CFE Sample CBT Question and Solutions

NOTE: The solutions presented here are not the full model solutions as published. There is no commentary and where there is more than one correct response, only one such response is presented.

ERM Sample CBT Question

(*14 points*) You are an actuary working in the Investment Department at JDY Life. JDY Life plans to offer a new indexed annuity product and is considering linking it to a composite index created by combining the ABC Fund and the XYZ Fund.

Your manager, Maggie, has asked for your assistance with understanding the correlation between the two funds for capital budgeting purposes and how it may change over time. Maggie has selected these two funds as she believes there may exist a natural hedge. She would ultimately like to model the joint distribution of the composite index using a copula and seeks an appropriate parameterization.

Maggie has provided you with the following information regarding the daily price change for each index over the past 30 days.

Time (t)	ABC Fund (j_t)	XYZ Fund (k_t)	$j_t k_t$	Rank of j_t	Rank of k_t
-30	-3.83%	-1.36%	0.052%	28	23
-29	2.55%	0.14%	0.004%	1	15
-28	2.11%	1.15%	0.024%	4	5
…	…	…	…	…	…
-3	-0.68%	0.64%	-0.004%	20	11
-2	0.50%	0.02%	0.000%	15	16
-1	-0.99%	-0.19%	0.002%	23	18

Arithmetic Mean	-0.109%	-0.155%	0.006%	15.50	15.50
Variance	0.034%	0.021%	0.001%	74.92	74.92
Covariance				18.45	

(a) (*3 points*) In order to calibrate a copula for modeling the joint distribution of the two funds, you first must determine an appropriate correlation metric. You consider the following correlation coefficients:

- Pearson's rho
- Spearman's rho
- Kendall tau

Additional analysis shows that there are 253 concordant pairs of index returns in the dataset.

(i) Calculate each correlation metric based on the data provided. Show all work.

The response for this part is to be provided in the Excel spreadsheet.

(ii) Describe the advantages and disadvantages of each metric that you should consider when selecting an appropriate correlation metric for parameterizing a copula.

ANSWER:

(b) (*5 points*) The first step in forecasting correlation between the two funds is to forecast the volatility of the individual funds. In order to forecast the volatility of each fund's value, you fit a GARCH(1,1) model, as shown below, to each series of fund returns:

$$h_t = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta h_{t-1}$$

The parameters for each model are provided in the following table:

Parameter	Fund ABC	Fund XYZ
α_0	0.0001	0.0002
α_1	0.1610	0.2570
β	0.7770	0.5650

You are also given the initial observation of each of the funds:

Time (t)	Fund ABC j_t	Fund XYZ k_t
0	5.01%	4.89%

(i) Determine which fund has the higher expected variance. Show all work.

The response for this part is to be provided in the Excel spreadsheet.

(ii) Assume that the t_0 conditional variances for each fund are equal to the empirical variances shown in the table on the previous page.

Calculate the conditional variances for the next two time steps, t=1 and t=2 for each fund using the GARCH(1,1) model. Show all work.

The response for this part is to be provided in the Excel spreadsheet.

(iii) Maggie explains that there are simple modifications that can be made to the basic GARCH(1,1) model in order to more accurately reflect the behavior of typical financial time series.

Explain why the following modifications may be implemented for volatility forecasting:

- Using the absolute value of the innovation term
- Using a separate parameter for positive shocks and negative shocks

ANSWER:

(c) (*3 points*) Maggie believes that the RiskMetrics approach is more appropriate than the GARCH model for forecasting the *covariance* between the two funds. She suggests using a risk decay factor of 0.95 and the empirical covariance as the t=0 conditional covariance.

The formula for forecasting the covariance using the RiskMetrics approach is given below.

$$h_{12,t+1} = \lambda h_{12,t} + (1-\lambda) r_{1,t} r_{2,t}$$

(i) Identify two arguments in favor of using the RiskMetrics approach for forecasting covariance.

ANSWER:

(ii) Calculate the forecasted t=1 Pearson correlation coefficient using the RiskMetrics approach and the results of part (b)(ii). Show all work.

The response for this part is to be provided in the Excel spreadsheet.

(d) (*3 points*) Based on the results of your analysis, your team plans to implement the forecasting model to assess short-term capital adequacy for the annuity block.

