INSTRUCTIONS TO CANDIDATES

**General Instructions**

1. This afternoon session consists of 6 questions numbered 10 through 15 for a total of 40 points. The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

**Written-Answer Instructions**

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

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Tournez le cahier d’examen pour la version française.
10. (7 points) You are a new actuary in the portfolio management team at INT Life, a large life insurance company in the US. Your manager has asked for your help in evaluating the risk and return of the long-duration investment portfolio. You first comment on new investment considerations.

(a) (1 point) Describe the relationship between a bond’s bid-ask spread and its liquidity risk.

To analyze your current bond portfolio, you decompose the bond-level market spreads into default, liquidity, and risk premia using a regression model:

\[ \text{OAS} = \alpha + \beta \ \text{ExpectedDefaultCost} + \gamma \ \text{ExpectedLiquidityCost} + \eta \]

An analyst at INT suggests that you can use market-quoted CDS to measure the expected default cost. You believe that there are issues with such an approach.

(b) (2 points)

(i) (1 point) Explain to your manager why spread decomposition is useful.

(ii) (1 point) Explain some of the issues with the use of CDS spreads in the context described above.

Your manager then informs you that INT recently entered the commercial real estate market, citing the following reasons:

1. INT wanted to stay competitive with HNR Life, a rival company, who has also recently entered this market.
2. Both companies have similar knowledge and resources, but the portfolio management team at INT feels it can demand a higher market value for the same investments, due to INT’s lower average credit risk than HNR.
3. Due to its lower market efficiencies, this asset class has more lucrative opportunities than REITs.
4. The potential of new information from one of its affiliates provides INT with a competitive advantage over HNR.

(c) (2 points) Evaluate the validity of each of these four reasons.
10. Continued

Finally, your manager is considering expansion to one additional alternative asset class not yet considered above. The team has been given the following goals to be achieved for any asset class to be added:

Goal 1: Help the portfolio outperform in times of financial crises
Goal 2: Opportunity to take advantage of an inefficient market
Goal 3: Support duration matching hedging activities

(d) (2 points) Recommend an asset class that could meet all of these goals.
11. (7 points) The Chief Investment Officer (CIO) at XYZ Insurance Company has been tasked with making equity investment recommendations to the investment committee.

The CIO presents the following 3 stocks to the committee for discussion:

<table>
<thead>
<tr>
<th></th>
<th><strong>Stock A</strong></th>
<th><strong>Stock B</strong></th>
<th><strong>Stock C</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector</td>
<td>Financial institution</td>
<td>Technology company</td>
<td>Financial institution</td>
</tr>
<tr>
<td>Index inclusion</td>
<td>Not included in any equity index</td>
<td>Just added to an equity index</td>
<td>Not included in any equity index</td>
</tr>
<tr>
<td>Dividend policy</td>
<td>Follows a strict policy that has paid a dividend every quarter in the company’s history</td>
<td>Considering paying a dividend for the first time</td>
<td>Regularly pays a stable dividend each quarter</td>
</tr>
</tbody>
</table>

Based on his research, the CIO expects Stock A to be included in a major index in the coming months.

(a) (1 point) Explain the strategy and rationale an arbitrageur could use to take advantage of the CIO’s expectation.

(b) (1 point) Describe factors that may cause this strategy to be unprofitable.

(c) (1 point) Describe Ambiguity Aversion and how it would impact the decision making of a dividend-focused investor deciding between investing in either Stock A or Stock B.

The investment committee is interested in Stock B but would like to see how Stock B would have impacted the portfolio’s overall return over the past three years. You are an equity analyst on the CIO’s team. The CIO asks to you to get the daily closing price data for Stock B and to simulate any missing data.

