GI ADV Model Solutions
Spring 2020

1.

Learning Objective:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcome:

(4c) Calculate the price for a casualty per occurrence excess treaty.

Source:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Calculate the values of *a* and *b* for each hazard group.

Commentary on Question:

Candidates did not always recognize that the equations to solve depend only on the NCCI factors, not on the treaty being exposure rated.

For Hazard Group J the first equation to solve is

$\frac{0.008}{0.032} = \frac{ELF_{1,000,000}}{ELF_{250,000}} = \frac{a(1,000,000)^{-b}}{a(250,000)^{-b}}$ or $0.25 = 4^{-b}$, which implies b =1.

Then, $0.008 = a(1,000,000)^{-1}$, which implies a = 8,000.

Similarly, for Hazard Group K,

$\frac{0.020}{0.060} = \frac{ELF_{1,000,000}}{ELF_{250,000}} = \frac{a(1,000,000)^{-b}}{a(250,000)^{-b}}$ or $(1/3) = 4^{-b}$, which implies

$b = \ln(3)/\ln(4) = 0.7925$.

Then, $0.020 = a(1,000,000)^{-0.7925}$, which implies a = 1,138.

(b) Calculate the loss cost rate for the treaty.

For Hazard Group J,

$ELF_{200,000} = 8,000(200,000)^{-1} = 0.04$, $ELF_{800,000} = 8,000(800,000)^{-1} = 0.01$.

For Hazard Group K,

$ELF_{200,000} = 1,138(200,000)^{-0.7925} = 0.07163$,

$ELF_{800,000} = 1,138(800,000)^{-0.7925} = 0.02388$.

1. Continued

Expected treaty losses are:
80,000(0.60)(0.04 – 0.01) = 1,440
120,000(0.60)(0.07163 – 0.02388) = 3,438
110,000(0.80)(0.04 – 0.01) = 2,640
90,000(0.80)(0.07163 – 0.02388) = 3,438

Total losses are 10,956.
Total premium is 400,000.
Loss cost rate = 10,956/400,000 = 2.74%.

(c) Explain how you would handle:

(i) Policy limits

(ii) Discounting

(i) Workers compensation policies do not have policy limits, hence no adjustment is needed.

(ii) Loss data should be requested on a full undiscounted basis.

2. Learning Objective:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcome:

(5c) Calculate an underwriting profit margin using the risk adjusted discount technique.

Source:

Ratemaking: A Financial Economics Approach, D'Arcy and Dyer

Solution:

(a) Calculate the premium for this policy.

The equation to solve is:

$$P = \frac{125}{0.99} + 25 + \frac{(P-25)(0.35)}{1.03} - \frac{125(0.35)}{0.99} + \frac{(100+P-25)(0.03)(0.35)}{1.03}$$

$$= 126.263 + 25 + 0.340P - 8.495 - 44.192 + 0.765 + 0.010P = 99.340 + 0.350P$$

The solution is: P = 99.340/0.65 = 152.83.

(b) Calculate the underwriting profit margin for this policy.

UPM = 1 – (125 + 25)/152.83 = 1.85%.

(c) State two drawbacks of the Risk Adjusted Discount Technique.

Commentary on Question:

Any two of the three items listed below are sufficient for full credit.

- There is no widely accepted approach for setting the risk-adjusted discount rate.
- It is difficult to allocate equity to policies.
- The method considers only one policy term.

3. **Learning Objective:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Source:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Solution:

(a) Calculate the maximum likelihood estimates of ULT_{2017}, ULT_{2018}, and ULT_{2019}.

The following values from the cumulative distribution function are needed:

$$G(6) = 1 - \exp\left[-(6/10.68)^{0.6069}\right] = 0.5058$$

$$G(18) = 1 - \exp\left[-(18/10.68)^{0.6069}\right] = 0.7466$$

$$G(30) = 1 - \exp\left[-(30/10.68)^{0.6069}\right] = 0.8461$$

Then,

$$ULT_{2017} = 17,000/0.8461 = 20,092$$

$$ULT_{2018} = 16,000/0.7466 = 21,430$$

$$ULT_{2019} = 12,000/0.5058 = 23,725$$

(b) Estimate the reserve for the three accident years combined.

