1. **Learning Objectives:**

2. The candidate will understand and be able to assess issues and concerns common to actuarial models and their development and management.

**Learning Outcomes:**

(2a) Describe Model Efficiency concepts and explain and apply both the representative scenarios and replicating liabilities techniques for improving Model Efficiency.

(2b) Explain and apply the technique for the compression of model data using the "Cluster Analysis Spatial Approach".

**Sources:**

LAM-132-19: Cluster Analysis: A Spatial Approach to Actuarial Modeling

Model Efficiency Study Results, Nov 2011

**Commentary on Question:**

*Commentary listed underneath question component.*

**Solution:**

(a)

(i) Describe the cluster modeling approach.

(ii) Describe potential drawbacks of this approach.

**Commentary on Question:**

*This part was done reasonably well. Many candidates went into great detail to describe the step by step process of how to perform cluster modelling in part (i) but missed describing what the cluster modelling approach actually is as a concept.*

(i) Cluster modelling is a technique to compress data into a subset of representative cells or policies to decrease model run times with limited loss of accuracy. The representative cells have the same characteristics to the full inforce. It is done by calculating the distance between policies and grouping small policies to nearby larger policies in an iterative process until the desired level of compression is reached.
1. Continued

(ii) Drawbacks of cluster modelling are that it can require considerable effort to setup and test. Accuracy of the results may be potentially reduced for tail metrics like CTE. Bias is also a concern.

(b) Calculate the age of the seriatim record created by the cluster modeling approach. Show all work.

**Commentary on Question:**

*Generally this question was done well. Most candidates displayed a good understanding of the cluster modelling process. A common mistake was to not consider that when policies are combined not only do the face amounts needs to be added together but so do the other variables; in this instance the PV(Profit) and Reserve.*

**Step 1. Calculate the distance between the policies**

<table>
<thead>
<tr>
<th>Policy</th>
<th>Age</th>
<th>Gender</th>
<th>Face</th>
<th>PV(Profit)</th>
<th>Reserve</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27</td>
<td>M</td>
<td>1000</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>55</td>
<td>M</td>
<td>1050</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>F</td>
<td>1100</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>81</td>
<td>F</td>
<td>1200</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

Iteration #1:

\[ \text{Distance policy } x \leftrightarrow y = \sqrt{(PV(Profit)_x - PV(Profit)_y)^2 + (Reserve_x - Reserve_y)^2} \]

1 \leftrightarrow 2 = \sqrt{(10 - 8)^2 + (6 - 9)^2} = 3.61
1 \leftrightarrow 3 = \sqrt{(10 - 12)^2 + (6 - 4)^2} = 2.83
1 \leftrightarrow 4 = \sqrt{(10 - 6)^2 + (6 - 7)^2} = 4.12
2 \leftrightarrow 3 = \sqrt{(8 - 12)^2 + (9 - 4)^2} = 6.40
2 \leftrightarrow 4 = \sqrt{(8 - 6)^2 + (9 - 7)^2} = 2.83
3 \leftrightarrow 4 = \sqrt{(12 - 6)^2 + (4 - 7)^2} = 6.71

Importance = size(Face) * distance to nearest policy

Policy 1 closest to 3: 1000*2.83=2830
Policy 2 closest to 4: 1050*2.83=2970
Policy 3 closest to 1: 1100*2.83=3113
Policy 4 closest to 2: 1200*2.83=3396
1. Continued

Since policy 1 has the least importance it gets mapped to policy 3.

Policy 3: Face now 1000+1100=2100, PV(Profit) 10+12=22, Reserve 6+4=10

Iteration #2:

Recalculate distances
2 ↔ 3 = \sqrt{(8 - 22)^2 + (9 - 10)^2} = 14.04
2 ↔ 4 = 2.83 same as before
3 ↔ 4 = \sqrt{(22 - 6)^2 + (10 - 7)^2} = 16.28

Recalculate importance
Policy 2 closest to 4: 1050*2.83=2970
Policy 3 closest to 2: 2100*14.04=29484
Policy 4 closest to 2: 1200*2.83=3396

Since policy 2 has the least importance it gets mapped to 4.