(i) Explain how model risk could arise while calibrating and implementing the models described above.

ANSWER:

(ii) Recommend three model validation best practices that could be incorporated to help manage and mitigate model risk. Justify your response.

ANSWER:

ERM Sample Solutions – Parts Answered in Word

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(i) Calculate each correlation metric based on the data provided. Show all work.

The solution for this part is provided in the Excel spreadsheet.

CFE Sample CBT Questions

(ii) Describe the advantages and disadvantages of each metric that you should consider when selecting an appropriate correlation metric for parameterizing a copula.

ANSWER:

Pearson's Rho has the advantage that it is easy to calculate, but it is possible to have highly dependent variables with 0 correlation (e.g. X and X^2).

Spearman's Rho has the advantage that it does not depend on marginal distribution of both values so it can be used to calibrate copulas from empirical data. However, it has the disadvantage that there is the additional complexity of ranking of observations.

Kendall's Tau has the advantage that it is robust to outliers, but it is time consuming (i.e. more computational power is required) for large data set

(b) (*5 points*) The first step in forecasting correlation between the two funds is to forecast the volatility of the individual funds. In order to forecast the volatility of each fund's value, you fit a GARCH(1,1) model, as shown below, to each series of fund returns:

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Explain why the following modifications may be implemented for volatility forecasting:

- Using the absolute value of the innovation term
- Using a separate parameter for positive shocks and negative shocks

ANSWER:

a. Since the basic GARCH(1,1) model incorporates the squared value of the current innovation, large shocks tend to have substantial impact on the forecasted volatility. Using the absolute value instead mutes the impact of large innovations.

b. The basic GARCH(1,1) is symmetric, only the magnitude of the deviation influences the forecast (as innovations are squared). For some series, large negative returns have a bigger/different effect on risk than do positive returns.

(c) (*3 points*) Maggie believes that the RiskMetrics approach is more appropriate than the GARCH model for forecasting the *covariance* between the two funds. She suggests using a risk decay factor of 0.95 and the empirical covariance as the t=0 conditional covariance.

The formula for forecasting the covariance using the RiskMetrics approach is given below.

$$h_{12,t+1} = \lambda h_{12,t} + (1-\lambda) r_{1,t} r_{2,t}$$

(i) Identify two arguments in favor of using the RiskMetrics approach for forecasting covariance.

ANSWER:

- The most general GARCH model for forecasting correlation has a prohibitively high number of required parameters (7). As additional risk

factors are added, number of parameters increases rapidly. RiskMetrics only has 1 estimated parameter, and is therefore more robust to estimation error.
- RiskMetrics approach has a relatively small number of effective observations, placing more importance on recent observations

(ii) Calculate the forecasted t=1 Pearson correlation coefficient using the RiskMetrics approach and the results of part (b)(ii). Show all work.

The solution for this part is provided in the Excel spreadsheet.

(d) (*3 points*) Based on the results of your analysis, your team plans to implement the forecasting model to assess short-term capital adequacy for the annuity block.

(i) Explain how model risk could arise while calibrating and implementing the models described above.

ANSWER:

1. Model must be assessed for appropriateness - does the chosen model (i.e. GARCH, RiskMetrics) accurately reflect the dynamics of the time series under investigation?

2. The validation of model results should be performed to assess that both inputs and outputs are accurate. Errors in input data could lead to misleading output.

(ii) Recommend three model validation best practices that could be incorporated to help manage and mitigate model risk. Justify your response.

ANSWER:

1. Employ a robust model validation procedure that includes a review of Design Use/Fit, Design Method/Processing, Data, Assumptions, Results, and Governance.

2. Designate a Model Steward: A model steward could help to ensure modeling best practices related to documentation, validation, and governance. The role of model steward can provide a segregation of duties between the activities connected with the initial model and the subsequent activities associated with model and system updates, change control management testing, and validation of these models.

3. Fully document the model and explain the intended purpose of the models and how the user's needs are addressed by those models.

CFE FD Sample CBT Question

(*8 points*) Mrs. Julia Reich, the Chief Risk Officer (CRO) of RPPC Dynasty Corporation, wants to understand the impact to the risk margin required to manage the Darwin Life (Case Study Section 7.6) business on an ongoing basis, given the following scenarios:

I. Corporate tax rates increase from 21% to 35%.
II. A Darwin Life data vendor suffers a highly publicized data breach.
III. Investment risk in the general account portfolio backing the new ULSG product increases.
IV. Darwin acquires Snappy Life Insurance Company (Case Study Section 8).