The reserve is 20,092 + 21,430 + 23,725 – (17,000 + 16,000 + 12,000) = 20,247.

(c) Estimate the scale factor, σ^2.

The estimated increments are:

20,092(0.5058) = 10,163
20,092(0.7466 – 0.5058) = 4,838
20,092(0.8461 – 0.7466) = 1,999
21,430(0.5058) = 10,839
21,430(0.7466 – 0.5058) = 5,160
23,725(0.5058) = 12,000.
Then,

3. Continued

$$\sigma^2 = \frac{1}{6-5}\left[\frac{(10,000-10,163)^2}{10,163}+\frac{(5,000-4,838)^2}{4,838}+\frac{(2,000-1,999)^2}{1,999}\right.$$
$$\left.+\frac{(11,000-10,839)^2}{10,839}+\frac{(5,000-5,160)^2}{5,160}+\frac{(12,000-12,000)^2}{12,000}\right]=15.39.$$

(d) Explain the difference between the process variance and the parameter variance.

Commentary on Question:
Some candidates used the definitions from the paper by Marshall, et al. The answer here should relate to this specific reserving situation.

- Process variance: The variance of the observed increments
- Parameter variance: A measure of the estimation error

4. **Learning Objective:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcome:

(1b) Test for the validity of these assumptions.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack

Testing the Assumptions of Age-to-Age Factors, Venter

Solution:

(a) State four of the other testable implications and how each can be tested.

Commentary on Question:

Any four of the following are sufficient for full credit. The method of testing must match the assumption being tested.

- Significance of factors – Use standard deviation estimates from a regression analysis
- Superiority versus alternative emergence patterns – Use sum of squared errors, adjusting for the number of parameters
- Linearity of model – Plot residuals against previous cumulatives
- Stability of factors – Plot residuals against time
- No particularly low or high diagonals – Use Mack's high-low diagonal test

(b) Demonstrate that the weighted average test statistic is –0.20.

Commentary on Question:

Many candidates did not re-rank the values for the earlier of the two periods prior to calculating the sum of squared differences.

T_1: Ranks are (1,3,2,4,5) and (5,2,4,1,3). Sum of squared differences is 34.

$T_1 = 1 - 6(34)/(5^3 - 5) = -0.7$.

T_2: Ranks are (4,2,3,1) and (3,2,1,4). Sum of squared differences is 14.

$T_2 = 1 - 6(14)/(4^3 - 4) = -0.4$.

T_3: Ranks are (3,2,1) and (3,1,2). Sum of squared differences is 2.

$T_3 = 1 - 6(2)/(3^3 - 3) = 0.5$.

T_4: Ranks are (2,1) and (2,1). Sum of squared differences is 0.

$T_4 = 1 - 6(0)(2^3 - 2) = 1$.

Weighted average is $T = [4(-0.7) + 3(-0.4) + 2(0.5) + 1(1)]/10 = -0.2$.

4. Continued

(c) State the conclusion that should be drawn from the test. Justify your answer.

The variance of T is $\frac{1}{(5\times 4)/2} = 0.1$. The observed value is $0.2/\sqrt{0.1} = 0.63$ standard deviations from zero. There is not sufficient evidence to reject the null hypothesis of no correlation. Hence, the chain ladder cannot be ruled out as an appropriate model.

5. Learning Objective:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcome:

(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Source:

As Application of Game Theory: Property Catastrophe Risk Load, Mango

Solution:

(a) Calculate each of the following using the Marginal Surplus method:

(i) The risk load multiplier

(ii) The risk load charged to Account Y

The risk load multiplier is $\lambda = 1.5(0.1)/1.1 = 0.13636$.

$E(X) = 0.07(200) + 0.02(400) + 0.09(500) + 0.02(1000) = 87$

$Var(X) = 0.07(200^2) + 0.02(400^2) + 0.09(500^2) + 0.02(1000^2) - 87^2 = 40{,}931$

$SD(X) = \sqrt{40{,}931} = 202.31$

$E(X+Y) = 0.07(300) + 0.02(600) + 0.09(2000) + 0.02(1700) = 247$

$Var(X+Y) = 0.07(300^2) + 0.02(600^2) + 0.09(2000^2) + 0.02(1700^2) - 247^2 = 370{,}291$

$SD(X+Y) = \sqrt{370{,}291} = 608.52$

The risk load for Account Y is $0.13636(608.52 - 202.31) = 55.39$.