Policy 4: Face now 1050+1100=2150, PV(Profit) 8+6=14, Reserve 9+7=16

Iteration #3:

With only policy 3 and 4 left they will be the same distance apart.
Since policy 4 has the greater face amount of the two, policy 3 will get mapped to policy 4.

Policy 4 is the representative policy. It has age 81.

(c) Critique the following statement:

“Reserve replication techniques are superior to cluster modeling.”

Commentary on Question:
Candidates typically got some credit but missed getting all credit by not giving a thorough response. Many critiqued reserve replication saying it was not superior but neglected to consider that cluster modelling has similar drawbacks. For example, saying reserve replication takes a fair amount of effort to get started was not sufficient to prove it is not superior since cluster modelling also has a similar drawback.
1. **Continued**

Reserve replication or using replicating liabilities is another technique, like cluster modelling, to compress inforce to help with model efficiency. Unlike cluster modelling it uses a function to be optimized and a set of constraints.

Both models can produce accurate results with good compression. Both suffer from bias. Both take time and effort to implement. Reserve replication may do better with tail scenarios than cluster modelling. Counterintuitively for reserve replication lower compression requires more effort.

I disagree with the statement. We cannot conclusively say reserve replication is superior.
2. **Learning Objectives:**
   1. The candidate will understand, evaluate and use stochastic models, generalized linear models ("GLMs"), multi-state, and transition matrix models. The candidate will demonstrate an understanding of their underlying methodologies, strengths, limitations, applications, technical challenges, and governance.

4. The candidate will understand the basic design and function of Economic Scenario Generators and EquityLinked Insurance Models.

**Learning Outcomes:**

(4b) With respect to Equity-Linked models:
- Describe and apply methods for modeling long-term stock returns and certain guarantee liabilities (GMMB, GMDB, GMAB).
- Describe and evaluate the Actuarial and Hedging risk metrics for GMAB and GMDB models.
- Describe and apply methods for modeling Guaranteed annuity options and Guaranteed Minimum Income Benefits (GMIB), and EIA guarantees.

**Sources:**
AI - Investment Guarantees, Hardy, 2003, Chapters 1, 2, 6, 7 (pp. 115-125), 8 (pp. 133-143), 12, 13

**Commentary on Question:**
*Overall objective of the question was to test on understanding of different indexation designs for equity indexed strategy, including the cost of hedging and payouts under this product.*

**Solution:**
(a) For the following indexation methods:

- Multi-Year Point-to-Point (PTP)
- Annual Ratchet
- High Water Mark

(i) Describe the characteristics of each method.

(ii) Compare the cost of each method.

**Commentary on Question:**
*Candidates generally performed well for this part of the question.*
2. Continued

(i)

- Multi-Year Point-to-Point (PTP) - Uses the closing index level divided by the opening level over a specified term. All interim levels during the term are ignored.

- Annual Ratchet - The indexation is calculated by the greater of the floor rate and the increase in the underlying index, locking in the annual indexation gains. May use simple or compound interest.

- High Water Mark - Uses the maximum index value over the term, typically taking policy anniversary values only (rather than index at maturity as used for PTP).

(ii)

- Generally, the High Water Mark will be the most expensive, followed by the Annual Ratchet and the PTP.

- The compound version of the Annual Ratchet is more expensive than the simple interest.

(b) Calculate the value of the guaranteed minimum annual interest feature using the Put-Call Parity. Show all work.

Commentary on Question:
This question was not well answered, many candidates were confused about the value to use for the stock price (S) and strike amount (K). Partial grades were given for the calculation when the right interest, call price and stock value were used.

