelters like the Cayman Islands where assets are exempt from certain direct taxes.

- (d) Recommend whether LMN should proceed with the SPV transaction based on your response to (c). Justify your response.

Commentary on Question:

Some candidates who did not perform well in answering parts a, b & c, performed better in part d.

I recommend for LMN to proceed with the SPV transaction.

- This will fulfill the goal of financing the acquisition with lower financing costs found in the market. Despite LMN's low credit rating, the security offered by the SPV will be attractive to the market since it is backed by a high performing asset.
- The relative ease of establishing the SPV will enable LMN to have the funds to close the acquisition in a shorter amount of time.

Alternate solution: I recommend to not move forward with the SPV.

- It can exacerbate the risks and financial condition of LMN when the SPV fails to perform.
- Also, LMN might hold a significant equity tranche if that doesn't get enough market traction.

6. Continued

- (e) Describe three other strategic considerations that LMN would need to address prior to initiating the SPV transaction.

Commentary on Question:

Similar commentary to part d: some candidates who did not perform well in answering parts a, b & c, performed better in part e.

- Is the overall risk well balanced? LMN's portfolio is highly concentrated on the real estate assets and there is an asset concentration risk in case the venture fails.
- Will SPV have expertise needed to manage the real estate assets?
- Ability to obtain capital from market at level needed - the CDO (collateralized debt obligation) might not get enough interest in the market and LMN will not be able to raise enough capital.

7. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.
3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.
4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

- (2a) Demonstrate how each of the financial and non-financial risks faced by an entity can be amenable to quantitative analysis including an explanation of the advantages and disadvantages of various techniques such as Value at Risk (VaR), stochastic analysis, and scenario analysis.
- (2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas.
- (3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.
- (3c) Analyze quantitative financial data and insurance data (including asset prices, credit spreads and defaults, interest rates, incidence, causes and losses) using modern statistical methods. Construct measures from the data and contrast the methods with respect to scope, coverage and application.
- (4a) Demonstrate and analyze applicability of risk optimization techniques and the impact of an ERM strategy on an organization's value. Analyze the risk and return trade-offs that result from changes in the organization's risk profile.

Sources:

ASOP 23: Data quality

Value-at-Risk, Jorion, 3rd ed., The new benchmark for managing financial risk, chapters 5 and 7

Commentary on Question:

The question focuses on understanding and analyzing data in the specific context of XYZ and its impact on the appropriate choice of a risk measurement metric.

In order to receive maximum points for each sub-question, the following considerations were taken into account:

7. Continued

a(i): The student had to describe valid concerns after analysing the data submitted, not only in theory, but referencing specific data concerns. Most of the students answered this question very well.

a(ii): The student had to identify two key disclosures. In order to receive maximum points, the student had to step back and generalize about two main aspects that would be part of an actuarial report if he/she were to actually work with this data, not just repeat previous comments, which some students did.

B: In order to receive maximum points, a student had to explain which risk metric is the most appropriate in this context. That is, not only list valid theoretical/generic statements as many students did but also explain how each metric would be suitable or not in the context of XYZ.

D: In order to receive maximum points, a student had to take the demand of the CFO into account and do some analysis to justify if it made sense. The recommendation could be in line with the analysis but also not the limitations of this method. Some students followed this line of approach while others simply continued with their previous analysis. Both analyses were considered appropriate if thoroughly and correctly developed.

Solution:

(a) You are reviewing ASOP 23 prior to performing your work.

- (i) Describe concerns you have with the data you have been provided.
- (ii) Identify two key data-related disclosures that you would include as part of your actuarial report on the profitability of the tornado insurance.

(i): Here is a list of valid concerns.

- Data is not a year apart, not from the same month.
- Some data points look suspicious: 99.99, 18.23 is repeated 3 times, outliers present
- Data stops for a few years - too short-period to assess trend.
- Sales data is not appropriate to assess claims, granularity concerns.
- How many companies contributed? Is that from similar companies/products? Are sources reliable? Are companies representative? Different scales used?
- Questions of seasonality, currency were raised, geography.
- What is the sampling method? Data truncated?
- What is the reliability of the consultant who submitted the data?

7. Continued

(ii) Here are some valid general disclosures.

- Reliance on others: data used was not audited as it was submitted from an external consultant.
- Data completeness: some adjustments would have to be made to the data for outliers, repeated values, missing data points
- Data reasonableness: limitations on how the data is used and assumptions made
- Assessment of the impact on results of using this data/modifying it
- Unresolved concerns: material defects of inappropriate data for the purpose

(b) Explain whether VaR or CTE is the more appropriate risk measure to use in this situation.

