1. **Learning Objectives:**

   1. The candidate will understand, evaluate and use stochastic, generalized linear, multi-state, projection and transition matrix models. The candidate will demonstrate an understanding of their underlying methodologies, strengths, limitations, and applications.

**Learning Outcomes:**

(1a) With respect to stochastic models:

- Explain and apply the stochastic modeling methodology, including measurement metrics (e.g., CTE).
- Describe and apply the theory and uses of real world versus risk neutral assumptions.
- Describe and apply the techniques of Monte Carlo simulation (including variance reduction and importance sampling).
- Describe and evaluate Random Number Generator models, and explain their uses, advantages, and theory.
- Describe and evaluate how stochastic models may be used to understand mortality and policyholder behavior risks and inform the use of reinsurance.
- Describe the technique of nested stochastic projections and explain why they are needed, and evaluate implementation issues.

(1c) With respect to multi-state and transition matrix models:

- Describe and apply the methodologies for constructing multi-state and transition models in an insurance context.

**Sources:**

- Stochastic Modeling is on the Rise, Product Matters, Nov 2016
- LAM-137-19: Multi-state Transition Models with Actuarial Application, sections 1 & 2

**Commentary on Question:**

*This question tests the candidates understanding of the advantages and risks of stochastic modeling, along with efficient methodologies for performing stochastic modeling. It also tests the candidate’s ability to build and use a general Linear Congruential Generator.*
1. Continued

Solution:
(a) Critique the following statements:

A. While the stochastic model will be useful for calculating the average outcome, there is no reliable method for analyzing tail risks given the low probability of them occurring.

B. We will be able to gain a better understanding of our existing single scenario stress test results by comparing with the stochastic distribution of results.

C. We assume a normal distribution for the variables projected by the stochastic model, as the normal distribution is relatively easy to use and therefore less risky than assuming a more complex distribution.

D. 10,000 scenarios should be produced each time the model is run, because having more scenarios always provides additional information about the shape of the distribution.

E. The model’s input parameters are calibrated using the past 2 years of historical experience, about 100 data points in total.

F. During model validation, it is normal to see the stochastic mean being more than double the deterministic best estimate.

Commentary on Question:
Most candidates performed well on this part.

Stronger candidates were able to provide thorough critique to demonstrate their understanding on the topics. No credit was given for simply stating True or False.

A. This statement is incorrect. While the stochastic set may be used to help determine the average, one key advantage of stochastic results is enabling tail analysis. Risk metrics such as VaR and CTE can be used to effectively analyze behavior in the tails of the distribution.

B. This is an accurate statement. Deterministic or stress test scenario results can be compared to the stochastic distribution of results to understand where they fall in the broader range of possible outcomes and help determine where they fall between specified percentiles.
1. Continued

C. This statement is incorrect. It is common to assume a simpler or more standard distribution like the normal distribution would closely fit many variables projected, but this often isn't the case. Distributions should be chosen based on what best describes available information, if possible. An inappropriate distribution can generate meaningless results and miss skewed outcomes, resulting in misleading tails.

D. This statement is incorrect. To avoid excessive run requirements, the number of scenarios necessary to reach the point at which additional iterations provide no additional information about the shape of the distribution should be determined. This can be done by testing increasing amounts of scenarios until the full distribution is realized.

E. There is insufficient credibility for the historical data points used. The model cannot be fully calibrated, and the actuary should carefully consider if the use of this model is appropriate.

F. This is a red flag and can possibly mean the model is mis-calibrated.

(b)

(i) Calculate the active or disabled status generated by the uniform random number sequence for the first 4 policyholders. Show all work.

(ii) You are also given that:

- The LCG model generated 0.727 for policyholder X at time 0
- Discount rate of 5%
- All cashflows are at the beginning of year

Calculate the time 0 actuarial present value of payments caused by this rider over the next two periods.

Commentary on Question:
Most candidates performed well on this part.

