INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 40 points.
   This exam consists of 8 questions, numbered 1 through 8.
   The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam GIADV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d’examen pour la version française.
**BEGINNING OF EXAMINATION**

1. *(5 points)* Mitigating Terrible Losses, Inc. (MTL) writes catastrophe insurance against three different low probability events for a large individual account. The three events are mutually independent. The following table displays the probability of each event and the loss paid should it occur.

<table>
<thead>
<tr>
<th>Event</th>
<th>( p )</th>
<th>( L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zombie Apocalypse</td>
<td>0.01</td>
<td>1,000</td>
</tr>
<tr>
<td>Alien Invasion</td>
<td>0.02</td>
<td>3,000</td>
</tr>
<tr>
<td>Robot Overlords</td>
<td>0.03</td>
<td>5,000</td>
</tr>
</tbody>
</table>

MTL calculates risk loads based on variance using a multiplier of \( \lambda = 0.0002 \).

(a) *(1 point)* Calculate the risk load for this account.

MTL adds a new, smaller, account for which MTL pays 200 in the event of a Zombie Apocalypse, 500 in the event of an Alien Invasion, and nothing in the event of Robot Overlords.

(b) *(1 point)* Calculate the risk load for the new account using the marginal variance method.

(c) *(1.5 points)* Demonstrate that when using the marginal variance method the risk loads for the existing account and the new account are not renewal additive.

MTL is considering the marginal surplus method with a required return of \( \gamma = 0.15 \). This requires a value for \( z \), the distribution percentage point corresponding to the acceptable probability that the actual result will require even more surplus than allocated.

(d) *(1.5 points)* Calculate the value of \( z \) that produces the same risk load for the new account as obtained using the marginal variance method.
2. (6 points) Casualty R Us Reinsurance Company is evaluating a proposed casualty per occurrence excess treaty covering the layer 2,000,000 excess of 1,000,000.

The following information has been provided:

<table>
<thead>
<tr>
<th>Subject Premium</th>
<th>Underlying Limit</th>
<th>Policy Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,000,000</td>
<td>0</td>
<td>1,000,000</td>
</tr>
<tr>
<td>4,000,000</td>
<td>0</td>
<td>2,000,000</td>
</tr>
<tr>
<td>5,000,000</td>
<td>0</td>
<td>3,000,000</td>
</tr>
<tr>
<td>8,000,000</td>
<td>1,000,000</td>
<td>2,000,000</td>
</tr>
<tr>
<td>9,000,000</td>
<td>1,000,000</td>
<td>3,000,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Policy Limit</th>
<th>Increased Limits Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000,000</td>
<td>1.00</td>
</tr>
<tr>
<td>2,000,000</td>
<td>1.16</td>
</tr>
<tr>
<td>3,000,000</td>
<td>1.28</td>
</tr>
<tr>
<td>4,000,000</td>
<td>1.38</td>
</tr>
</tbody>
</table>

The expected loss ratio is 60%.

(a) (3 points) Calculate the expected losses in the layer using an exposure rating approach.

A swing plan has been proposed with the following features:
- Retro Premium = (Actual Layer Losses) x 100 / 80
- Provisional Rate = 20%
- Maximum Premium = 37.5% x Subject Premium
- Minimum Premium = 12.5% x Subject Premium

The following information has been provided:

<table>
<thead>
<tr>
<th>Range of Loss Cost</th>
<th>Probability</th>
<th>Average Loss Cost in Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10%</td>
<td>0.10</td>
<td>4%</td>
</tr>
<tr>
<td>10-30%</td>
<td>0.75</td>
<td>19%</td>
</tr>
<tr>
<td>30%+</td>
<td>0.15</td>
<td>44%</td>
</tr>
</tbody>
</table>

(b) (2 points) Calculate the expected loss ratio on this swing plan.

(c) (0.5 points) Explain the concept of “balance” in the context of swing plans.

(d) (0.5 points) Explain why the provisional rate may be too low.
3. (4 points) You are employing the methodology set forth in “A Framework for Assessing Risk Margins.”

Marshall, et al. summarize the methodology in use at the time as follows:

- Coefficients of variation for individual portfolios are determined.
- Correlations between individual portfolios are determined.
- The above values are determined separately for outstanding claim liabilities and premium liabilities.
- A statistical distribution is selected and combined with the above to determine the aggregate risk margin.

(a) (2 points) State four concerns expressed regarding this methodology.

(b) (2 points) Explain how each concern stated in part (a) is addressed by the framework discussed in the paper.
4. \textit{(8 points)} You are interested in determining unpaid claim estimates. The triangle of paid claims data you are working with, by accident year and development year, is presented below. It is assumed that all claims are fully developed after four years.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Development Year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>8,000</td>
</tr>
<tr>
<td>2</td>
<td>10,000</td>
</tr>
<tr>
<td>3</td>
<td>12,000</td>
</tr>
<tr>
<td>4</td>
<td>14,000</td>
</tr>
</tbody>
</table>

One of the models suggested by Venter is the parameterized BF (Bornhuetter and Ferguson) model, where the expected emerged loss is a lag factor $f(d)$ times an accident year parameter $h(w)$. An iterative method can be used to find the $f(d)$ and $h(w)$ values that minimize the sum of the squared residuals.

