

# Long-Term Actuarial Mathematics Exam Spring 2020

# Important Exam Information:

Exam Registration	Candidates may register online or with an application.
Introductory Study Note	The Introductory Study Note has a complete listing of all study notes as well as errata and other important information.
<u>Past Exams</u>	Past MLC Exams from 2000 – April, 2018 are available on the SOA website. The LTAM Exam from October 2018 and April 2019 are available on the SOA website.
<u>Updates</u>	Candidates should be sure to check the Updates page on the exam home page periodically for additional corrections or notices.

Recognized by the Canadian Institute of Actuaries.

# 1. Topic: Long-term insurance coverages (2-8%)

## Learning Objectives

The Candidate will understand the key features of long-term insurance coverages.

#### Learning Outcomes

- a) Describe the long-term coverages in insurance (life, health, and general), annuities, and retirement benefits (e.g. pensions, retiree health care, etc.)
- b) Describe the similarities and differences between the long-term coverages identified in Learning Outcome 1a.
- c) Describe the appropriate models to be used to calculate expected present values, premiums or contributions, and reserves for each long-term coverage.

# 2. Topic: Survival models and their estimation (15-25%)

#### **Learning Objectives**

The Candidate will understand key concepts concerning parametric and non-parametric (tabular) and multi-state models including single life, or multiple life, and multiple decrements.

#### **Learning Outcomes**

- a) Explain and interpret survival models and transitioning between states.
- b) Calculate and interpret standard functions including survival and mortality probabilities, force of mortality, and complete and curtate expectation of life.
- c) Calculate nonparametric estimates of survival models using the Kaplan-Meier and Nelson-Aalen formulas for seriatim data and adaptations for grouped data.
- d) Calculate, using both seriatim and grouped data, maximum likelihood estimates of transition probabilities assuming constant transition intensity during fixed age intervals.
- e) Calculate the variances of and construct confidence intervals for the estimators in parts c) and d).
- f) Calculate transition intensities exactly, or estimate transition intensities using large sample approximations.
- g) Describe and apply simple longevity models.
- For models dealing with multiple lives and/or multiple states, explain the random variables associated with the model and calculate and interpret marginal and conditional probabilities.
- i) Construct and interpret select and ultimate survival models.
- j) Describe the behavior of Markov chain models, identify possible transitions between states, and calculate and interpret the probability of being in a particular state and transitioning between states.
- k) Apply to calculations involving these models appropriate approximation methods for fractional ages based on uniform distribution of deaths or constant force.

## 3. Topic: Present Value Random Variables (10-20%)

#### Learning Objectives

The Candidate will be able to perform calculations on the present value random variables associated with benefits and expenses for any of the models in Learning Objective 2.

#### Learning Outcomes

The Candidate will be able to:

- a) Calculate and interpret probabilities, means, variances, and percentiles.
- b) Calculate and interpret the effect of changes in underlying assumptions such as mortality and interest.
- c) Apply appropriate approximation methods such as uniform distribution of deaths, constant force, Woolhouse, and Euler.

## 4. Topic: Premium Calculation (15-30%)

#### **Learning Objectives**

The Candidate will be able to use and explain premium-calculation methodologies.

#### **Learning Outcomes**

- a) Calculate and interpret probabilities, means, variances, and percentiles of random variables associated with a premium, including loss-at-issue random variables.
- b) Calculate premiums based on the equivalence principle, the portfolio percentile premium principle, and profit testing.
- c) Using the models in Learning Objective 2, calculate and interpret the effect of changes in benefits or underlying assumptions such as decrements, morbidity, expenses, and interest.
- d) Apply appropriate approximation methods such as uniform distribution of deaths, constant force, Woolhouse, and Euler.

# 5. Topic: Reserves (20-30%)

#### Learning Objectives

The Candidate will understand reserves for insurances and annuities for models in Learning Objectives 2 and 4.

## **Learning Outcomes**

- a) Calculate and interpret the following reserve types:
  - Net premium
  - Modified
  - Gross premium
  - Expense
- b) Calculate and interpret probabilities, means, variances, and percentiles of random variables associated with these reserves, including future-loss random variables.
- c) Calculate and interpret common profit measures such as expected profit, actual profit, gain, gain by source and period, internal rate of return, profit margin, and break-even year.
- d) Apply appropriate approximation methods such as uniform distribution of deaths, constant force, Woolhouse, and Euler.