To receive full credit, the candidate b(i) calculated the active or disabled status; partial credit was given for each of the following:

- using the correct formula for general linear congruential generator
- generates first 4 X(i) numbers in sequence
- applies uniform distribution (Xi/M) for each policyholder
- applies correct logic to Ui to determine active/disabled status for each policyholder
1. Continued

b(ii) calculated the actuarial present value of the payment at time zero. The candidate could receive full credit for either of two alternative calculation methods.

Partial credit was given for each of the following:

- correctly identifies initial Active Status for this policyholder
- correctly identifies status change paths, probability and payment patterns associated with each path (or each time-period)
- calculates the PV of payments for each path or time-period
- calculates the correct PV total for all possible paths with payments

b(i) Given:
Multiplier = 2
Increment = 5
Modulus = 11
Initial Seed = 88

Solution:
- \( X(i) = (aX(i-1) + c) \mod(M) \)
- Status = Active if \( U_i < 0.8 \)

<table>
<thead>
<tr>
<th>Policyholder</th>
<th>X(i)</th>
<th>( U_i )</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5 = Mod (2*88+5,11)</td>
<td>0.455=5/11</td>
<td>Active</td>
</tr>
<tr>
<td>2</td>
<td>4 = Mod (2 * 5+5,11)</td>
<td>0.364=4/11</td>
<td>Active</td>
</tr>
<tr>
<td>3</td>
<td>2 = Mod (2 * 4+5,11)</td>
<td>0.182=2/11</td>
<td>Active</td>
</tr>
<tr>
<td>4</td>
<td>9 = Mod (2 * 2+5,11)</td>
<td>0.818=9/11</td>
<td>Disabled</td>
</tr>
</tbody>
</table>

(ii) Given:
\[ Q = \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix} \]

Solution:
Initial status: 0.727 is less than 0.80, therefore this policyholder starts in active state.
1. Continued

**Method 1**

<table>
<thead>
<tr>
<th></th>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path 1</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>0.81 = 0.9*0.9</td>
</tr>
<tr>
<td>Path 2</td>
<td>A</td>
<td>A</td>
<td>D</td>
<td>0.09 = 0.9*0.1</td>
</tr>
<tr>
<td>Path 3</td>
<td>A</td>
<td>D</td>
<td>A</td>
<td>0.03 = 0.3*0.1</td>
</tr>
<tr>
<td>Path 4</td>
<td>A</td>
<td>D</td>
<td>D</td>
<td>0.07 = 0.1*0.7</td>
</tr>
</tbody>
</table>

Corresponding Payments,

<table>
<thead>
<tr>
<th>Path</th>
<th>t=1</th>
<th>t=2</th>
<th>PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Path 2</td>
<td>0</td>
<td>900</td>
<td>73.47 = 0.09*900/1.05^2</td>
</tr>
<tr>
<td>Path 3</td>
<td>900</td>
<td>0</td>
<td>25.71 = 0.03*900/1.05</td>
</tr>
<tr>
<td>Path 4</td>
<td>900</td>
<td>850</td>
<td>113.97 = 0.07*[900/1.05+850/1.05^2]</td>
</tr>
</tbody>
</table>

Total AVP = 213.15

**Method 2**

Making payment at time 1

- Probability = 0.1 from A -> D
  
  85.71 = 0.1*(850+50)/1.05

Making payment at time 2

- Expense needs to be paid
  
  Probability = 0.09 from A -> A -> D
  
  73.47 = 0.9*0.1*(50+850)/1.05^2

- Expense does NOT need to be paid
  
  Probability = 0.07 from A -> D -> D
  
  53.97 = 0.1*0.7*850/1.05^2

Total AVP = 213.15
2. **Learning Objectives:**

2. The candidate will understand and be able to assess issues and concerns common to actuarial models and their development and management.

**Learning Outcomes:**

(2a) Describe Model Efficiency concepts and explain and apply both the representative scenarios and replicating liabilities techniques for improving Model Efficiency.

