Spring 2020, Multiple Choice Solutions

MC1: Answer C

MC2: Answer D

\[ 0.8 \cdot q_{[80]+0.6} = 1 - \frac{l_{[80]+1.4}}{l_{[80]+0.6}} = 1 - \frac{0.6l_{[80]+1} + 0.4l_{82}}{0.4l_{[80]} + 0.6l_{[80]+1}} \]

\[ = 1 - \frac{0.6(37,700) + 0.4(33,862)}{0.4(40,695) + 0.6(37,700)} = 1 - 0.9297 = 0.070 \]

MC3: Answer D

Present Value of Premiums = Present Value of Benefits

\[ P \left( 1 + p_x \cdot v + 2 \cdot p_x \cdot v^2 \right) = 100,000 \left( q_x \cdot v + 2 \cdot q_{x+1} \cdot v^2 + 3 \cdot p_x \cdot q_{x+2} \cdot v^3 \right) \]

\[ P = 100,000 \left( \frac{0.03v + (2)(0.97)(0.05)v^2 + (3)(0.97)(0.95)(0.07)v^3}{1 + 0.97v + (0.97)(0.95)v^2} \right) = 27,711.05 \div 2.73523 = 10,131 \]

The easiest way to get the reserve is to use the recursive formula

\[ V = \frac{P(1+i) - q_x \cdot \text{(Death Benefit)}}{p_x} = \frac{(10,131)(1.06) - (0.03)(100,000)}{0.97} = 7978 \]

MC4: Answer E

The probability of withdrawal or death between age 60 and 61 is \( e^{-\mu \cdot t} \cdot \mu^t \) = 0.07 = 0.93239.

The probability of retirement at 61 is 0.80.

The probability of withdrawal between age 61 and 62 is \( \int_0^1 \mu_{x+t} e^{-0.07t} dt = 0.05 \left( \frac{1-e^{-0.07}}{0.07} \right) = 0.048290 \)

\[ \Rightarrow \text{Prob of withdrawal between ages 61 to 62 is } (0.93239)(0.8)(0.048290) = 0.036020 \]

\[ \Rightarrow \text{Expected number of withdrawals} = 3000 \times 0.036020 = 108.06 \]
MC5: Answer C

\[ H(y_3) = H(y_4) + \frac{d_4}{r_5} \Rightarrow 0.57000 = 0.29727 + \frac{3}{r_5} \Rightarrow r_5 = 11 \]

\[ r_4 = r_5 + 9 + d_4 = 21 \]

MC6: Answer C

\[ C_a(40, t) = at^3 + bt^2 + ct + d; \quad C'_a(40, t) = 3at^2 + 2bt + c \]

\[ C_a(40, 0) = d = \varphi(40, 0) = 0.035 \quad \text{and} \quad C'_a(40, 0) = \varphi'(40, 0) = c = 0 \]

\[ C_a(40, 3) = 27a + 9b + 3c + d = \varphi(40, 3) = 0.005 \Rightarrow 27a + 9b = -0.03 \]

\[ C'_a(40, 3) = 27a + 6b + c = \varphi'(40, 3) = 0 \Rightarrow 27a + 6b = 0 \]

\[ \Rightarrow 3b = -0.03 \Rightarrow b = -0.01 \Rightarrow a = 0.06 / 27 = 0.00222 \]

\[ \Rightarrow C(40, 2) = 0.00222(8) - 0.01(4) + 0.035 = 0.0128 \]

MC7: Answer A

\[ p_{xy} = 0.77; \quad \text{Pr}[(y) \text{ survives,} (x) \text{ dies}] = 0.9 - 0.77 = 0.13 \]

\[ \Rightarrow \text{required probability} = 0.77 \times 0.13 = 0.1001 \]

MC8: Answer E

The premium for the second year and later is \( P_{x+y}^{EP} = \frac{S \cdot A_{x+y+1}}{\tilde{a}_{[80]} + t} = \frac{S(1 - d\tilde{a}_{[80]+1})}{\tilde{a}_{[80]+1}} \)

\[ =\frac{100,000[1-(0.05/1.05)(12.3355)]}{12.3355} = 3344.78 \]

\[ V_2 = 100,000 A_{10} - 3344.78 a_{10} = (100,000)(0.42818) - (3344.78)(12.0083) = 2653 \]

MC9: Answer A

\[ EPV = 50,000 p^{01}_{80} v + 50,000 p^{00}_{80} p^{01}_{81} v^2 + 50,000 p^{01}_{80} p^{12}_{81} v^2 + 100,000 p^{02}_{80} v + 100,000 p^{00}_{80} p^{02}_{81} v^2 \]

\[ = 15,964 \]
MC10: Answer C
Present Value of Premiums = Present Value of Benefits

\[ 12P \dddot{a}_{65:10}^{(12)} = 20E_{65} \times 50000 \times \dddot{a}_{65}^{(2)} \]

\[ \dddot{a}_{65:30}^{(12)} = 1.0002(7.8435) - 0.46651(1 - 0.55305) = 7.6366 \]

\[ \dddot{a}_{65}^{(2)} = 1.00015(6.7993) - 0.25617 = 6.5441 \]

\[ P = \frac{(0.24381)(50,000)(6.5441)}{(12)(7.6366)} = 870.54 \]