- CTE is a more conservative metric to use given the fact that tornadoes are infrequent but have huge financial impact. It exceeds VaR in this case.
- Contrary to VaR, it is not risk blind, and would be in line with the desired risk tolerance of 100 million dollars as stated. It is also better for a five-year time horizon and the going-concern of XYZ.
- CTE has useful properties – monotocity, translation, invariance, homogeneity and sub-additivity – which are relevant in this case since XYZ desires to combine two fat-tailed type of businesses, for which ultimate knowledge of loss needs to be known, not only the point at which extreme loss can be anticipated to occur, which the 90% VaR would indicate.

(c) Recommend whether or not XYZ should offer the tornado product. Justify your response.

Since the CFO imposes his choice, the following VaR of the combined portfolio has to be calculated.

Thus, the portfolio VaR would be:

The total VaR for independent risks is the root of the sum of squared VaRs = $(75^2 + 60^2)^{.5}$, that is 96 million dollars.

Based on this result, since it is lower than the desired risk tolerance of 100 million dollars, this project should be undertaken.

However, the independence assumption should be questioned as these events tend to happen concurrently. In addition, there might agency cost involved here as the CFO might have some financial incentive to pursue this new product.

7. Continued

If one were to base his decision using the CTE instead, this new product would clearly not be undertaken as such since it would exceed the desired risk tolerance over the 5-year time horizon. The profitability of the product would be harder to justify unless some risk management approach were put in place – ex. Reinsurance – to reduce the impact of the tornado tail event on XYZ.

8. Learning Objectives:

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.
3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.
4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

- (1a) Explain risk concepts and be able to apply risk definitions to different entities.
- (1c) Identify and assess the potential impact of risks faced by an entity, including but not limited to market risk, currency risk, credit risk, counterparty risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk, project risk and strategic risk.
- (3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.
- (3b) Analyze and evaluate the properties of risk measures (e.g., Delta, volatility, duration, VaR, TVaR, etc.) and their limitations.
- (4a) Demonstrate and analyze applicability of risk optimization techniques and the impact of an ERM strategy on an organization's value. Analyze the risk and return trade-offs that result from changes in the organization's risk profile.
- (4h) Analyze funding and portfolio management strategies to control equity and interest rate risk, including key rate risks. Contrast the various risk measures and be able to apply these risk measures to various entities. Explain the concepts of immunization including modern refinements and practical limitations.
- (4i) Analyze the application of Asset Liability Management and Liability Driven Investment principles to Investment Policy and Asset Allocation.

8. Continued

- (4k) Apply best practices in risk measurement, modeling and management of various financial and non-financial risks faced by an entity.

Sources:

ERM-330-17: Barton Waring Liability-Relative Investing I

Commentary on Question:

This question expected candidates to understand and apply dual dollar duration risk measures to immunize the surplus of a defined benefit pension plan from real rate and inflation rate risks, using the dual duration exposures of Equity, Nominal Bonds, and TIPS.

Solution:

- (a)
- (i) Demonstrate whether the illustrated plan is duration matched.
 - (ii) Demonstrate whether the plan is dollar-duration matched.
 - (iii) Explain why dollar-duration matching should be the preferred objective.

Commentary on Question:

Most candidates received full credit on parts (i) and (ii). Full credit for (iii) required differentiating the characteristic of duration and dollar-duration matching, describing stable surplus as the preferred objective and mentioning that duration-matching is inadequate for immunization when a plan surplus or deficit exists.

No credit was lost by candidates who failed to multiply by .01 in part (ii) since the analysis being asked for was unaffected by this term.

- (i) Duration of liability = 9.8.
Asset duration = (Equity %)(Equity duration) + (Bond %)(Bond duration)
= $0.6 \cdot 3 + 0.4 \cdot 20 = 1.8 + 8 = 9.8$

Therefore the plan is duration matched.

- (ii) Dollar Duration of L = (Liability amount)(Liability duration)(.01) =
(\$100m)(9.8)(.01) = \$9.8m
Dollar Duration of assets = (Asset amount)(Asset duration)(.01) =
(\$80m)(9.8)(.01) = \$7.84m

Therefore the plan is not dollar-duration matched

8. Continued

- (iii) The objective of liability relative investing is to immunize the dollar amount of surplus. Dollar-durations measure the change in surplus (or deficit) of the Plan from a 1% change in interest rate, whereas duration only measures the % change in wealth. If the assets are greater than or less than the liability then simply matching duration will not protect the dollar amount of surplus.
- (b) Match each of the three assets classes: equity, nominal bonds and TIPS, with one of the dual dollar-duration profiles in the graph. Justify your response.