(2h) Describe and evaluate the guidance in the Actuarial Standards of Practice

(2i) Describe and evaluate issues related to open and closed coding of models, and methods for addressing those issues and concerns

(2j) Describe and evaluate considerations around the governance of expert judgment in actuarial modelling

**Sources:**

Assumption Governance, The Actuary, Jan 2021

ASOP 56: Modeling, Sections 3 & 4


LAM-132-19: Cluster Analysis: A Spatial Approach to Actuarial Modeling

**Commentary on Question:**

*Commentary listed underneath question component.*

**Solution:**

(a) XYZ uses a compliance-based Assumption Governance framework with the main objective of satisfying regulatory requirements.

(i) Critique the use of a compliance focused approach to Assumption Governance.

(ii) Recommend an alternative Assumption Governance framework for XYZ and describe the benefits they would realize.

**Commentary on Question:**

*Candidates scored about half the points available on average. Points were dropped for missing some of the requisite items,*
2. Continued

(i) Problems with compliance focused approach
- feelings of simply completing a checklist will not engage employees, could lead to retention issues
- resources are used on things not necessarily adding value to the company
- may remove actuarial judgment from the process – losing opportunities for improvement to that process
(ii) - employees should be encouraged to use actuarial judgment in the process
- implementing a more strategic focus
- more employee engagement may lead to less turnover
- should improve communication between the teams
- can allocate more resources to process improvement – possibly with new technologies

(b) XYZ’s modeling function is decentralized into small teams that independently maintain a model for each product. XYZ would like to create a centralized modeling function to improve efficiency, with a specific focus on optimizing new product development.

(i) Recommend the approach XYZ should follow when creating their centralized modeling function. Justify your answer.

(ii) Identify the relevant considerations and recommended practices of ASOP 56 that XYZ needs to consider when creating the centralized modeling function. Justify how they apply to XYZ’s approach.

Commentary on question
On part (i) many candidates did quite well – the question was looking for a reusable library approach as opposed to copying existing code. On part (ii) many candidates did very well – some however were not conversant with ASOP 56 and a number of candidates simply omitted this part

(i) XYZ should focus on building reusable libraries rather than starting from scratch or copying existing code
- will reduce resources required going forward
- less duplication in effort
- may lessen testing required
- should improve control of assumptions

(ii) Key provisions of ASOP 56
- reliance on models developed by others
- actuary should understand the model and disclose the extent of the reliance
2. Continued

- reliance on experts who developed the models
- evaluation and mitigation of model risks
- documentation
- of the model’s purpose and the testing results

(c) Critique the recommendations made in the report. Justify your answer.

A. Introducing segment boundaries is not advised as it will increase run time and add more constraints.

B. When mapping policies, we start with the pair with the lowest distance (i.e. policy pair #1 and #4) and map to the policy with higher face amount between the pair (i.e. map policy #4 to policy #1).

C. When there are pairs with similar distance (such as policy pairs #1 and #3 and policy pairs #3 and #4), to avoid bias, the company can use a random number generator to determine which pairs of policies should be mapped first.

D. The UL cluster model is calibrated and uses the same weights for account value as the variable annuity cluster model with compression ratio of 50-to-1.

E. A small cluster model size is preferred over larger cluster model sizes.

Commentary on Question:
Points were dropped by not actually providing a critique (Incorrect, false or, in some cases, partially true were accepted). A critique without backing was awarded no points. Some points were dropped for an incomplete explanation.

A – this is incorrect
   - increasing the number of segments will actually reduce run time
   - run time savings approximately proportional to the sum of squares of policies in each segment

B – this is incorrect – not just distance – importance needs to be calculated to determine mapping priority

C – this is incorrect – again, importance needs to be calculated
2. Continued

D – this is incorrect
- substantial differences between the two products will
  necessitate recalibration with the new policy exposures.
- different variables may require different weights

E – this is incorrect
- a smaller cluster size is not always better – must consider
  the trade-off between run time savings and better model fit with more cells
- depends on the intended use of the model
3. **Learning Objectives:**

The candidate will understand the principles of Asset-liability Management ("ALM"), and be able to describe and evaluate various techniques for addressing the mitigation of risk.

