1.

$$d_{41}^{(3)} = \ell_{41}^{(\tau)} q_{41}^{(3)}$$
$$\ell_{41}^{(\tau)} = \ell_{40}^{(\tau)}(1 - q_{40}^{(1)} - q_{40}^{(2)} - q_{40}^{(3)})$$

If $q_{40}^{(1)}$ increases by 0.01, then the change in $\ell_{41}^{(\tau)} = -0.01\ell_{40}^{(\tau)}$.
The change in $d_{41}^{(3)} = -0.01\ell_{40}^{(\tau)} q_{41}^{(3)} = -0.01 \times 15{,}000 \times 0.10 = -15$

ANSWER: B

2.

$${}_{3.5}p_{[61]} - {}_{3.5}p_{[60]+1} = {}_{0.5}p_{64}({}_{3}p_{[61]} - {}_{3}p_{[60]+1})$$

$$= \left(\frac{\ell_{65}}{\ell_{64}}\right)^{0.5} \left(\frac{\ell_{64}}{\ell_{[61]}} - \frac{\ell_{64}}{\ell_{[60]+1}}\right)$$

$$= \left(\frac{4016}{5737}\right)^{0.5} \left(\frac{5737}{8654} - \frac{5737}{9600}\right)$$

$$= 0.05466$$

ANSWER: C

3.

$${}_{5}p_{x}^{01} = \int_0^5 {}_{t}p_x^{\overline{00}} \mu_{x+t}\, {}_{5-t}p_x^{\overline{11}} dt = \int_0^5 e^{-(\mu^{01}+\mu^{03})t} \mu^{01} e^{-(5-t)(\mu^{12}+\mu^{13})} dt$$
$$= \int_0^5 e^{-(0.01+0.02)t}(0.01)e^{-(5-t)(0.30+0.40)} dt = \int_0^5 e^{-(0.03)t}(0.01)e^{-(5-t)(0.7)} dt$$
$$= (0.01)e^{-3.5}\int_0^5 e^{0.67t} dt = (0.01)e^{-3.5}\left(\frac{e^{0.67(5)} - 1}{0.67}\right) = 0.01239568$$

ANSWER: C

4.

Healthy lives' probabilities:

Probability of a Healthy life at time 0 being Healthy at:

Time 1: 0.9

Time 2: (0.9)(0.9) = 0.81

Probability of a Sick life at time 1 that was Healthy at time 0 being Healthy at:

Time 2: (0.05)(0.30) = 0.015

Probability of being Healthy at time 1: 0.9

Probability of being Healthy at time 2: 0.81 + 0.015 = 0.825

Sick lives' probabilities:

Probability of a Healthy life at time 0 being Sick at:

Time 1: 0.05

Probability of a Sick life at time 1 that was Healthy at time 0 being Sick at:

Time 2: (0.05)(0.60) = 0.03

Probability of a Healthy life at time 1 that was Healthy at time 0 being Sick at:

Time 2: (0.90)(0.05) = 0.045

Probability of being Sick at time 1: 0.05

Probability of being Sick at time 2: 0.03 + 0.045 = 0.075

APV (Healthy): $500(e^{-0.5\times 0.04} + 0.9e^{-1.5\times 0.04} + 0.825e^{-2.5\times 0.04}) = 1287.138812$

APV (Sick):
$5000(0.05e^{-1.5\times 0.04} + 0.075e^{-2.5\times 0.04}) = 574.755165$

Total APV: 1287.14 + 574.76 = 1861.90

ANSWER: D

5. The present value random variable $PV = 100{,}000e^{-0.05T},\ 2 \le T \le 10$ is a decreasing function of T so that its 90th percentile is

$100{,}000e^{-0.05p}$ where p is the solution to $\int_2^p 0.4t^{-2}\,dt = 0.10$.

$$\int_2^p 0.4t^{-2}\,dt = -0.4\left(\frac{t^{-1}}{-1}\right)\Bigg|_2^p = 0.4\left(\frac{1}{2}-\frac{1}{p}\right) = 0.10$$

$$p = 4$$

$$100{,}000e^{-0.05\times 4} = 81{,}873.08$$

ANSWER: A

6. The expected present value is:

$$\ddot{a}_{\overline{5}|} + {}_5E_{55}\ddot{a}_{60} = 4.465106 + 0.70810 \times 11.1454 = 12.35716$$

The probability that the sum of the undiscounted payments will exceed the expected present value is the probability that at least 13 payments will be made. This will occur if (55) survives to age 67. The probability is therefore:

$$ {}_{12}p_{55} = \frac{\ell_{67}}{\ell_{55}} = \frac{7{,}201{,}635}{8{,}640{,}861} = 0.83344$$

ANSWER: C

7.