Commentary on Question:

Most candidates did well on this part.

Equity is represented by profile 2 as equities are much more sensitive to changes in real interest rates than to changes in inflation but do maintain a small amount of inflation sensitivity.

Nominal bonds are represented by profile 1 since their sensitivity to real rates and inflation are equal.

TIPS are represented by profile 3 since they are fully immunized from inflation changes and therefore have an inflation duration of 0.

- (c) Identify which dual-duration pair best matches SLIC's DB Pension Plan liabilities. Justify your response.

Commentary on Question:

As Plan X and Plan Y have the same real rate duration, full credit on this question required referencing provisions of SLIC's plan that would contribute to determining its relative inflation duration. Some candidates incorrectly stated that plans with inflation-linked benefits have higher inflation durations. Several candidates inappropriately justified their responses by referring to the plan's asset allocation instead of its liability profile.

Plan Y has a much higher exposure to inflation than Plan X. Inflation duration for retired lives of Plans with full COLA is practically zero. Non-COLA plans with a final average pay benefit are sensitive to wage inflation and have higher inflation duration for retirees and lower inflation duration for actives. SLICS's DB Plan is a final-average pay benefit, without COLA, and has more retirees' liability than actives'. Therefore SLICS's DB Plan is closer to Plan Y than Plan X.

8. Continued

- (d) Calculate the missing values in the table. Show your work.

Commentary on Question:

Some candidates got the correct answers but did not receive full credit because they did not show their work. The model answers do not include the .01 term that would normally be used to calculate dollar durations since these factors would cancel out in the calculations.

To immunize surplus the dual dollar durations of assets and liabilities must match.

$$\$D_{L,r} = (\$100m)(15) = \$1,500m$$

$$\$D_{L,I} = (\$100m)(4) = \$400m$$

Equities make up (.6)(\\$80m) = \\$48m of the asset portfolio and contribute dual dollar durations of:

$$\$D_{e,r} = (\$48m)(20) = \$960m$$

$$\$D_{e,I} = (\$48m)(4) = \$192m$$

Therefore, the bond portfolio must contribute dual dollar durations of:

$$\$D_{b,r} = \$D_{L,r} - \$D_{e,r} = \$1,500m - \$960m = \$540m$$

$$\$D_{b,I} = \$D_{L,I} - \$D_{e,I} = \$400m - \$192m = \$208m$$

As TIPS have an inflation duration of 0, the full amount of bond inflation duration is attributable to nominal bonds.

$$\$D_{n,I} = \$D_{b,I} = \$208m$$

Since nominal bonds have equal inflation and real durations, the amount of real interest rate dollar duration that the TIPS must contribute are:

$$\$D_{t,r} = \$D_{b,r} - \$D_{n,r} = \$D_{b,r} - \$D_{n,I} = \$540m - \$208m = \$332m$$

Row 1

$$\% \text{ Bonds} = 40\%$$

$$\% \text{ TIPS} = (\% \text{ Bonds}) - (\% \text{ Nominal}) = 40\% - 43.3\% = -3.3\%$$

$$\$D_{n,I} = (\$ \text{ Nominal Bonds}) * (\text{Nominal bond duration}) = (\$ \text{ Asset value}) * (\text{Nominal bond } \%) * D_n$$

$$\$208m = \$80m * .433 * D_n$$

$$D_n = 6.00$$

$$\$D_{t,r} = (\$ \text{ TIPS}) * (\text{TIPS real duration}) = (\$ \text{ Asset value}) * (\text{TIPS } \%) * D_{t,r}$$

$$\$332m = (\$80m)(-.033)(D_{t,r})$$

$$D_{t,r} = -125.8$$

8. Continued

Row 2

$$\begin{aligned} \$D_{n,I} &= (\$ \text{ Nominal Bonds}) * (\text{Nominal bond duration}) = (\$ \text{ Asset value}) * \\ &(\text{Nominal bond } \%) * D_n \end{aligned}$$

$$\$208\text{m} = \$80\text{m} * .2 * D_n$$

$$D_n = 13.00$$

$$\$D_{t,r} = (\$ \text{ TIPS}) * (\text{TIPS real duration}) = (\$ \text{ Asset value}) * (\text{TIPS } \%) * D_{t,r}$$