(b) Calculate each of the following using the Marginal Variance method:

(i) The risk load multiplier that produces the same risk load for the combined portfolio as that obtained using the Marginal Surplus method

(ii) The risk load charged to Account Y

The risk load multiplier is $\lambda = 0.13636/608.52 = 0.0002241$.

The risk load for Account Y is $0.0002241(370{,}291 - 40{,}931) = 73.81$.

(c) Demonstrate that the Marginal Variance method is not renewal additive.

The total risk load is 0.0002241(370,291) = 82.98.

$E(Y) = 0.07(100) + 0.02(200) + 0.09(1500) + 0.02(700) = 160$

$Var(Y) = 0.07(100^2) + 0.02(200^2) + 0.09(1500^2) + 0.02(700^2) - 160^2 = 188{,}200$

The risk load for Account X is $0.0002241(370{,}291 - 188{,}200) = 40.81$.

The sum of the risk loads is 73.81 + 40.81 = 114.62 which is not equal to 82.98.

6. **Learning Objective:**

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(2c) Describe methods to assess this uncertainty.

Source:
A Framework for Assessing Risk Margins, Marshall, et al.

Solution:

(a) Define each of the following sources of internal systemic risk:

- Specification error
- Parameter selection error
- Data error

Specification error: The error that can arise from an inability to build a model that is fully representative of the underlying insurance process.

Parameter selection error: The error that can arise because the model is unable to adequately measure all predictors of claim cost outcomes or trends in these predictors.

Data error: The error that can arise due to poor data, unavailability of data, or inadequate knowledge of the portfolio being analyzed.

(b) Calculate the internal systemic risk coefficient of variation for premium liabilities for both lines combined.

Risks 1, 2, and 4 are specification error, risk 6 is parameter selection error, and risks 3 and 5 are data error.
Motor score: 0.3(3+2+4)/3 + 0.5(4) + 0.2(2+5)/2 = 3.6. Motor CoV = 10%.
Home score: 0.3(4+4+5)/3 + 0.5(6) + 0.2(3+5)/2 = 5.1. Home CoV = 5.5%.
Combined CoV is
$\sqrt{0.2^2(0.1)^2 + 0.8^2(0.055)^2 + 2(0.5)(0.2)(0.8)(0.1)(0.055)} = 0.0567 = 5.67\%.$

(c) Provide a situation where this a priori assumption may not hold.

If there is unclosed or contractually bound future business then premium liabilities will have additional uncertainty due to needing to estimate the associated premium or exposure.

7. Learning Objective:

3. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcome:

(3e) Explain Table M and Table L construction in graphical terms.

Source:

The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Solution:

(a) Express the Table M charge, $\varphi(r)$, as an integral, where *r* is the entry ratio, *Y* is a continuous random variable representing actual loss in units of expected loss, and $f(y)$ is the probability density function of *Y*.

Commentary on Question:

The alternative integral, $\varphi(r) = \int_r^\infty [1 - F(y)]dy$ *was an acceptable answer.*

$$\varphi(r) = \int_r^\infty (y - r) f(y)dy$$

(b) State the fundamental relation connecting $\psi(r)$ and $\varphi(r)$.

$$\psi(r) = \varphi(r) + r - 1$$

(c) Complete the following Table M for this risk by evaluating the integral from part (a) to complete the second column and using the fundamental relation from part (b) to complete the third column:

r	$\varphi(r)$	$\psi(r)$
0.00	1.000	0.000
0.20	0.800	0.000
0.40	0.600	0.000
0.60	0.402	0.002
0.80		
1.00		
1.20		
1.40	0.002	0.402
1.60	0.000	0.600
1.80	0.000	0.800
2.00	0.000	1.000