MC11: Answer B

\[ EPV \text{ Premiums: } 0.95P \dddot{a}_{50:10} = (0.95)(8.0550)P = 7.65225P \]

\[ EPV \text{ Benefits and expenses: } 2,000,000 \cdot 10E_{50}A_{60:30} + 500,000 \cdot 30E_{50} + 260 \cdot 10E_{50} \dddot{a}_{60:50} \]
\[ = (2,000,000)(0.60182)(0.41040 - 0.29508) + (500,000)(0.34824)(0.50994) \]
\[ + (260)(0.60182)(12.3816) = 229,532 \]

\[ \Rightarrow P = \frac{229,532}{7.65225} = 29,995 \]

MC12: Answer D

\[ EPV = 1000\left(1 + \frac{1}{0.5}p_{98}v + p_{98}v^2 + \frac{1}{1.5}p_{99}v^3\right) \]

\[ 0.5p_{98} = 1 - \frac{1}{3}q_{98} = 0.920955 \]

\[ p_{99} = 1 - q_{99} = 0.762866 \]

\[ 1.5p_{99} = 0.762866(1 - \frac{1}{3}q_{100}) = 0.696168 \]

\[ \Rightarrow EPV = 1000 \left[1 + \frac{0.920955}{1.05} + \frac{0.762866}{(1.05)^2} + \frac{0.696168}{(1.05)^3}\right] = 3170 \]
MC13: Answer A

Present Value of Premium = Present Value of Benefits \( \Rightarrow P \overline{a}_{55:10}^{00} = 50,000 \overline{a}_{55:10}^{01} \)

\[
\overline{a}_{55:10}^{00} = 10.1228 - 0.74091 \times v^{10} \times 6.6338 - 0.11682 \times v^{10} \times 0.0395 = 7.1026
\]

\[
\overline{a}_{55:10}^{01} = 2.3057 - 0.74091 \times v^{10} \times 2.8851 - 0.11682 \times v^{10} \times 8.8123 = 0.361405
\]

\[
\Rightarrow P = \frac{(50,000)(0.361405)}{7.1026} = 2544
\]

MC14: Answer D

Due to the interest rate change, we will find the reserve at the end of the 10\(^{th}\) year and then use the recursive formula to find the answer.

\[
\begin{align*}
_{10}V &= 1000A_{70} - [(0.9)(36) - 5] \ddot{a}_{70} = (1000)(0.42818) - [(0.9)(36) - 5](12.0083) = 99.15 \\
_{10}V &= \left( \frac{V + P - Expenses}{1 + i} \right) - (DeathBenefit)(q_{69}) = \left( \frac{V + (0.9)(36) - 5(1.03) - (1000)(0.009294)}{1 - 0.009294} \right) \\
\Rightarrow \_9V &= \left( \frac{(1000)(0.009294) + (99.15)(1 - 0.009294))}{1.03} \right) + 5 - 0.9 \times 36 = 76.99
\end{align*}
\]

MC15: Answer A

\[
A_{80}^{(4)} = 0.25 q_{80} v^{0.25} + 0.25 P_{80} v^{0.25} A_{80,25}^{(4)}
\]

\[
= 0.008164 v^{0.25} + 0.991836 v^{0.25} A_{80,25}^{(4)}
\]

\[
\Rightarrow A_{80,25}^{(4)} = \frac{0.66227 - 0.008085}{0.991836 \times 0.990243} = 0.666069
\]

MC16: Answer B

Profit = 100(\(_4V + 0.89P\))(1.05) - 100,000 - 99(13,529) = 87,418
MC17: Answer B

\[ P_t^{(0)} = (V^{(0)} + 0.98 P)(1.08) - p_{x+1}^{01} (30,000 + V^{(1)}) - p_{x+1}^{00} (V^{(0)}) \]

\[ = 6933.6 - 1642.0 - 4921.5 = 370.1 \]

MC18: Answer E

\[ NC = 0.02 \times 90,000 \times 20 \times E_{45} \times \delta_{63}^{(12)} = 8479.0 \]

MC19: Answer B

Let S denote the salary from age 45 to age 46.

\[ S \left( \frac{s_{88} + s_{99}}{2s_{45}} \right) \times 0.016 \times 15 \]

\[ RR = \frac{S \left( \frac{s_{99}}{s_{45}} \right)}{S \left( \frac{s_{88}}{s_{45}} \right)} = 0.238 \]

MC20: Answer A

\[ \frac{d}{dt} V^{(i)} = \delta_i V^{(i)} - 50,000 - \mu_{x}^{10} (V^{(0)} - V^{(i)}) - \mu_{x}^{12} (-V^{(i)}) \]

At \( t = 15 \) we have:

\[ \frac{d}{dt} V^{(i)} = (0.04879) (300,860) - 50,000 - 0.00087 (40,942 - 300,860) + 0.04754 (300,860) \]

\[ = -20,792 \]