$$\$332\text{m} = (\$80\text{m})(.2)(D_{t,r})$$

$$D_{t,r} = 20.8$$

Row 3

$$\$D_{t,r} = (\$ \text{ TIPS}) * (\text{TIPS real duration}) = (\$ \text{ Asset value}) * (\text{TIPS } \%) * D_{t,r}$$

$$\$332\text{m} = (\$80\text{m})(\text{TIPS } \%)(16.8)$$

$$\text{TIPS } \% = 24.7\%$$

$$\% \text{ TIPS} = (\% \text{ Bonds}) - (\% \text{ Nominal})$$

$$24.7\% = 40\% - (\% \text{ Nominal})$$

$$\text{Nominal } \% = 15.3\%$$

$$\begin{aligned} \$D_{n,I} &= (\$ \text{ Nominal Bonds}) * (\text{Nominal bond duration}) = (\$ \text{ Asset value}) * \\ &(\text{Nominal bond } \%) * D_n \end{aligned}$$

$$\$208\text{m} = \$80\text{m} * .153 * D_n$$

$$D_n = 17.00$$

Row 4

$$\% \text{ TIPS} = (\% \text{ Bonds}) - (\% \text{ Nominal})$$

$$33.5\% = 40\% - (\% \text{ Nominal})$$

$$\text{Nominal } \% = 6.5\%$$

$$\begin{aligned} \$D_{n,I} &= (\$ \text{ Nominal Bonds}) * (\text{Nominal bond duration}) = (\$ \text{ Asset value}) * \\ &(\text{Nominal bond } \%) * D_n \end{aligned}$$

$$\$208\text{m} = \$80\text{m} * .065 * D_n$$

$$D_n = 40$$

$$\$D_{t,r} = (\$ \text{ TIPS}) * (\text{TIPS real duration}) = (\$ \text{ Asset value}) * (\text{TIPS } \%) * D_{t,r}$$

$$\$332\text{m} = (\$80\text{m})(.335)(D_{t,r})$$

$$D_{t,r} = 12.4$$

8. Continued

Nominal Bond Duration	Nominal Bond %	TIPS Real Rate Duration	TIPS %
6.00	43.3%	-125.8	-3.3%
13.00	20.0%	20.8	20.0%
17.00	15.3%	16.8	24.7%
40.00	6.5%	12.4	33.5

- (e) Select one of the four Nominal Bond/TIPS allocations as being the most practical to implement. Justify your choice.

Commentary on Question:

Full credit was given for selecting Row 2 or Row 3 in the table above with reasonable justification. Credit was also given for reasonable selections with appropriate justifications where candidates had incorrect results in (d).

Sample solution:

Row 3 is the best choice since it is made up of entirely long positions and has similar bond and TIPS durations which should be easiest to find in the market.

Row 1 has a short position in TIPS which is disallowed by many investment policies and would be difficult to transact in practice. Row 4 requires an extremely long nominal bond duration which is not practical.

- (f) Explain whether you think SLIC should pursue dual duration analysis to assist in setting its DB Plan asset allocation strategy.

Commentary on Question:

Full credit was available for either recommendation as long as appropriate justification was provided which references the specific perspective of SLIC. Most candidates failed to give answers specific to SLIC. Sample solutions have more detail than required for full credit.

Sample solution in favor:

SLIC should pursue dual duration analysis in order to reflect the low inflation duration of the active employees in this final average pay plan. Since 52% of the participants are active and 40% of the liability is for active employees, the plan's inflation duration will be significantly less than its real rate duration. If dual duration analysis is not performed the company will not have a full understanding of the mismatch between assets and liabilities and will be unable to properly immunize the existing deficit.

8. Continued

Sample solution against:

SLIC should NOT pursue dual duration analysis. SLIC does not have a COLA adjustment in its DB Plan, so inflation exposure is not that great. Real and inflation duration exposures for assets are not readily available, especially for equity. There seems to be disagreement even about how to measure Equity dual durations, so SLIC could get widely different results. More of SLIC's pension liability is retirees, where inflation duration is similar to real interest duration, so Nominal Bonds (and duration) is adequate. Higher investment cost, complexity, and transaction fees – might need to use futures and forwards to get the high dual duration exposures needed. Optimization does not discriminate well between highly correlated assets; solution may be suspect.