$$P\ddot{a}_{50:\overline{10|}} = P(IA)^{1}_{50:\overline{10|}} + {}_{10}E_{50}A_{60}$$
$$\ddot{a}_{50:\overline{10|}} = \ddot{a}_{50} - {}_{10}E_{50}\ddot{a}_{60} = 17.0 - 0.60 \times 15.0 = 8$$
$$A_{60} = 1 - d\ddot{a}_{60} = 1 - \left(\frac{0.05}{1.05}\right)15 = 0.285714$$
$$P\left(\ddot{a}_{50:\overline{10|}} - (IA)^{1}_{50:\overline{10|}}\right) = {}_{10}E_{50}A_{60}$$
$$P = \frac{{}_{10}E_{50}A_{60}}{\ddot{a}_{50:\overline{10|}} - (IA)^{1}_{50:\overline{10|}}} = \frac{0.6 \times 0.285714}{8 - 0.15} = 0.021838$$
$$100P = 2.18$$

ANSWER: D

8. $L_0 = 100{,}000v^T - 926\bar{a}_{\overline{T|}} = \left(100{,}000 + \frac{926}{\delta}\right)e^{-\delta T} - \frac{926}{\delta}$

Since L_0 is a decreasing function of T , the 25th percentile of L_0 is $L_0(t)$ where t is such that $\Pr[T_{35} \le t] = 0.25$ or $\Pr[T_{35} > t] = 0.75$.

$$\frac{\ell_{35+t}}{\ell_{35}} = 0.75$$
$$\ell_{35+t} = 0.75\ell_{35} = 0.75 \times 9{,}420{,}657 = 7{,}065{,}493$$
$$\ell_{67} < 7{,}065{,}493 < \ell_{68}$$
$$t = (67-35) + s$$
$$\ell_{67+s} = s\ell_{68} + (1-s)\ell_{67}$$
$$7{,}065{,}493 = 7{,}018{,}432s + 7{,}201{,}635(1-s)$$
$$s = 0.74312$$
$$t = 32.74312$$
$$L_0(32.74312) = \left(100{,}000 + \tfrac{926}{\ln(1.06)}\right)e^{-32.74312\ln(1.06)} - \tfrac{926}{\ln(1.06)} = 1305.4469$$

ANSWER: E

9.

$$P\ddot{a}_{40:40:\overline{10}|} = 1{,}000{,}000\ {}_{45}E_{40:40}\ddot{a}_{85:85}$$

$$\ddot{a}_{40:40:\overline{10}|} = \ddot{a}_{40:40} - {}_{10}E_{40:40}\ddot{a}_{50:50}$$

$$= 13.6036 - 1.06^{10} \times 0.53667^2 \times 11.6513 = 7.59397$$

$${}_{45}E_{40:40} = \left(\frac{\ell_{85}}{\ell_{40}}\right)^2 \times (1.06)^{-45} = 0.004658216$$

$$7.59397P = 1{,}000{,}000 \times 0.004658216 \times 3.1855$$

$$P = 1954.02$$

ANSWER: B

10.

$$\frac{d}{dt}\,{}_tV = \delta_t\,{}_tV + P_t - e_t - (S_t + E_t - {}_tV)\mu_{x+t}$$

At $t = 30.5,$

$$292 = 0.05\,{}_{30.5}V + 100 - 0 - (10{,}000 + 0 - {}_{30.5}V) \times 0.038$$

$$572 = {}_{30.5}V(0.05 + 0.038)$$

$${}_{30.5}V = 6500$$

ANSWER: D

11.

$$G\ddot{a}_{45:\overline{10|}} = HA_{45} + G + 0.05G\ddot{a}_{45:\overline{10|}} + 80 + 10\ddot{a}_{45} + 10\ddot{a}_{45:\overline{10|}}$$

$$G = \frac{HA_{45} + 80 + 10\left(\ddot{a}_{45} + \ddot{a}_{45:\overline{10|}}\right)}{0.95\ddot{a}_{45:\overline{10|}} - 1}$$

$$\ddot{a}_{45:\overline{10|}} = \ddot{a}_{45} - {}_{10}E_{45}\ddot{a}_{55} = 14.1121 - 0.52652 \times 12.2758 = 7.6486$$

$$G = \frac{HA_{45} + 80 + 10(14.1121 + 7.6486)}{(0.95 \times 7.6486) - 1}$$

To get f, set $H = 0$ so that

$$f = \frac{80 + 10(14.1121 + 7.6486)}{(0.95 \times 7.6486) - 1} = \frac{297.607}{6.26617} = 47.49$$

ANSWER: E

12.

$$G = 0.35G + 2000\left(\frac{0.1}{1.08} + \frac{0.9 \times 0.1}{1.08^2} + \frac{0.9 \times 0.9 \times 0.1}{1.08^3}\right)$$

$$0.65G = 468.107$$

$$G = 720.16$$

ANSWER: D

13.

The expected present value of the premiums is:

$$P\ddot{a}_{55:55:\overline{10}|} = P\ddot{a}_{55:55} - P\left({}_{10}E_{55}\right)^2 \left(1.06\right)^{10} \ddot{a}_{65:65}$$
$$= P(10.4720 - 0.48686^2 \times 1.06^{10} \times 7.8552)$$
$$= 7.13755P$$

The benefit is 6,000 per year to each of them while they are alive, but while they are both alive they must, between them, return 2,000 since the benefit is only 10,000.