You decide to use principal component analysis (PCA) to simulate the missing return data for Stock B. You select 4 finance stocks (Stocks D, E, F and G) that are very similar to Stock B and perform PCA on the standardized daily returns of all 5 stocks over the past year. The results of this PCA are as follows:
11. Continued

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Stock</th>
<th>Eigenvectors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>W1  W2  W3  W4  W5</td>
</tr>
<tr>
<td>2.46</td>
<td>B</td>
<td>0.45 -0.19 -0.63 0.10 0.59</td>
</tr>
<tr>
<td>0.72</td>
<td>D</td>
<td>0.45 -0.19 0.53 0.69 0.04</td>
</tr>
<tr>
<td>0.71</td>
<td>E</td>
<td>0.48 -0.15 -0.35 -0.03 -0.79</td>
</tr>
<tr>
<td>0.58</td>
<td>F</td>
<td>0.40 0.91 0.05 -0.04 0.05</td>
</tr>
<tr>
<td>0.52</td>
<td>G</td>
<td>0.45 -0.26 0.44 -0.71 0.16</td>
</tr>
</tbody>
</table>

You perform a second PCA on the standardized daily returns of Stocks D, E, F, and G over the two years preceding the past year. The results of this PCA are as follows:

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Stock</th>
<th>Eigenvectors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>W1  W2  W3  W4</td>
</tr>
<tr>
<td>2.2269</td>
<td>D</td>
<td>0.54 -0.22 -0.30 -0.76</td>
</tr>
<tr>
<td>0.6830</td>
<td>E</td>
<td>0.52 0.04 -0.61 0.60</td>
</tr>
<tr>
<td>0.6229</td>
<td>F</td>
<td>0.46 0.80 0.39 -0.05</td>
</tr>
<tr>
<td>0.4672</td>
<td>G</td>
<td>0.48 -0.56 0.62 0.26</td>
</tr>
</tbody>
</table>

You know the number of principal components that are needed to simulate the returns for stock B must account for at least 75% of the total variation in the system of the five stocks.

A sample of the standardized daily returns for stocks D, E, F, and G:

<table>
<thead>
<tr>
<th>Standardized Daily Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years Ago</td>
</tr>
<tr>
<td>1.50</td>
</tr>
<tr>
<td>2.00</td>
</tr>
<tr>
<td>3.00</td>
</tr>
</tbody>
</table>

(d) \(3 \text{ points}\)  Simulate the standardized daily return for stock B over the two years preceding the past year.

(e) \(1 \text{ point}\)  Explain an effective way to convert a PCA simulated standardized daily return to prices.
12. (8 points) Your company, ABC Trust, is providing a one-year loan of $200,000 to a highly rated company in six months and funding it by borrowing at the 12-month LIBOR rate. The loan interest rate is 6% annually.

The Chief Risk Officer (CRO) of ABC Trust recommends a hedge program by entering a caplet position to lock in a minimum of 1.5% interest rate spread for these transactions.

(a) (1 point) Describe the caplet that achieves the CRO’s recommendation.

ABC Trust has been using the CIR++ short-rate model to price interest rate derivatives, but the CRO strongly recommends a market model.

(b) (1 point) Compare and contrast the properties of the CIR++ and the CIR short-rate models.

(c) (1 point) Describe the advantages of using a market model over a short rate model when pricing interest rate derivatives.

You are given the following information:
- The current one-year forward rate for borrowing six months from now, F(0,0.5,1.5) = 4%.
- The six-month spot rate is 4% (annual compounding).
- The current implied volatility of all forward rates is flat at 30%.
- There is 0% probability of default on the loan.
- There are no transaction costs, bid-ask spreads or other expenses associated with the loan or any capital market instruments.

(d) (2 points) Calculate the price of the caplet the CRO recommends using Black’s formula.

(e) (2 points) Calculate the minimum profit that ABC Trust could earn on the loan after implementing the CRO’s hedge recommendation.