Put – Call Parity:
Call (at time 0) – Put (at time 0) = Stock (at time 0) – K (at maturity) \( \exp[- r (T-t)] \)

\( T-t = 3, \) 3 – year maturity (given)
Risk Free rate = 4% (given)
Annual Guarantee Rate = 3% (given)
Call = 45 (given) with the same Term and Strike as the Product
Since the Term and Strike of the Call option is the same as the product, participation rate should not be applied.

Base on the Formula above the Put Value:
Call + K x (1+r)-(T-t) – Stock
Where Strike Value (K) = Guarantee at maturity = 2000 x 1.03^3 = 2,185.45
45 + 2,185.45 x 1.04 ^ -3 -2000 = (12.14).

The value is negative since the guarantee growth rate is less than the risk-free rate.
2. Continued

(c) Calculate the EIA surrender value at the end of each policy year. Show all work.

**Commentary on Question:**

Most candidates performed generally well on this part, the following common mistakes occurred: deducting surrender charge at the end of period (year 3), not capturing the guarantee interest (3%) or applying the participation rate after the cap. Most candidates that demonstrated all their work usually performed significantly better.

<table>
<thead>
<tr>
<th>Period</th>
<th>Index</th>
<th>Calculated Return (A)</th>
<th>Calculated Rate with participation (B) = A x 70%</th>
<th>Cap /Floor Cap =10% Floor = 0%</th>
<th>Fund with EIA rate (Previous year x return) (C)</th>
<th>GV rate</th>
<th>GV (Previous year x return) (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12,500</td>
<td>0</td>
<td>2,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11,000</td>
<td>-12%</td>
<td>-8.4%</td>
<td>0%</td>
<td>2,000</td>
<td>3%</td>
<td>2,060</td>
</tr>
<tr>
<td>2</td>
<td>12,750</td>
<td>15.9%</td>
<td>11.1%</td>
<td>10%</td>
<td>2,200</td>
<td>3%</td>
<td>2,122</td>
</tr>
<tr>
<td>3</td>
<td>13,300</td>
<td>4.3%</td>
<td>3.0%</td>
<td>3.0%</td>
<td>2,266</td>
<td>3%</td>
<td>2,186</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period</th>
<th>EIA amount (C)</th>
<th>GV (D)</th>
<th>Maximum of EIA and GV</th>
<th>Surrender Charge</th>
<th>Surrender Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2,000</td>
<td>2,000</td>
<td>2,000</td>
<td>100</td>
<td>1,960</td>
</tr>
<tr>
<td>1</td>
<td>2,000</td>
<td>2,060</td>
<td>2,060</td>
<td>100</td>
<td>2,100</td>
</tr>
<tr>
<td>2</td>
<td>2,200</td>
<td>2,122</td>
<td>2,200</td>
<td>100</td>
<td>2,100</td>
</tr>
<tr>
<td>3</td>
<td>2,266</td>
<td>2,186</td>
<td>2,266</td>
<td>None since maturity</td>
<td>2,266</td>
</tr>
</tbody>
</table>

(d) SLC Life plans to use a traditional deterministic valuation for the EIA product. Critique this decision.

**Commentary on Question:**

Candidates performed well and were able to critique the use of deterministic valuation. Points were also given for benefits of stochastic valuation.

Do not recommend using deterministic valuation method for the following reason:
- Any deterministic single path used to model extreme equity market return will lack credibility
- Results are difficult to interpret and communicate to senior management
- Does not capture the full risk profile, in particular tail scenarios
- Does not provide essential qualitative or quantitative information

(e) Describe four assumptions required to value the above EIA product under a risk-neutral stochastic framework.
2. **Continued**

**Commentary on Question:**
Candidate performed poorly for this part of the question, in particular the conditions implicit in risk-neutral framework were listed instead of the assumptions specific to valuing the Equity Indexed Annuity product. Partial credits were given for common assumptions between generic risk-neutral framework and assumptions to value the EIA product.