9. Learning Objectives:

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.
2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.
3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

Learning Outcomes:

- (1c) Identify and assess the potential impact of risks faced by an entity, including but not limited to market risk, currency risk, credit risk, counterparty risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk, project risk and strategic risk.
- (2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas.
- (2h) Construct approaches to modeling various risks and evaluate how an entity makes decisions about techniques to model, measure and aggregate risks including but not limited to stochastic processes.
- (3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.
- (3c) Analyze quantitative financial data and insurance data (including asset prices, credit spreads and defaults, interest rates, incidence, causes and losses) using modern statistical methods. Construct measures from the data and contrast the methods with respect to scope, coverage and application.

Sources:

Value at Risk, Jorion, Chapter 8, Multivariate Models

Value at Risk, Jorion, Chapter 11, VAR Mapping; Managing Investment Portfolios

Maginn and Tuttle 3rd Edition, Chapter 6, Section 5 Other Fixed Income Strategies

Commentary on Question:

This question tests students' understanding of using multivariate models to calculate aggregate VaR based on multiple risk factors. Many candidates failed to attempt the question, possibly because it was the last question of the exam. Of those that did provide a response, most did relatively well on the earlier parts which were direct applications of the source material but struggled to apply their knowledge on the later parts where a deeper understanding was required.

9. Continued

Solution:

- (a) For each of the three treasury zero-coupon maturity terms:
- (i) Calculate the 1-year Yield VaR at the 95% confidence level.
 - (ii) Calculate the 1-year Returns VaR at the 95% confidence level.

Show your work.

Commentary on Question:

This part was a straightforward calculation from the source material. Most candidates received full credit.

- (i) Yield VaR = $\alpha(95\%) \times \text{Yield Volatility}$; $\alpha(95\%) = 1.645$
 $\text{VaR}(3\text{yr}) = 1.645 \times 1.06\% = 1.74\%$
 $\text{VaR}(7\text{yr}) = 1.645 \times 0.98\% = 1.61\%$
 $\text{VaR}(20\text{yr}) = 1.645 \times 0.88\% = 1.45\%$
- (ii) Returns VaR = $D^* \times \text{Yield VaR}$
 $\text{VaR}(3\text{yr}) = 2.96 \times 1.74\% = 5.16\%$
 $\text{VaR}(7\text{yr}) = 6.84 \times 1.61\% = 11.03\%$
 $\text{VaR}(20\text{yr}) = 19.46 \times 1.45\% = 28.17\%$

- (b)
- (i) Derive the liability cash flows that were mapped to each treasury zero-coupon maturity term.
 - (ii) Calculate the SPIA liability modified duration.

Show your work

Commentary on Question:

This part was straightforward and most candidates received at least some credit.

- (i) Cash Flow = Market Value * $(1 + \text{Yield})^{\text{term}}$
3yr: $40 \times (1.015)^3 = 41.83$
7yr: $33 \times (1.023)^7 = 38.69$
20yr: $27 \times (1.028)^{20} = 46.91$
- (ii) Liability Duration = $(\sum_{\text{term}} \text{Cash Flow Map PV} \times \text{Modified Duration}) / \text{Total Cash Flow Map PV} = (40 \times 2.96 + 33 \times 6.84 + 27 \times 19.46) / 100 = 8.695$

9. Continued

- (c)
- (i) Calculate the modified duration of Portfolio A and Portfolio B.
 - (ii) Calculate the 1-year Yield VaR for Portfolio B at the 95% confidence level.
 - (iii) Calculate the 1-year Returns VaR for Portfolio B at the 95% confidence level.

Show your work.

Commentary on Question:

This part was a straightforward calculation from the source material. Although candidates generally did well on part (i), most either did not attempt parts (ii) and (iii) or incorrectly applied the formula from the reading. Some candidates pursued an alternate approach to part (ii) using the Var-Covar matrix; full credit was given for either method.

- (i) Calculate asset portfolio durations using the same formula as liability duration in part (b).
Portfolio A: $(50.46 * 2.96 + 26.76 * 6.84 + 31.55 * 19.46)/108.77 = 8.701$
Portfolio B: $(20.14 * 2.96 + 74.44 * 6.84 + 23.52 * 19.46)/118.1 = 8.692$
- (ii) Diversified VaR = $\sqrt{\text{Yield VaR}' * \text{correlation matrix} * \text{Yield VaR}}$
Yield VaR' = Cash Flow Map PVs * Yield VaRs from (a)(i)
Yield VaR' = $[20.14 * 1.74\% \quad 74.44 * 1.61\% \quad 23.52 * 1.45\%]$
Yield VaR' = $[0.3512 \quad 1.2000 \quad 0.3405]$
Yield VaR' * correlation matrix =
 $[0.3512 * 1.00 + 1.2000 * 0.97 + 0.3405 * 0.83$
 $0.3512 * 0.97 + 1.2000 * 1.00 + 0.3405 * 0.92$
 $0.3512 * 0.83 + 1.2000 * 0.92 + 0.3405 * 1.00]$
= $[1.7978 \quad 1.8539 \quad 1.7360]$
Yield VaR' * correlation matrix * Yield VaR = $(1.7978 * 0.3512 + 1.8539 * 1.2000 + 1.7360 * 0.3405) = 3.45$
Diversified VaR = $\sqrt{3.45} = 1.857$
- (iii) Returns VaR = $D^* * \text{Yield VaR} = 8.692 * 1.857 = 16.138$