**Learning Outcomes:**

(3a) With respect to Asset-Liability Models:

- Describe and apply the fundamental elements of the theory and practice of ALM in an insurance company, including assessing the dangers of mismatched assets and liabilities.
- Describe and demonstrate how ALM can be used to identify and manage product and asset risks, including:
  - Major product risks for which ALM can be a useful tool for their management.
  - Using ALM as a means to manage interest rate risk, equity risk, and risks from optionality.
- Describe how common insurance contracts and variations generate embedded options in an insurer's balance sheet, and assess basic strategies for managing exposures created by such embedded options.
- Describe and apply the basic concepts of cash flow matching, immunization, duration/convexity matching, segmentation.
- Describe and apply Key Rate Durations (KRD) and their use in evaluating interest rate sensitivities of portfolios, including understanding the derivation of KDRs, the profiles of KDRs for selected major asset types, and assessing KRDs in a portfolio context.
- Describe and evaluate the Goldman Sachs' ALM/Strategic Asset Allocation approach for integrating ALM into an enterprise's risk and financial management framework.
- Describe and evaluate ALM modeling considerations in the context of modeling risk aggregation, dependency, correlation of risk drivers and diversification.

**Sources:**

- LAM-117-14: Key Rate Durations: Measures of Interest Rate Risk
- LAM-118-14: Revisiting the Role of Insurance Company ALM within a Risk Management Framework
- LAM-131-19: Life Insurance Accounting, Asset/Liability Management Ch 22

**Commentary on Question:**

*Candidates are expected to demonstrate solid understanding of the use of key rate duration and other ALM concepts often used in practice.*
3. Continued

Solution:
(a) Explain the six core processes of an ERM framework as discussed in the Goldman Sachs’ paper Revisiting the Role of Insurance Company ALM within a Risk Management Framework.

Commentary on Question:
The candidate is expected to retrieve the 6 core processes in a ERM framework outlined in LAM-118-14 ("Revisiting the Role of Insurance Company ALM within a Risk Management Framework"). To earn the full credit, the Candidate need to not only list the correct identification but also need to elaborating/adding context for each process being list.

1. Risk governance
Formal and informal communication about day-to-day risk management and decision making is critical.

2. Risk and capital measurement
Organizations must define economic risk measures and overlay specific regulatory/rating agency capital constraints.

3. Risk budgeting
Controlling the level of risk acceptable is a holistic, top-down approach. This includes determining which risks will be actively or passively managed, as well as risks that will be managed through hedging or reinsurance.

4. Liquidity risk management
Liquidity needs should be defined both at the aggregate level as well as by each line of business, implicitly "charging" lines of business with higher liquidity needs.

5. ALM and SAA (Strategic Asset Allocation)
Defined risk and capital measures, allocated risk budgets and defined liquidity constraints are all inputs to determine ALM constraints and overall SAA.

6. Risk reporting
Regular and routine monitoring of risks includes sensitivity, stress tests, multiple measures of risks and contingency planning. Risk reporting needs to be a living, nimble process that feeds back into the risk governance.
3. Continued

(b) You are given the following information on one of PBJ Life’s asset segments supporting a single liability payment in Year 19:

<table>
<thead>
<tr>
<th>Liability</th>
<th>Single payout in Yr 19</th>
<th>MV (Smillion)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets (Zero Coupon Bonds (ZCB))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5yr ZCB</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>10yr ZCB</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>30yr ZCB</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

You are given the following interest rate shock scenarios:

<table>
<thead>
<tr>
<th>Interest Rate Shock</th>
<th>5yr</th>
<th>10yr</th>
<th>30yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1 (parallel shock)</td>
<td>-50bp</td>
<td>-50bp</td>
<td>-50bp</td>
</tr>
<tr>
<td>Scenario 2 (Steepening shock)*</td>
<td>-50bp</td>
<td>-25bp</td>
<td>+150bp</td>
</tr>
</tbody>
</table>

*Linearly interpolate shock at different terms, if necessary.