- Risk-Free Rate of Return: valuing the option under a risk-neutral stochastic model requires an assumption for risk-free returns
- Index Volatility: To model index returns stochastically
- Lapse: Since payouts are path-dependent, payouts will depend on when lapses occur
- Other Liability Assumptions: For example, mortality to project liabilities cashflows appropriately
3. **Learning Objectives:**

3. The candidate will understand the principles of Asset-liability Management ("ALM") and Projection Models, and be able to describe and evaluate various techniques for addressing the mitigation of risk.

**Learning Outcomes:**

(3a) With respect to Asset-Liability Models:

- Describe and apply the fundamental elements of the theory and practice of ALM in an insurance company, including assessing the dangers of mismatched assets and liabilities.
- Describe and demonstrate how ALM can be used to identify and manage product and asset risks, including:
  - Major product risks for which ALM can be a useful tool for their management.
  - Using ALM as a means to manage interest rate risk, equity risk, and risks from optionality.
- Describe how common insurance contracts and variations generate embedded options in an insurer's balance sheet, and assess basic strategies for managing exposures created by such embedded options.
- Describe and apply the basic concepts of cash flow matching, immunization, duration/convexity matching, segmentation.
- Describe and apply Key Rate Durations (KRD) and their use in evaluating interest rate sensitivities of portfolios, including understanding the derivation of KDRs, the profiles of KDRs for selected major asset types, and assessing KRDs in a portfolio context.
- Describe and evaluate the Goldman Sachs' ALM/Strategic Asset Allocation approach for integrating ALM into an enterprise's risk and financial management framework.
- Describe and evaluate ALM modeling considerations in the context of modeling risk aggregation, dependency, correlation of risk drivers and diversification.

**Sources:**

Chapter 22 of Life Insurance Accounting, Asset/Liability Management

Ch. 16 of ALM Management of Financial Institutions, Tilman, 2003

**Commentary on Question:**

The goal of this question is for candidates to show an understanding of the objectives of ALM, evaluate methods and metrics used in ALM, identify options embedded in insurance products, and show how derivatives can be used to manage interest rate risk. Candidates generally did well in part (a) and (c).
3. Continued

Solution:
(a) Critique these strategies and propose improvements if needed.

Commentary on Question:
This question tested the candidates’ understanding of the objectives, evaluation methods, and metrics used in ALM. Candidates generally did well. For full credit, candidates must clearly explain why the proposed strategy is not a good strategy.

Critique:
- There is maturity mismatch as assets have longer duration than certain liabilities.
- Duration measures small changes in interest rates and assumes a parallel shift in the yield curve.
- Statutory cash flows has a deliberate bias towards conservatism as it includes provisions for adverse deviation.

Propose changes:
- Create separate segments for products with longer duration liabilities (e.g. Life) and shorter duration liabilities (e.g. Annuities) and use assets with similar durations in each segment.
- Use convexity as well as duration to protect against a wider range of interest movements or use key rate durations to address non-parallel shifts.
- Use an economic basis as it gives the purest representation of asset/liability dynamics.

(b) (i) Describe the embedded options of this feature from YYZ’s perspective.

(ii) Explain the ALM risks created.

(iii) Propose risk mitigation strategies.

Commentary on Question:
This question required the candidate to identify options embedded in insurance products. Many candidates failed to recognize and explain the two options that policy loans exhibit. Candidates need to provide at least two reasonable mitigation strategies in order to receive full credit on part iii.

The policy loan feature is combined of put and call options. It gives the policyholder rights to sell the policy for its cash value when interest rates are above the maximum policy loan rate. It also gives the policyholder rights to buy the policy back at a guaranteed price when interest rates are below the maximum loan rates.
3. Continued

ALM risk created includes loss of investment income if the funds requested under the option were invested at an interest rate higher than the maximum policy loan rate. Also, if a large portion of the assets are longer duration fixed income assets, market values will decrease when interest rates rise. If interest rates are higher than the maximum policy loan rate, there will be a large number of policy loan requests that will require liquidating assets at depressed prices. When interest rates decline, funds borrowed to earn higher rates will return to the insurer, but lower rates may mean that assets are repurchased at prices higher than the ones received when they were liquidated.