9. Continued

- (d)
- (i) Calculate the 1-year 95% $VaR(dP)$ for Portfolio A incorporating the convexity adjustment.
 - (ii) Calculate the excess return of the bond portfolio over the liabilities for each of Portfolio A and Portfolio B.
 - (iii) Calculate the 1-year 95% VaR due to credit spreads for each of Portfolio A and Portfolio B.

Show your work.

Commentary on Question:

This part required candidates to understand the relationship between VaR of yields vs. returns explained in the source material and to then apply that knowledge, directly or approximately, to an analogous situation with credit spreads vs. spread returns. Most candidates did not attempt this part. Of those who did attempt it, most were successful with part (i) and struggled with parts (ii) and (iii) which required deeper understanding.

- (i) As given, convexity adjustment is $-(0.5 * C * P) * VaR(dy)^2$ where $VaR(dy)$ is the Yield VaR before scaling up for P
1-year 95% $VaR(dP) = 14.902 - 108.77 * 67.16/2 * (1.713/100)^2 = 13.83$
- (ii) Excess return = Portfolio return – Liability return
Liability return = $(40 * 1.5\% + 33 * 2.3\% + 27 * 2.8\%)/100 = 2.115\%$
Since Portfolio A has a 1% static spread over Treasury,
Portfolio A return = $(49 * 2.5\% + 25 * 3.3\% + 26 * 3.8\%)/100 = 3.038\%$
Since Portfolio B has a 2% static spread over Treasury,
Portfolio B return = $(19 * 3.5\% + 65 * 4.3\% + 16 * 4.8\%)/100 = 4.228\%$
Portfolio A excess return = $3.038\% - 2.115\% = 0.923\%$
Portfolio B excess return = $4.228\% - 2.115\% = 2.113\%$
- (iii) Similar to Returns VaR:
 $VaR(\text{Spread Returns}) = P * |D_{\text{spread}}| * VaR(\text{Spread})$,
where $VaR(\text{Spread}) = \alpha * Vol(\text{Spread})$
For Portfolio A, $VaR(\text{Spread}) = 1.645 * 35\% = 0.57575$;
 $VaR(\text{Spread Returns}) = 100 * 1\% * 9.09 * 0.57575 * 0.01 = 0.052336$

For Portfolio B, $VaR(\text{Spread}) = 1.645 * 30\% = 0.4935$;
 $VaR(\text{Spread Returns}) = 100 * 2\% * 7.99 * 0.4935 * 0.01 = 0.078861$

9. Continued

- (e) Recommend one of Portfolio A or Portfolio B as providing better market risk/reward merits relative to the SPIA liabilities portfolio. Justify your choice on the basis of your analysis.

Commentary on Question:

Candidates could have used their calculation results from any of parts (b)-(d) to compare the two portfolios. Full credit was given for correct analysis based on each candidate's calculated (or assumed) values, even if the values themselves were incorrect.

Compare on 3 bases:

- Duration and Convexity: From parts (b) and (c), Liability duration is 8.695, Portfolio A duration is 8.701 and Portfolio B duration is 8.692 → Portfolio B is a slightly better duration match, but Portfolio A has higher convexity than liabilities → Portfolio A is preferable overall
- Yield VaR and Returns VaR: From part (c), Portfolio A Yield VaR is 1.713, Portfolio B Yield VaR is 1.857 → Portfolio A is preferable. Also from part (c), Portfolio A Returns VaR is 14.902, Portfolio B Returns VaR is 16.138 → Portfolio A is preferable
- Excess return: From part (d), Portfolio A has 0.923% excess return, Portfolio B has 2.113% excess return → Portfolio B is preferable

Overall, recommend Portfolio A due to lower VaR and better duration/convexity match and in spite of lower excess return.