(a) (*2 points*) Assess the likely impact to a market value margin (MVM) calculation for each of I-IV.

ANSWER:

A member of the Board of Directors comments, "MVM's 'one-year shock' method is not appropriate for setting margins for longer duration businesses such as life insurance."

(b) (*1 point*) Critique the Board member's comment.

ANSWER:

Darwin wants to estimate the capital required for the new ULSG product for sales in 2018-2020. As part of that, management assumes a $200,000 MVM for ULSG sales over the next three years. Darwin has assumed the following:

- ULSG premium is 50% of current projected UL first-year premium
- Present value of ULSG claims and surrenders are 10% of ULSG premium
- Solvency Capital Requirement (SCR) for each of the next three years is 8% of the present value of claims and surrenders
- 3% annual swap rate (flat yield curve)

(c) (*3 points*) Calculate the cost of capital charge implied in Darwin's MVM assumption. Show your work.

The response for this part is to be provided in the Excel spreadsheet.

CFE Sample CBT Questions

(d) (2 points) Recommend three changes to Darwin's MVM estimation approach. Justify your recommendation.

ANSWER:

CFE FD Sample CBT Solution

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IV. Darwin acquires Snappy Life Insurance Company (Case Study Section 8).

(a) (*2 points*) Assess the likely impact to a market value margin (MVM) calculation for each of I-IV.

ANSWER:

I. No impact. The liabilities on the balance sheet are set on a pre-tax basis and hence the MVM should be pre-tax as well. The tax rate is irrelevant.

II. Increase. Operational risk is non-hedgable. The expected and worst-case liability is likely to increase and as a result increase the MVM.

III. No or little impact. Investment risk is typically hedgeable. In fact, Darwin already uses derivatives and credit default swaps (CDS) to manage the interest and credit risk in the GA.

IV. Increase, at least initially. Operational risk is likely to increase during and after the acquisition. The MVM may ultimately decrease over the medium to long-term if synergies from combining life insurance operations are realized.

A member of the Board of Directors comments, "MVM's 'one-year shock' method is not appropriate for setting margins for longer duration businesses such as life insurance."

(b) (*1 point*) Critique the Board member's comment.

ANSWER:

The Board member is wrong, or just confused. The idea that the MVM only takes account of one-year worth of risk and everything after the first year is ignored is incorrect.

The change in liability value is between expected future liability cash flows and a tail scenario. The tail scenario cash flows reflect not what we can observe in any one year but rather how far off we can be in estimating our expected liability cash flows over their entire life.

Darwin wants to estimate the capital required for the new ULSG product for sales in 2018-2020. As part of that, management assumes a $200,000 MVM for ULSG sales over the next three years. Darwin has assumed the following:

- ULSG premium is 50% of current projected UL first-year premium
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(c) (*3 points*) Calculate the cost of capital charge implied in Darwin's MVM assumption. Show your work.

The response for this part is to be provided in the Excel spreadsheet.

(d) (2 points) Recommend three changes to Darwin's MVM estimation approach. Justify your recommendation.

ANSWER:

Consider aggregation in some form

- Since MVM is typically calculated for each line of business where the products have similar risk profiles, calculate MVM for the entire UL block or across all Darwin life insurance business (UL + Trad Life + Term).

Extend the time horizon

- The projected SCR should be from time-zero to runoff. A three-year time horizon is not particularly meaningful for long-tailed business such as life insurance, so it should be extended.

Refine or sensitivity test basic assumptions

- The simplistic ULSG cashflow estimates are likely inappropriate. Fundamentally, a better proxy measure for non-hedgable mortality risk may be NAR. Also, consider alternative discount rate(s) and their impact.

CFE SDM Sample CBT Question 1

(a) (*1 point*) Define discrete optimization.

ANSWER:

Information on Blue Jay Air (BJA) can be found in Section 2 of the Case Study.

As part of its international expansion strategy, BJA is assessing how to maximize profit margins by strategically designing seating arrangements for its international flights.