Potential risk mitigation strategies include

- Have both long term and short term assets in the portfolio backing the liabilities.
- Adjust dividends to reflect policy loan activities, i.e. allow policy loan assets to back liabilities.
- Design policy loan with variable loan rates.

(c) YYZ is considering selling a Universal Life product that credits 50 basis points above LIBOR. YYZ will back the liability with the following assets:

- 10-year bond yielding 4%
- Interest rate swap that receives LIBOR and pays 3% fixed

Recommend if YYZ should sell this product. Justify your response.

Commentary on Question:

The question required the candidate to analyze how derivatives can be used to manage interest rate risk. Candidates were given full credit if they could show that a fixed spread is maintained and make a recommendation. Candidates could either recommend or not recommend to sell this product with a valid concern such as duration or cash flow mismatch, but the following analysis needed to be included in their assessment.

Use a portfolio consisting of the 10-year bond and Interest Rate Swap. The bond earns 4% and YYZ pays 3% under the swap agreement thus meeting the 1% spread target. After crediting the LIBOR + 50bps, YYZ left with 50 bps of net profit regardless of interest rate environment. Therefore, I recommend selling this product.
4. Learning Objectives:

1. The candidate will understand, evaluate and use stochastic models, generalized linear models ("GLMs"), multi-state, and transition matrix models. The candidate will demonstrate an understanding of their underlying methodologies, strengths, limitations, applications, technical challenges, and governance.

Learning Outcomes:

(1a) With respect to stochastic models:
- Explain and apply the stochastic modeling methodology, including measurement metrics (e.g., CTE).
- Describe and apply the theory and uses of real world versus risk neutral assumptions.
- Describe and apply the techniques of Monte Carlo simulation (including variance reduction and importance sampling).
- Describe and evaluate Random Number Generator models, and explain their uses, advantages, and theory.
- Describe and evaluate how stochastic models may be used to understand mortality and policyholder behavior risks and inform the use of reinsurance.
- Describe the technique of nested stochastic projections and explain why they are needed, and evaluate implementation issues.

Sources:
Stochastic Analysis of Long Term Multiple-Decrement Contracts, SoA, Clark, 2008 (exclude appendices)

Stochastic Modeling is on the Rise, Product Matters, Nov 2016


Commentary on Question:
Commentary listed underneath question component.

Solution:

(a) Describe the benefits of using stochastic modeling techniques instead of traditional deterministic techniques for non-financial risks.

Commentary on Question:
Candidates generally did well on part (a). They more often included the full risk profile (multiple scenarios) than the interactions between risks.
4. Continued

Stochastic modeling and analysis provides a full risk profile for the company. Stochastically generated scenarios introduce volatility around the best estimate, instead of only looking at one single scenario. Interactions between various risks and tail events can also be analyzed, which can lead to better business decisions. For example, it would be easier to study the effectiveness of various reinsurance programs.

(b) Recommend variables that should be modeled stochastically and variables that should be modeled deterministically based upon the above results. Justify your answer.

Commentary on Question:
Candidates did well on part (b). The question was intended to lead candidates to see lapses as not meriting stochastic testing in this case, but candidates did earn credit when they gave strong justifications for stochastic lapses, such as recognizing the interaction between mortality and lapses.

Base mortality should be modeled stochastically. The results indicate that mortality is very sensitive to fluctuations and has a large impact on surplus.

A catastrophic mortality variable should also be modeled stochastically. Results indicate that there is a lot of risk in the two outbreak events. Stochastic modeling can help understand the company's exposure to tail risks caused by extreme events which may have low frequency but high severity.