(a) \textit{(1 point)} Calculate the natural starting values for $f(1), f(2), f(3)$ and $f(4)$ implied from the standard chain ladder method. These values should sum to 1.

(b) \textit{(2 points)} Calculate starting values for $h(1), h(2), h(3)$ and $h(4)$ using the $f(d)$ values from part (a).

The values of $h(1), h(2), h(3)$ and $h(4)$ that minimize the sum of the squared residuals are 21,209, 20,633, 23,626 and 30,435, respectively.

(c) \textit{(2 points)} Calculate the values of $f(1), f(2), f(3)$ and $f(4)$ that minimize the sum of the squared residuals.

(d) \textit{(1 point)} Estimate the unpaid claims for each of accident years 2, 3 and 4 using the parameterized BF model.

(e) \textit{(1 point)} Estimate the unpaid claims for each of accident years 2, 3 and 4 using the standard chain ladder method.

You now estimate the variability of the chain ladder unpaid claim estimates using Mack’s method.

(f) \textit{(1 point)} Estimate the proportionality constants $\alpha_1^2$, $\alpha_2^2$, and $\alpha_3^2$. 
5. (4 points) You are given the following triangle of cumulative paid losses:

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Months of Development</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>2015</td>
<td>5,000</td>
</tr>
<tr>
<td>2016</td>
<td>6,000</td>
</tr>
<tr>
<td>2017</td>
<td>7,000</td>
</tr>
<tr>
<td>2018</td>
<td>8,000</td>
</tr>
</tbody>
</table>

You apply Clark’s stochastic reserving model using the LDF method and a Pareto distribution with cumulative distribution function \( G(x) = 1 - \left( \frac{\theta}{x + \theta} \right)^\alpha \).

Clark lists several advantages to using parametric curves to describe the expected emergence pattern.

(a) (0.5 points) Provide two advantages that are relevant for this triangle.

(b) (0.5 points) Explain why the Cape Cod method is generally preferred to the LDF method.

The maximum likelihood estimate of \( \theta \) is 21.897 and of \( \alpha \) is 4.3335.

(c) (0.5 points) Calculate \( ULT_{2017} \).

(d) (1 point) Estimate the expected payments in 2020 for accident year 2018.

Clark lists the following key assumptions:

(i) Incremental losses are independent of all other incremental losses.
(ii) The emergence pattern is the same in all accident years.
(iii) Variance estimates are based on the Rao-Cramer lower bound.

(e) (1.5 points) Identify which of these assumptions are not made by Mack. Justify your responses.
6. \( (4 \text{ points}) \) Consider the following three layers:

Layer 1: 100 excess of 0
Layer 2: 100 excess of 100
Layer 3: \( \infty \) excess of 200

Let \( t_1, t_2, \text{ and } t_3 \) be the trend factors that apply to the three layers, respectively, when trend of 10% is applied to a given probability distribution.

Consider the following six possibilities:

(i) \( t_1 < t_2 < t_3 \)
(ii) \( t_1 < t_3 < t_2 \)
(iii) \( t_2 < t_1 < t_3 \)
(iv) \( t_2 < t_3 < t_1 \)
(v) \( t_3 < t_1 < t_2 \)
(vi) \( t_3 < t_2 < t_1 \)

For each of (i) through (vi), either

- Identify a probability distribution with positive probability at exactly two discrete points (loss sizes) for which the inequality is satisfied; or
- Explain why there is no probability distribution for which the inequality is satisfied.
7. (4 points) You are applying the Target Total Rate of Return Model and are given the following information:

- The risk-free rate is 2%.
- The insurer’s beta is 1.5.
- The market risk premium is 6%.
- Premiums are 850,000.
- Equity is 500,000.
- Investable assets are 1,200,000.
- The insurer’s investment return is 7%.

(a) (1 point) Calculate the target total rate of return using the Capital Asset Pricing Model (CAPM).

(b) (1.5 points) Calculate the underwriting profit margin.

(c) (1 point) Explain how the existence of catastrophe risk makes the use of CAPM problematic for insurers.

(d) (0.5 points) Explain the relationship between the internal rate of return and net present value when employing discounted cash flow analysis.
8. (5 points) Specialist Reinsurance Company has proposed offering a finite risk cover without reinstatements with the following terms:

- Annual Premium: 50,000,000
- Occurrence Limit: 200,000,000
- Profit Commission: 95% after 10% margin on Annual Premium
- Additional Premium: 60% of (Loss + Margin – Annual Premium)

Any single loss will fully exhaust the limit.

(a) (2 points) Calculate the net profit or loss for Specialist assuming:

(i) No losses

(ii) One or more losses

(b) (1 point) Calculate the rate on line for an equivalent traditional risk cover.

Specialist wishes to modify the terms of the proposed finite risk cover so that the rate on line for an equivalent traditional risk cover is 15%.

(c) (1.5 points) Calculate the Additional Premium percentage required to match the rate on line of 15%.

(d) (0.5 points) State the two conditions that a finite reinsurance arrangement must fulfill for a ceding company to consider it insurance.

**END OF EXAMINATION**
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