6. Topic: Pension Plans and Retirement Benefits (10-15%)

# Learning Objectives

The Candidate will understand how the models from previous Learning Objectives apply to pension plans and retirement benefits.

# Learning Outcomes

- a) Describe and compare defined contribution and defined benefit pension plans including final salary and career average earning plans.
- b) Describe retiree health care plans.
- c) Identify and interpret the common states and decrements for pension plans, and the parametric and tabular models, including Markov chain models, associated with these decrements.
- d) Given particular participant data, plan provisions, and valuation assumptions, apply the models mentioned in learning outcome 6c to defined benefit pension plans and calculate and interpret replacement ratios, accrued benefits, gain or loss, and their expected values with adjustments such as the early retirement reduction factor.
- e) Given particular participant data, plan provisions, and valuation assumptions, calculate and interpret the actuarial accrued liability and the normal cost for a defined benefit plan under the projected unit credit (PUC) cost method and the traditional unit credit (TUC) cost method.
- f) Identify and interpret the assumptions and methods for retiree health care plans. Given particular participant data, plan provisions, and valuation assumptions, calculate and interpret the expected present value of future benefits, accumulated postretirement benefit obligation (APBO), and the normal cost or service cost for retiree health care plans.
- g) Calculate and interpret the effect of changes in underlying valuation assumptions such as mortality, discrete salary increase changes, other decrements and interest on the quantities mentioned in learning outcomes 6d, 6e, and 6f.
- h) Apply appropriate approximation methods such as uniform distribution of deaths, constant force, Woolhouse, and Euler.

#### Resources

- Actuarial Mathematics for Life Contingent Risks, 2<sup>nd</sup> Edition, 2013, Dickson, D., Hardy, M., Waters, H., Cambridge University Press, ISBN: 978-1-10704-407-4. Exercises are considered part of the required readings.
  - Chapters 1 10 and Chapter 12
  - o Excluding Sections 1.8, 3.12, 4.8, 5.14, 6.10, 7.3.5, 7.6, 7.7, 7.10, 8.14, 9.8, 10.8, & 12.9

#### • Two study notes:

- LTAM-21-18 Supplementary Note on Long Term Actuarial Mathematics. The entire note is on the syllabus, However, Section 4.6 references Monte Carlo simulations and references Chapter 11 of Actuarial Mathematics for Life Contingent Risks. Candidates will not be expected to complete Monte Carlo simulations and does not need to review Chapter 11 of Actuarial Mathematics for Life Contingent Risks.
- LTAM-22-19 Chapters 10, 11, and 14 from Loss Models, From Data to Decisions, 5<sup>th</sup> edition, 2018 by Klugman, Panjer, and Willmot. Chapters 10 and 11 are provided for background reading. Chapter 14 is required reading, except for Sections 14.4 and 14.6.

You may also use the third edition of Actuarial Mathematics for Life Contingent Risks.

Dickson, C.M.D., Hardy, M.R., and Waters, H.R. (2020), *Actuarial Mathematics for Life Contingent Risks*, **Third Edition**, Cambridge: Cambridge University Press

- Chapters 1 11, Chapter 13 and Chapter 19 excluding 1.12, 2.7, 3.13, 4.8, 5.14, 6.8, 7.2.5, 7.5, 7.6, 7.9, 7.10, 8.11, 9.7, 10.8, 11.13, 13.8, 13.10 and 19.5
- The student should also exclude information on De Moivre's Law on Page 42 and the sub-section on Woolhouse's formula and state dependent annuities beginning on Page 309 and ending on Page 312.

The third edition of *Actuarial Mathematics for Life Contingent Risks* includes the material from the Long Term Actuarial Mathematics Supplementary Note so there is only one study note.

<u>LTAM-22-19</u> Chapters 10, 11 and 14 from *Loss Models, From Data to Decisions,* 5<sup>th</sup> *edition,* 2018 by Klugman, Panjer, and Willmot. Chapters 10 and 11 are provided for background reading. Chapter 14 is required reading, except for Sections 14.4 and 14.6.

- Notation and Terminology used on Exam LTAM
- Exam LTAM Tables

Excel Workbook for Exam LTAM Tables (These spreadsheets were used to develop the tables used for the LTAM exam and is provided for educational purposes only. The workbook will not be available at the LTAM exam.)