International flights will have two ticket types: Economy Class, and Comfort Class. The following information is known about the international flight strategy:

Average Ticket Price	Economy Class: $500 Comfort Class: $900
Plane Capacity	300 Economy Class seats *OR* 200 Comfort Class seats *OR* Any combination in between

The goal is for BJA to maximize the amount of revenue received from ticket sales per flight. However, BJA's internal policy is to ensure that all flights have at least 280 total seats available.

Additionally, in order to ensure that demand is met, BJA believes that there should be at least 3 times more Economy Class seats available than Comfort Class seats per flight. Industry research suggests that there is negligible risk of underselling a flight if this condition is met.

(b) (*2 points*)

(i) State the objective function.

(ii) State the constraint functions.

ANSWER:

(c) (*4 points*) For the optimization problem defined in part (b):

(i) Sketch the feasible region with binding constraints clearly labelled.

The response for this part is to be provided in the Excel spreadsheet.

(ii) Calculate the optimal solution. Show your work.

ANSWER:

In an effort to further boost profit margins, BJA is considering removing the internal requirement of having a 280-seat capacity per flight. However, there are additional regulatory restrictions in place that require every commercial flight to have a total seat capacity of at least 80% of the total possible seat capacity.

(d) (*1 point*) State the new constraint functions.

ANSWER:

(e) (3 *points*) Calculate the optimal solution for the discrete optimization problem defined in part (d). Show your work

A sketch of the feasible region should be in the Excel spreadsheet.

ANSWER:

(f) (*2 points*) Critique whether the decision models in parts (b) and (d) are appropriate for BJA.

ANSWER:

CFE SDM Sample CBT Solution for Question 1

(a) Define discrete optimization.

ANSWER:

An optimization model is a discrete (or integer) optimization model if all of the decision variables are required to be positive integers, and all of the constraints and the objective function are linear functions.

A regular linear optimization model has the added flexibility that the decision variables need not be integers. A discrete linear optimization model is similar to a regular linear optimization model, but with the added constraint that all input variables must be positive integers.

(b)

(i) State the objective function.

(ii) State the constraint functions.

ANSWER:

i) Objective function: max 500x + 900y = C, where x is the number of Economy Class seats, and y is the number of Comfort Class seats

ii) Constraint functions:

x + 1.5y <= 300
x + y >= 280 (alternatively: -x - y <= 280)
x - 3y >= 0 (alternatively: -x + 3y <= 0)
x, y >= 0
where x, y are elements of P (where P is the set of positive integers)

(c) For the optimization problem defined in part (b):

(i) Sketch the feasible region with binding constraints clearly labelled.

(ii) Calculate the optimal solution. Show your work.

ANSWER:

Per the sketch, we need to find intersect of x + y = 280 and x + 1.5y = 300

x = (280 – y)

(280 – y) + 1.5y = 300 → y = 20/0.5 = 40

x = (280 – 40) = 240

Optimal solution occurs when x = 240, and y = 40

At this optimal point the objective function is:

C = 500*(240) + 900*(40) = 156,000

(d) State the new constraint functions.

ANSWER:

The constraint functions are also unchanged, except we need to remove the constraint x + y >= 280 and replace with the constraint x + y >= 240 (i.e. 240 = 0.8*300).

Therefore, new constraint functions are as follows:

x + 1.5y <= 300

x + y >= 240 (alternatively: -x - y <= 240)

x - 3y >= 0 (alternatively: -x +3y <= 0)

x, y >= 0

where x, y are elements of P (where P is the set of positive integers)

(e) (3 *points*) Calculate the optimal solution for the discrete optimization problem defined in part (d). Show your work

A sketch of the feasible region should be in the Excel spreadsheet.

ANSWER:

Per the sketch, we need to find intersect of x - 3y = 0 and x + 1.5y = 300

x = (0 + 3y) = 3y

(3y) + 1.5y = 300 → y = 300/4.5 = 66.66

x = (3*66.66) = 200

The optimal solution would occur when x = 200, and y = 66.66, but this violates the requirement for x and y to be positive integers (i.e. we cannot have a fractional number of seats).

Without violating the integer requirement, optimal solution occurs at x = 201 and y = 66, at this optimal point the objective function is:

C = 500*(201) + 900*(66) = 159,900

(f) (*2 points*) Critique whether the decision models in parts (b) and (d) are appropriate for BJA.