7. **Continued**

The integral can be evaluated as follows:

$$\varphi(r) = \int_r^\infty (y-r)\frac{1}{0.2\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{y-1}{0.2}\right)^2\right]dy$$

$$= 0.2\int_{\frac{r-1}{0.2}}^\infty (0.2x+1-r)\frac{1}{0.2\sqrt{2\pi}}\exp\left[-\frac{1}{2}x^2\right]dx$$

$$= 0.2\int_{\frac{r-1}{0.2}}^\infty x\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}x^2\right]dx + \int_{\frac{r-1}{0.2}}^\infty (1-r)\phi(x)dx$$

$$= -\frac{0.2}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}x^2\right]\Bigg|_{\frac{r-1}{0.2}}^\infty + (1-r)\left[1-\Phi\left(\frac{r-1}{0.2}\right)\right]$$

$$= \frac{0.2}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{r-1}{0.2}\right)^2\right] + (1-r)\left[1-\Phi\left(\frac{r-1}{0.2}\right)\right].$$

Then, $\varphi(0.8) = 0.2166$, $\varphi(1.0) = 0.0798$, and $\varphi(1.2) = 0.0166$.

Using the formula from (b),

$\psi(0.8) = 0.2166 + 0.8 - 1 = 0.0166$, $\psi(1) = 0.0798 + 1 - 1 = 0.0798$, and $\psi(1.2) = 0.0166 + 1.2 - 1 = 0.2166$.

8. Learning Objective:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4a) Calculate the price for a proportional treaty.

(4d) Apply an aggregate distribution model to a reinsurance pricing scenario.

Source:

Basics of Reinsurance Pricing, Clark

Solution:

(a) Explain why adjustable features are common in proportional reinsurance treaties.

There will often be disagreement between the ceding company and reinsurer about the expected loss ratio and the appropriate ceding commission. An adjustable feature can resolve these differences.

(b) Illustrate with examples how the following adjustable features work:

(i) Sliding scale commission

(ii) Profit commission

(iii) Loss corridor

Commentary on Question:

Some candidates provided definitions without providing a specific example that illustrated the approach. The examples presented here are illustrative. Any similar example would earn full credit.

Sliding scale commission: Commission of 25% at a loss ratio (LR) above 65% that slides 1:1 to be 35% at a 55% LR, then 0.5:1 to be 45% at a 35% LR, then constant. So, at a 40% LR, it is 15% below 55% and so at 0.5:1 adds 7.5% to the commission, for a commission of 35% + 7.5% = 42.5%.

Profit commission: Begin with 100%, then subtract the actual LR (e.g., 55%, for 45% left), then subtract the ceding commission (e.g., 25%, for 20% left), then subtract a margin (e.g., 10%, for a profit of 10%). The profit commission is a fixed percentage (e.g., 50%) of the profit, so in this case the profit commission is 5%.

8. Continued

Loss corridor: The ceding company reassumes a portion of the liability if the LR exceeds a certain amount. For example, the corridor may be 75% of the layer from 80% to 90%. If the actual LR is 84%, the ceding company will reassume 75% of 84% – 80%, or 3%.

(c) Identify one disadvantage of each of the following approaches:

(i) Empirical distribution

(ii) Single distribution model

Commentary on Question:
One disadvantage from each list is sufficient for full credit.

Empirical distribution disadvantages:

- Does not account for all possible outcomes
- Cannot be modified to reflect future changes
- If using Bornhuetter-Ferguson or Cape Cod, may provide an artificially smooth sequence of loss ratios

Single distribution model disadvantages:

- No provision for a positive probability at zero
- Difficult to reflect changes in per-occurrence limits
- Cannot split into frequency and severity components

(d) Describe one approach to pricing the impact of a carryforward provision.

Commentary on Question:
Either of the two approaches is sufficient for full credit.

- Include the carryforward from past years and estimate its effect on the current year only
- Look at the long run of the contract and extend the modification, incorporating a reduction in the variance

(e) Identify one problem with the approach described in part (d).

Commentary on Question:
The problem needed to match the approach described in the previous part. Either of the responses for the second approach is sufficient for full credit.

- The first approach ignores the potential effect on later years.
- The second approach has no obvious way to reduce the variance.
- The second approach does not reflect the possibility that the contract may not renew.