(i) For each of the given interest rate shock scenarios, calculate the impact on surplus. Show all work.

(ii) Critique the following statement:

“The asset portfolio has the same effective duration as the liability, therefore exposure to interest rate risk is perfectly immunized.”

Commentary on Question:
For part (i) the candidate must estimate the rate shock for year 19 by Linearly interpolating the year 10 and 30 shocks; to earn the fully credits, the candidates need to calculate the impact for each scenario and show all work. For example, for scenario #1, Candidates only get partial credit if candidate skip the calculation and jump to the conclusion of no impact because of the parallel shock and matched duration.

Part (i):
Assets:

\[
\begin{align*}
\text{MV}(1) & = 20 \\
\text{MV}(2) & = 30 \\
\text{MV}(3) & = 50 \\
\end{align*}
\]

Duration:

\[
\begin{align*}
\text{D}(1) & = 5 \\
\text{D}(2) & = 10 \\
\text{D}(3) & = 30 \\
\end{align*}
\]
3. Continued

Liability:
\[ \text{MV(L)} = 100 \]
\[ \text{D(L)} = 19 \]

Key Rate Duration (KRD)
\[ \text{KRD}(i) = \frac{\text{MV}(i)}{\sum \text{MV}(i)} \times \text{D}(i) \]

Change in (Asset or Liability)
\[ = - \text{KRD} \times \text{Shock} \]

Change in net Surplus = Change in Asset - Change in Liability

Linearly interpolate Shock for year 19
\[ = 30\text{yr Shock} \times \frac{(19-10)}{(30-10)} + 10\text{yr Shock} \times \frac{(30-19)}{(30-10)} \]
\[ = 0.015 \times 0.45 + (-0.0025) \times 0.55 \]
\[ = 0.005375 \]

### MV ($million)

<table>
<thead>
<tr>
<th>Liability</th>
<th>KRD - 5yr</th>
<th>KRD - 10yr</th>
<th>KRD - 19yr</th>
<th>KRD - 30yr</th>
<th>Effective Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td></td>
<td>19</td>
<td>19</td>
<td></td>
<td>19</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>3</td>
<td>15</td>
<td></td>
<td>19</td>
</tr>
</tbody>
</table>

### Change in MV

<table>
<thead>
<tr>
<th>Scenario 1</th>
<th>Parallel shock -50bp</th>
<th>5yr</th>
<th>10yr</th>
<th>19yr</th>
<th>30yr</th>
<th>Change in MV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Change in liability</td>
<td>0</td>
<td>0</td>
<td>-0.005</td>
<td>-0.005</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td>Change in assets</td>
<td>0.5</td>
<td>1.5</td>
<td>0</td>
<td>7.5</td>
<td>9.5</td>
</tr>
<tr>
<td></td>
<td>Change in net surplus</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scenario 2</th>
<th>Unparallel shock</th>
<th>5yr</th>
<th>10yr</th>
<th>19yr</th>
<th>30yr</th>
<th>Change in MV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Change in liability</td>
<td>0</td>
<td>0</td>
<td>-0.2125</td>
<td>0</td>
<td>-10.2125</td>
</tr>
<tr>
<td></td>
<td>Change in assets</td>
<td>0.5</td>
<td>0.75</td>
<td>0</td>
<td>-22.5</td>
<td>-21.25</td>
</tr>
<tr>
<td></td>
<td>Change in net surplus</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td>-11.0375</td>
</tr>
</tbody>
</table>

Conclusion:
Scenario 1: Immunized to parallel shocks given no mismatch in total duration.
Scenario 2: Mismatch in KRD causes income volatility under unparallel interest rate movements. Asset & liability are NOT fully immunized.
3. Continued

Part (ii):
It is true that the assets and liability have the same effective duration, however effective duration assumes that spot curve shifts will be parallel. As demonstrated in part (i), the portfolio is immunized to parallel shocks in the yield curve (scenario 1); however, the portfolio is not immunized to non-parallel shocks (scenario 2) due to mismatched KRDs (key rate durations).