Mortality improvement and base lapse results indicate these two variables are insensitive to changes; these two variables should be modeled deterministically. It is likely not worth the time or effort to model these two variables stochastically. The company could continue to test these two variables using stress testing.

(c) BWI runs its stochastic model and compares the results to its deterministic model as follows:

<table>
<thead>
<tr>
<th>Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic Best Estimate</td>
</tr>
<tr>
<td>Average of all Stochastic Results</td>
</tr>
</tbody>
</table>

Interpret the results as they relate to the validity of the stochastic model.
4. **Continued**

**Commentary on Question:**

To receive full credit, candidates needed to clearly state that the results suggest the stochastic model is not valid. Candidates frequently cited Jensen’s inequality here, some used it to justify the validity of the stochastic model, but the magnitude of the difference is not consistent with that explanation. Some candidates also stated that other measures from the stochastic results (e.g. CTE-70) would be “better” than the average stochastic result, which is not relevant to the attempt to reconcile the stochastic model with the best estimate model.

The results should raise a red flag; a stochastic average that differs greatly from the deterministic best estimate may indicate a problem with the stochastic model, as you would expect them to be close.

It is possible that one or more of the stochastically modeled variables is not being modeled properly. Each stochastically modeled variable should be calibrated and tested individually to see how it flows through the model.

If multiple catastrophic events (pandemics, natural disasters, etc.) are being modeled in a single catastrophic mortality variable, this should be examined to ensure the risks are interacting with each other as intended.

Evaluate how each reinsurance arrangement would impact BWI’s cash flows under each of the following situations:

(i) If a pandemic event occurred.

(ii) If future mortality experience is better than expected in the years following the pandemic.

**Commentary on Question:**

Most candidates could explain the basics of how each of these three reinsurance arrangements work; however, it was rare that a candidate received full credit.

Some candidates simply ranked the three impacts, instead of explaining each individually. These candidates received partial credit.

Many did not include the reinsurance premium as a cash flow.

For the first reinsurance arrangement, some referenced the $250,000 as if it were a cap on the total claim amount, instead of a cap on each policy.

Some failed to realize that the second arrangement was 100% reinsured.
4. Continued

In part (ii), many candidates didn’t recognize that the mortality improvement immediately follows a pandemic, which would raise YRT premiums or potentially result in loss carry forward.

A. Excess YRT

(i) BWI pays a premium to the reinsurer. In a pandemic, mortality increases severely, and many more death claims are paid than expected. For each policy with a face amount greater than $250,000, BWI receives a reimbursement from the reinsurer for the (Face Amount - $250,000).

(ii) The reinsurer is likely to increase the YRT premium paid by BWI following the pandemic. Future mortality improvement would mean fewer death claims are paid. The reinsurer still gives BWI a reimbursement for claims over $250,000.

B. 100% YRT with Experience Refund

(i) BWI pays a large premium to the reinsurer to cover all its mortality risk. Because every policy is 100% reinsured, BWI would be fully reimbursed by the reinsurer for all death claims paid. Because the actual death claim experience is higher than anticipated during the pandemic, the reinsurer will not refund any premiums to BWI.

(ii) YRT premiums are likely to increase. If there is a loss carry over provision, no refund in future years, until loss carry forward is extinguished. If there is no loss carry over provision, BWI is likely to receive an experience refund during periods with mortality improvement.

C. Stop Loss

(i) BWI pays a relatively small premium to the reinsurer to cover its mortality risk. The reinsurer would reimburse BWI for claims in excess of an attachment point limit set in the reinsurance contract. Because the level is probably set very high, BWI still pays a lot of claims in a pandemic scenario.

(ii) BWI will still pay the small premium to the reinsurer. They will likely not receive any reimbursements during a time of improving mortality. Depending on the treaty and the attachment point, BWI might have to pay the reinsurer a refund of previous claims because of the better experience.