ANSWER:

Aside from seating arrangements, the decision models from parts b) and d) do not capture other considerations which are essential to BJA and their stated strategic objectives. For example, BJA's intent on developing service differentiation, i.e. by catering to the business traveler, is not at all addressed by this model as it stands. Further, BJA's desire to distinguish itself by providing a higher standard of flight travel experience and safety is not something that the current optimization model accounts for.

The current model is too simplistic in the sense that it only looks at revenue maximization as the end goal. It does not consider BJA's strategic initiatives, its reputation, nor the demand for its services at different seating class levels. BJA management should consider developing complementary decision-making models (quantitative or qualitative where appropriate) to help them ensure that they are making holistic decisions which are well aligned with BJA's medium to long-term business objectives.

CFE SDM Sample CBT Question 2

(*12 points*) Information on Blue Jay Tire (BJT) can be found in section 3 of the Case Study.

(a) (*1 point*)

(i) Identify three risks that impact how many tires BJT supplies.

(ii) Identify three risks that impact demand for tires produced by BJT.

ANSWER:

You are asked to develop a model to explain how oil price and number of tire recalls affect the price of tires produced by BJT.

(b) (*3 points*) Sketch a causal loop diagram of the model requested by Beaudry.

The response for this part is to be provided in the Excel spreadsheet.

(c) (*1 point*) Recommend a way to mitigate risk based on the dynamic model developed in part (b). Justify your answer.

ANSWER:

BJT plans to build additional production capacity to support its growth. It will require an initial investment and is expected to become profitable after three years.

(d) (*3 points*) Sketch a causal loop diagram to illustrate how adding additional capacity will impact profit. The nodes in the diagram should include, but not be limited to: Units Sold, Profit, and Production Capacity.

The response for this part is to be provided in the Excel spreadsheet.

(e) (*2 points*) Evaluate the impact on sustainability and stability of the model due to adding production capacity. Justify your answer using the model in part (d).

CFE Sample CBT Questions

ANSWER:

(f) (*2 points*) Recommend a strategy to mitigate the risk of instability and improve sustainability. Justify your answer.

ANSWER:

CFE SDM Sample CBT Question 2 Solutions

(a)

(i) Identify three risks that impact how many tires BJT supplies.

(ii) Identify one risk that impacts demand for tires produced by BJT.

ANSWER:

Supply: commodity risk, manufacturing risk (tire recall), labor risk (union negotiation), legal risk (minimum wage), distributor risk, insurance risk, environment risk, reputational risk, political risk, & currency risk

Demand: economic risk (oil prices)

(b) Sketch a causal loop diagram of the model.

The response for this part is to be provided in the Excel spreadsheet.

(c) Recommend a way to mitigate risk based on the dynamic model developed in part (b). Justify your answer.

ANSWER:

Since oil price is given (external) to BJT, tire recall is the only risk factor that the management can mitigate its impact to the business. Recommend finishing the tire recall process quickly to reduce delays and mitigate negative impact. Alternatively, candidates could recommend improving the quality control process.

(d) Sketch a causal loop diagram to illustrate how adding additional capacity will impact BJT's profit. The nodes in the diagram should include, but not be limited to: Units Sold, Profit, and Production Capacity.

The response for this part is to be provided in the Excel spreadsheet.

(e) Evaluate the impact on sustainability and stability of the model due to adding production capacity. Justify your answer using the model in part (d).

ANSWER:

Building additional production capacity will introduce delay in time to meet increased demand of tire. Until BJT builds new plants and/or adds extra production capacity to existing facilities, an increased demand puts an upward pressure to price. But it will take time and bring in disequilibrium in the short run. Meanwhile competition might fill up a supply gap if they have redundant capacity.

When BJT completes building additional production capacity, if demand does not change, BJT will meet demand and have large profits. The tire market will be in equilibrium.

If demand comes down, BJT will have excess production capacity. This might drive down price and introduce discrepancy between demand and supply. So, the tire market might experience disequilibrium and instability. This might cause the oscillatory behavior of the market.

(f) Recommend a strategy to mitigate the risk of instability and improve sustainability. Justify your answer.

ANSWER:

Recommend increasing production capacity in phases responding to the increase of tire demand. This strategy will meet demand increase in the short term without over-producing tires.