(c) Three of PBJ’s major product lines are Non-Participating Single Pay Whole Life Insurance, Fixed Annuities, and Variable Annuities. You are reviewing the use of derivatives to manage risk.

(i) Critique the following statement:

“Since all premium has been paid on the Whole Life product, there are no embedded options in the product from the perspective of the policyholder or the company.”

(ii) PBJ’s Fixed Annuity has the following features:

<table>
<thead>
<tr>
<th>Current rate of return</th>
<th>3.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guaranteed minimum rate</td>
<td>3.0%</td>
</tr>
<tr>
<td>Surrender charge</td>
<td>5% in year 1, grading to 0% in year 6</td>
</tr>
</tbody>
</table>

Critique the following statement:

“For the fixed annuity product, PBJ only needs to be concerned about the policyholder’s embedded options if interest rates fall.”

(iii) PBJ wants to protect its variable annuity against equity market risk. The primary revenues on this product are M&E fees on the account value, collected at the end of the year. Account values vary directly with the equity market. PBJ purchases one-year S&P Index put options to hedge their equity risk. You are given the following:

<table>
<thead>
<tr>
<th>Account Value – beginning of year</th>
<th>5,000,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedge Notional Amount</td>
<td>Total AV</td>
</tr>
<tr>
<td>M&amp;E charge</td>
<td>2%</td>
</tr>
<tr>
<td>Current S&amp;P Index = Strike Price</td>
<td>1800</td>
</tr>
<tr>
<td>Premium for each put option</td>
<td>300</td>
</tr>
<tr>
<td>Change in S&amp;P Index – end of year</td>
<td>-10%</td>
</tr>
</tbody>
</table>
3. Continued

At the end of the year your colleague argues the cost of the put option was too high and PBJ did not benefit from it. Determine whether the cost of the put option is in excess of the benefit it provides. Justify your answer.

Commentary on Question:
For part (iii), to earn full credit, the candidate should provide justification by showing the calculations and explaining that the company lost less money than they would have without the options.
The Revenue source is the M&E charge; therefore, the option cost and payoff should be calculated based on that rather than the account value. Most candidate did not get this done right.
Most of the candidate focus on the negative result (Gain / Loss) and did not recognize the benefit of reducing the loss that was brought by the put option.

Part (i):
This is not true. Whole Life (WL) policies have a cash surrender value. From the policyholder’s perspective, they have the option to surrender their policy for the guaranteed level of cash. This is a put option on the value of the policy. WL also offers policy loans. The policyholder has the option to borrow cash against the policy at a pre-determined interest rate. This is a put option on fixed-rate bonds.
From the company’s perspective, if a policyholder surrenders their policy, the company has the right to receive the difference between the reserve and the cash surrender value (which will reduce the company's reserve on the balance sheet). This acts as a callable bond.

Part (ii):
This is not true. The company also should be concerned if interest rates rise. If rates rise rapidly and the company does not react quickly enough, policyholders could surrender their policies despite the surrender charge (put on the value of the policy). The surrender charge period is only 5 years, so any policy older than 5 years has no penalty for surrendering. Also, if higher rates in the market are attractive enough, the relatively low surrender charge may not deter policyholders from surrendering.
The company may have purchased long-duration assets to back their annuities; yields on these assets would not change in an increasing interest rate environment. If the company increases their crediting rate, they can suffer spread compression between the rates earned on the assets and the rates credited to policyholders.
3. Continued

Part (iii):
Account Value (AV) = 5,000,000
M&E Charge (M&E %) = 2%
Current Index (Strike Price) = 1,800
Option Premium = 300