The CRO specifies the following volatility surface for her recommended Lognormal Forward-LIBOR model:

<table>
<thead>
<tr>
<th>Instant. Vols</th>
<th>Time: ( t \in (0,T_0) )</th>
<th>( (T_0,T_1) )</th>
<th>( (T_1,T_2) )</th>
<th>( \ldots )</th>
<th>( (T_{M-2},T_{M-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fwd Rate: ( F_i(t) )</td>
<td>( \sigma_{i,1} )</td>
<td>Dead</td>
<td>Dead</td>
<td>\ldots</td>
<td>Dead</td>
</tr>
<tr>
<td>( F_1(t) )</td>
<td>( \sigma_{1,1} )</td>
<td>\sigma_{2,2}</td>
<td>Dead</td>
<td>\ldots</td>
<td>Dead</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \sigma_{M,2} )</td>
<td>( \sigma_{M,3} )</td>
<td>\ldots</td>
<td>( \sigma_{M,M} )</td>
</tr>
<tr>
<td>( F_M(t) )</td>
<td>( \sigma_{M,1} )</td>
<td>( \sigma_{M,2} )</td>
<td>( \sigma_{M,3} )</td>
<td>\ldots</td>
<td>( \sigma_{M,M} )</td>
</tr>
</tbody>
</table>

(f) (1 point) Suggest two alternative assumptions to simplify her specification of the volatility surface.
13. (7 points) XYZ Corp issued a one-year, zero-coupon bond today, and has no other debt. You are modeling this bond using the Merton model.

You have the following information regarding XYZ Corp:

\[ F = \text{face amount of the debt} = 1000 \]
\[ A = \text{today’s fair value of the firm’s assets} = 1200 \]
\[ \sigma_A = \text{the volatility of the asset value of the firm} = 0.1 \]
\[ r = \text{the continuously compounded risk-free rate} = 0.03 \]

(a) (1 point) Determine a numerical upper bound for the probability of default over a 1-year time horizon, if market participants are risk-adverse.

(b) (2 points) Calculate the annual effective credit spread of the one-year zero coupon bond.

Your colleague wishes to compare the Merton model against a Markov chain model for the probability of default.

Your colleague assumes that XYZ Corp can be in 4 states. From ‘best’ to ‘worst’, the states are A, B, C, and D (for Default). The firm starts in state A and transitions every 0.5 years according to the following migration matrix:

\[
M = \begin{bmatrix}
A & B & C & D \\
0.93 & 0.02 & 0.03 & 0.02 \\
0.03 & 0.94 & 0.02 & 0.01 \\
0.02 & 0.08 & 0.80 & 0.09 \\
0 & 0 & 0.01 & 0.99 \\
\end{bmatrix}
\]

(c) (2 points) Describe four problems with the migration matrix M.

Your colleague decides to use Monte-Carlo simulation to simulate 10,000 scenarios of the state of XYZ Corp after one year. He describes his methodology in this way:

“For each scenario, I generate two random numbers uniformly selected between 0 and 1. Then, for each 0.5 year period, given the starting state S, the ending state E is the one such that

- the probability of being in state S or better is greater than the random number R
- the probability of being in state S or worse is smaller than R”

(d) (1 point) Critique your colleague’s methodology.
13. **Continued**

You and your colleague agree on an estimated probability of default of 0.05. You observe that the market price of the bond today is 920 and debtholders expect to earn an annualized rate of return of 5%.

(e) * (1 point) Calculate the loss given default ratio, using the estimated probability of default.

You assigned Analyst A to research the expected impact on the investment portfolio if you reallocate 5% of the portfolio from the existing equities to non-leveraged US direct real estate investment. Analyst A is unsure about the index that best represents the characteristics of such an investment.

(a) (1 point) List the attributes the analyst should look for in the appropriate index to represent direct real estate.