The expected present value of benefits is therefore:

$$= 2 \times 6{,}000 \times {}_{10}E_{55} \times \ddot{a}_{65} - 2{,}000\left({}_{10}E_{55}\right)^2 \left(1.06\right)^{10} \ddot{a}_{65:65}$$
$$= 2 \times 6{,}000 \times 0.48686 \times 9.8969 - 2{,}000 \times 0.48686^2 \times 1.06^{10} \times 7.8552$$
$$= 51{,}151.96$$

Using the equivalence principle, we get

$$P = \frac{51{,}151.96}{7.13755} = 7{,}166.60$$

ANSWER: B

14.

$$AV_6 = \left(AV_5 + P_6 \times (1 - 0.035) - 100 - \left(\frac{1{,}000{,}000 \times 50}{1{,}000 \times 1.06} \right) \right) \times 1.04$$
$$= (100{,}000 + 25{,}000 \times 0.965 - 100 - 47{,}169.81) \times 1.04$$
$$= 76{,}855.19 \times 1.04 = 79{,}929.40$$

The cash surrender value at the end of year 6 is:

$$AV_6 \times (1 - 0.06) = 79{,}929.40 \times 0.94 = 75{,}133.63$$

ANSWER: E

15.

The asset share per policy at the end of the 11th year is:

$$AS_{11} = \frac{\left(AS_{10} + P - 50 - 0.15P\right)\times 1.08 - 0.005\times 100{,}000}{(1-0.005)}$$

$$= 11{,}312.36$$

ANSWER: E

16.

Expected expense financial impact – Expected expenses + Foregone interest

$$100 + .07\times 100 = 107$$

Actual expense financial impact – Actual expenses + Foregone interest

$$75 + .07\times 75 = 80.25$$

Gain from expenses:

$$107 - 80.25 = 26.75$$

ANSWER: C

17.

$$\text{NPV} = \text{PreContractExp} + \sum_{k=1}^{3} \left(CF_k \right) v^k \,_{k-1}p_{55}$$

$$\text{NPV} = \text{PreContractExp} +$$

$$\sum_{k=1}^{3} \left(\text{StartingRes}_k + GP_k - E_k + \text{InvEarn}_k - \text{ExpDeathBenefits}_k - \text{ExpReserveCosts}_k \right) v^k \,_{k-1}p_{55}$$

$$\text{PreContractExp} = -100$$

$$CF_1 = 75 - 20 + 2.80 - 10.0 - 64.35 = -16.55$$

$$CF_2 = 65 + 75 - 20 + 6.00 - 15.0 - 123.13 = -12.13$$

$$CF_3 = 125 + 75 - 20 + 9.00 - 21.0 = 168.00$$

$$NPV = -100 - 16.55 \times 1.1^{-1} \times 1 - 12.13 \times 1.1^{-2} \times 0.99 + 168 \times 1.1^{-3} \times 0.99 \times 0.985$$

$$= -100 - 15.05 - 9.92 + 123.08$$

$$= -1.89$$

ANSWER: B

18.

$$APV = \sum_{x=63}^{65} B_x \frac{d_x^{(r)}}{\ell_{50}} v^{x+.5-50}$$

$$= 20{,}000 \times \frac{1{,}350}{29{,}919} \times 1.05^{-13.5} + 25{,}000 \times \frac{2{,}006}{29{,}919} \times 1.05^{-14.5}$$

$$+ 30{,}000 \times \frac{4{,}448}{29{,}919} \times 1.05^{-15.5}$$

$$= 467.05 + 826.19 + 2{,}093.65$$

$$= 3386.89$$

ANSWER: A

19.

$$\text{AAL}_{2017} = AB_{2017} \times {}_{20}E_{45} \times \ddot{a}_{65}^{(12)}$$

$$AB_{2017} = 15 \times 0.02 \times 63{,}000 \times 1.05^{14} = 37{,}420.71$$

$$\text{AAL}_{2017} = 37{,}420.71 \times 0.15 \times 11 = 61{,}744.17$$

ANSWER: A

20.

$$V_0 = 0.02 \times YOS_0 \times S_0 \times \frac{\ell_{65}}{\ell_{51}} \times (1+i)^{-14} \times \ddot{a}_{65}^{(12)}$$

$$= 0.02 \times 10 \times 68{,}700 \times \frac{7{,}533{,}964}{8{,}897{,}913} \times 1.06^{-14} \times 9.4316 = 48{,}531.69$$

The normal cost contribution is:

$$NC = V_0\left(\frac{S_1}{S_0} \times \frac{YOS_1}{YOS_0} - 1\right) = 48{,}531.69\left(\frac{70{,}400}{68{,}700} \times \frac{11}{10} - 1\right)$$

$$= 48{,}531.69 \times 0.1272 = 6{,}173.23$$

ANSWER: A