Ending Index (Underlying Price) = Current Index × (1+ Change in Index %)
= 1,800 × (1-10%)
= 1,620

Cost of Put Option
= (AV / Current Index) × M&E % × Option Premium
= (5,000,000 / 1,800) × 2% × 300
= 16,667

Put Option Payoff
= Max {0, (AV / Current Index) × M&E % × (Strike Price- Underlying Price)}
= Max (0, (5,000,000 / 1,800) × 2% × (1,800 – 1,620)}
= Max (0, 2,777.78 × 2% × 180)
= Max (0, 10,000)
= 10,000

Alternatively,
If no put options purchased:

M&E revenue Beginning of Year (BOY)
= AV × M&E%
= 5,000,000 × 2%
= 100,000

M&E revenue collected End of Year (EOY)
= AV × (1+ Change in Index%) × M&E%
= 5,000,000 × (1-10%) × 2%
= 90,000

M&E Revenue Lost
= M&E Revenue EOY – M&E Revenue BOY
= 90,000 – 100,000
= - 10,000

Put Option Payoff = - M&E Revenue Lost if no put options
= - (-10,000) = 10,000
3. Continued

Net Gain / Loss
= Payoff - Cost
= 10,000 - 16,667
= - 6,667

The colleague's statement is not correct. The total cost of the option is 16,667 and the option payoff is 10,000. Even though the company lost 6,667, they would have lost 10,000 if they did not have the put options. Buying the put options resulted in a smaller loss for the company (i.e. saved the company 3,333)
4. **Learning Objectives:**

4. The candidate will understand the basic design and function of Economic Scenario Generators and Equity Linked Insurance Models.

**Learning Outcomes:**

(4a) With respect to Economic Scenario Generators:
- Describe the need for ESGs and explain the structure of ESG models and components.
- Describe and apply basic default free interest rate models, including one-factor continuous time models.
- Assess the propriety of a particular ESG model and related assumptions for particular applications.

**Sources:**
Economic Scenario Generators: A Practical Guide, 2016, Ch. 1, 2, 4.1, 5, 6, 9, 10, 11.1 & 11.3

LAM-139-19: Simulation of a Guaranteed Minimum Annuity Benefit, Freedman, 2019
Excel Model - Stochastic Simulation of a GMAB Option (Accompanies Simulation of a GMAB)

**Commentary on Question:**
*This question was designed to test the candidate’s basic understanding of using Economic Scenario Generators (ESGs) for market consistent (risk neutral) valuation of financial guarantees and identify limitations of the models*

**Solution:**

(a) Critique the following statements from QRF’s Model Risk Management group regarding the ESG model.

(i) The real-world scenarios can provide more intuitive results than the risk-neutral scenarios

(ii) Correlation between modeled economic and financial market variables are set and held constant at model development.

(iii) Spread inputs for a real estate investment trust in our portfolio were not readily available in the market. The actuary estimated the spreads based on fixed income bond spreads from an Oil & Gas bond with the same term-to-maturity.

(iv) The model is calibrated using historical data from 2010 to 2019.
4. **Continued**

**Commentary on Question:**
Company QRF has a Universal Life portfolio and uses an ESG to model the guarantees and for stress testing. The candidate is asked to comment on a number of statements.

*Candidates did fairly well on this question.*

(i) The candidate needs to state if they agree or disagree with the statement. No points are given if the candidate just lists out facts about real-world and risk-neutral scenarios. No points are given if the candidate simply stated agree or disagree without explanations.

If the candidate agrees that real-world is more intuitive, then they need to explain that real-world scenarios reflect market dynamics, risk and stylized facts. These scenarios are calibrated to historical experience so the model output is more realistic. Risk-neutral scenarios are less intuitive as the risk-free rate is not realistic and are calibrated to current market prices and mainly used for market consistency (it is a mathematical way to arrive a closed-form solution).