The Foundation is required to make a $50,000 distribution each year. You are considering changes to the current portfolio to maximize the amount available for distribution. Bonds are to be at least 75% of the portfolio and allocations to any asset class are to be in 5% increments. You have also received the below table of expected cash flows for each asset class under two different economic scenarios:

<table>
<thead>
<tr>
<th></th>
<th>Boom</th>
<th>Recession</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of occurrence</td>
<td>80%</td>
<td>20%</td>
</tr>
<tr>
<td>Cash Flows per 1% of portfolio allocation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>Equities</td>
<td>700</td>
<td>300</td>
</tr>
<tr>
<td>Direct Real Estate</td>
<td>800</td>
<td>0</td>
</tr>
<tr>
<td>REIT</td>
<td>600</td>
<td>200</td>
</tr>
</tbody>
</table>

(b) (2 points) Determine an asset allocation that would maximize the expected cash flows, while ensuring the required distribution is met under both scenarios.

You received two real estate investment proposals from another analyst, Analyst B. The first proposal is a direct investment in a privately owned office building. The second proposal is an investment in a publicly listed real estate company that specializes in the office building sector.

(c) (2 points) Compare the two proposals:

(i) from the liquidity perspective;
(ii) from the diversification perspective and;
(iii) from the acquisition and operating costs perspective.
14. Continued

Analyst B indicated that he generally recommends a direct investment in real estate over an indirect investment in real estate. He shared that the reason is he has recently read some blogs and seen some advertisements regarding the benefits of direct investments.

Analyst C also recommended a direct investment in real estate. Analyst C indicated the she has found that investment in a publicly listed real estate company has always produced superior returns, in spite of a recent downturn in returns over the last quarter.

(d) (1 point) Describe behavioral biases exhibited by Analyst B and Analyst C.
15.  (5 points) You have been recently hired by TUV, a new asset management firm. Your first task is to design a performance attribution process for the firm, which it does not have currently.

Although your firm only has a small team managing a small portfolio at the moment, the CFO wants the process to be flexible so it can handle a variety of assets and management styles.

(a)  (1 point) Discuss the characteristics of a flexible performance attribution model.

The following represents the investment decision making process in your firm:

- The Chief Investment Officer is responsible for making sector selection.
- The portfolio manager will be responsible for selecting individual securities in each sector.
- The Chief Investment Officer’s and portfolio manager’s bonuses depend on their respective contributions to the portfolio’s outperformance.

Based on the firm’s current portfolio, you designed a performance attribution process and prepared the following attribution report for a certain time period.

<table>
<thead>
<tr>
<th>Sector</th>
<th>W_p</th>
<th>W_b</th>
<th>R_p</th>
<th>R_b</th>
<th>Outperformance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Asset Allocation</td>
</tr>
<tr>
<td>Large Value</td>
<td>30%</td>
<td>35%</td>
<td>8%</td>
<td>6%</td>
<td>-0.30%</td>
</tr>
<tr>
<td>Large Core</td>
<td>25%</td>
<td>30%</td>
<td>7%</td>
<td>8%</td>
<td>-0.25%</td>
</tr>
<tr>
<td>Mid Value</td>
<td>C</td>
<td>20%</td>
<td>5%</td>
<td>3%</td>
<td>0.15%</td>
</tr>
<tr>
<td>Small Core</td>
<td>20%</td>
<td>15%</td>
<td>D</td>
<td>9%</td>
<td>0.45%</td>
</tr>
</tbody>
</table>

(b)  (1 point) Calculate A, B, C and D from the table above.

The CFO, after seeing this report, made the following comment:

“The Small Core sector has a much higher benchmark return than the Mid Value sector, yet this report tells me it is a good decision to overweight the Mid Value sector since asset allocation is positive. Clearly any allocation to the Mid Value sector is a bad choice since overweighing the Small Core sector gave higher outperformance.”
15. Continued

(c) *(1 point)* Critique the CFO’s comment including a recommended improvement to the performance attribution process.

(d) *(2 points)* Recalculate the contribution from asset allocation for each sector using your improved performance attribution process from part (c) above.

**END OF EXAMINATION**
Afternoon Session
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