If the candidate disagrees, then they need to explain that real-world is intuitive when used for capital, risk management and produce company forward view (earnings) projections as it reflects market dynamics, risk and stylized facts. Real-world scenarios calibrate to historical experience so the model output is more realistic. Risk-neutral scenarios are more intuitive for pricing/valuation as they are calibrated to current market prices for market consistency and provide closed form solutions.

(ii) This is not correct. The relationship between economic and financial market variables changes over time and depends on the situation.

(iii) It is not appropriate to use an Oil and Gas bond as a proxy for a REIT. Corporate bond spreads are firm and industry specific and based on default probability. Also need to consider credit rating of the proxy bond.

(iv) This is a period of sustained low interest rates and the calibration period used is not long enough. Should include a longer period and include periods of higher interest rates. Also, expert opinion is needed on future interest rate trends.
4. Continued

(b)  

(i) Calculate the market-consistent value of the cost of guarantee.

(ii) Using a quantitative measure, analyze the appropriateness of the scenario reduction techniques. Recommend changes if needed.

**Commentary on Question:**

The candidate is required to determine the market consistent cost of guarantee on UL products, given reserves calculated under scenarios with and without a minimum crediting rate. The candidate is also required to assess whether the scenario reduction technique is appropriate. 

The majority of candidates correctly calculated the market consistent cost of the guarantee. However, only a few candidates correctly concluded that the number of scenarios was too small and more scenarios were necessary. This was determined by calculating the standard deviation of the expected cost, and was not well done by the candidates.

(i) **Expected cost of the guarantee = reserves with minimum crediting rate guarantee - reserves without minimum crediting rate guarantee**

<table>
<thead>
<tr>
<th>Scenario Set #</th>
<th>Reserves with minimum crediting rate guarantee</th>
<th>Reserves without minimum crediting rate guarantee</th>
<th>Expected cost of guarantee reserve</th>
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</tr>
</tbody>
</table>
4. Continued

(i) Market consistent cost of the guarantee = average over 10 scenarios of the expected cost of the guarantee = 32.5

The technique is appropriate if the standard deviation of the expected cost is relatively small. Standard deviation = 18.0. As a percentage of the market consistent cost, this is 55% which is fairly large.

Therefore, the recommendation is to increase the number of scenarios.

A. Your student compiled the following list of observations and considerations. Assess each of the findings.

(i) Probabilities of default will rise with general economic conditions as evidenced in 2009 and 2020.

(ii) Widening corporate spreads will always track realized default rates.

(iii) Corporate spreads gross of defaults fluctuated more because of 2008 and 2020 recessions.

Commentary on Question:
The candidate is presented with a number of charts to analyze the impacts on ESG parameters:

1. Corporate default rates
2. Yield spreads between corporate and US Treasury bonds
3. Yield curve spot rates for US Treasuries

The candidate is then asked to assess a number of observations and findings based on the charts given.

The majority of candidates scored well on this section partly because the answers required either a true or false response, followed up with an explanation. In general, points were given based on whether the candidate answered true or false, and additional points were given if the explanation backing up the observation was reasonable. If the explanation cancelled out the answer, then no points were given.

A: TRUE. Probabilities of default will rise as economic conditions worsen, as evidenced by the financial tsunami in 2009 and COVID-19 in 2020

B: FALSE, as widening corporate spreads do not always track to realized default rates.
4. Continued

C: TRUE, defaults during stressed economic environment can be volatile, which impacts spread.

D: TRUE, lower rated bonds are more exposed to change in economic conditions.

E: FALSE, since interest rates can be negative and should be considered in the ESG.

F: FALSE. An inverted yield curve is when short-maturity yields are higher than longer-maturity yields. There is a period of inverted yield curve between time 0 and 1 only.

G: This is FALSE. A 10-year look back period is too short since it captures a period of very low rates. Should include a longer look back period. Also make use of expert judgement in determining the look back period. COVID-19 is a one-off event that could skew results.