

# Long-Term Actuarial Mathematics Exam

FALL 2021

## Important Exam Information—Updated 9/29/21:

The Fall 2021 LTAM Exam will include 20 multiple-choice questions worth 40 points and written-answer questions worth 56 points. The exam will be taken at Prometric testing centers. The multiple-choice questions will be answered on the computer, while the written-answer questions will be displayed on the computer and answered in a paper answer booklet to be provided and collected by Prometric. Paper versions of the exam and tables will not be provided. Candidates will be provided with a scratchpad.

The examination is four hours in length. The examination clock during the exam will provide a total of four hours and fifteen minutes. The additional fifteen minutes may be used at the exam taker's discretion for such things as reading time, breaks, or additional work time on the exam.

Candidates should also note that there has been a change in the required reading from last session in that Chapter 18 of *Actuarial Mathematics for Life Contingent Risks* replaces the previously used study note.

### [Exam Registration](#)

Candidates may register online or with an application.

### [Introductory Study Note](#)

The Introductory Study Note has a complete listing of all study notes as well as errata and other important information.

### [Past Exams](#)

Past MLC Exams from 2000 – April, 2018 are available on the SOA website. The LTAM Exam from October 2018 through October 2020 are available on the SOA website.

### [Updates](#)

Candidates should be sure to check the Updates page on the exam home page periodically for additional corrections or notices.

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**1. Topic: Long-term insurance coverages (2-8%)**

**Learning Objectives**

The Candidate will understand the key features of long-term insurance coverages.

**Learning Outcomes**

The Candidate will be able to:

- a) Describe the long-term coverages in insurance (life, health, and general), annuities, and retirement benefits (e.g. pensions, retiree health care, etc.)
- b) Describe the similarities and differences between the long-term coverages identified in Learning Outcome 1a.
- c) Describe the appropriate models to be used to calculate expected present values, premiums or contributions, and reserves for each long-term coverage.

**2. Topic: Survival models and their estimation (15-25%)**

**Learning Objectives**

The Candidate will understand key concepts concerning parametric and non-parametric (tabular) and multi-state models including single life, or multiple life, and multiple decrements.

**Learning Outcomes**

The Candidate will be able to:

- a) Explain and interpret survival models and transitioning between states.
- b) Calculate and interpret standard functions including survival and mortality probabilities, force of mortality, and complete and curtate expectation of life.
- c) Calculate nonparametric estimates of survival models using the Kaplan-Meier and Nelson-Aalen formulas for seriatim data and adaptations for grouped data.
- d) Calculate, using both seriatim and grouped data, maximum likelihood estimates of transition probabilities assuming constant transition intensity during fixed age intervals.
- e) Calculate the variances of and construct confidence intervals for the estimators in parts c) and d).
- f) Calculate transition intensities exactly, or estimate transition intensities using large sample approximations.
- g) Describe and apply simple longevity models.
- h) For models dealing with multiple lives and/or multiple states, explain the random variables associated with the model and calculate and interpret marginal and conditional probabilities.
- i) Construct and interpret select and ultimate survival models.
- j) Describe the behavior of Markov chain models, identify possible transitions between states, and calculate and interpret the probability of being in a particular state and transitioning between states.
- k) Apply to calculations involving these models appropriate approximation methods for fractional ages based on uniform distribution of deaths or constant force.

### 3. Topic: Present Value Random Variables (10-20%)

#### Learning Objectives

The Candidate will be able to perform calculations on the present value random variables associated with benefits and expenses for any of the models in Learning Objective 2.

#### Learning Outcomes

The Candidate will be able to:

- a) Calculate and interpret probabilities, means, variances, and percentiles.
- b) Calculate and interpret the effect of changes in underlying assumptions such as mortality and interest.
- c) Apply appropriate approximation methods such as uniform distribution of deaths, constant force, Woolhouse, and Euler.

### 4. Topic: Premium Calculation (15-30%)

#### Learning Objectives

The Candidate will be able to use and explain premium-calculation methodologies.

#### Learning Outcomes

The Candidate will be able to:

- a) Calculate and interpret probabilities, means, variances, and percentiles of random variables associated with a premium, including loss-at-issue random variables.
- b) Calculate premiums based on the equivalence principle, the portfolio percentile premium principle, and profit testing.
- c) Using the models in Learning Objective 2, calculate and interpret the effect of changes in benefits or underlying assumptions such as decrements, morbidity, expenses, and interest.
- d) Apply appropriate approximation methods such as uniform distribution of deaths, constant force, Woolhouse, and Euler.

**5. Topic: Reserves (20-30%)**

**Learning Objectives**

The Candidate will understand reserves for insurances and annuities for models in Learning Objectives 2 and 4.

**Learning Outcomes**

The Candidate will be able to:

- a) Calculate and interpret the following reserve types:
  - Net premium
  - Modified
  - Gross premium
  - Expense
- b) Calculate and interpret probabilities, means, variances, and percentiles of random variables associated with these reserves, including future-loss random variables.
- c) Calculate and interpret common profit measures such as expected profit, actual profit, gain, gain by source and period, internal rate of return, profit margin, and break-even year.
- d) Apply appropriate approximation methods such as uniform distribution of deaths, constant force, Woolhouse, and Euler.

**6. Topic: Pension Plans and Retirement Benefits (10-15%)**

**Learning Objectives**

The Candidate will understand how the models from previous Learning Objectives apply to pension plans and retirement benefits.

**Learning Outcomes**

The Candidate will be able to:

- a) Describe and compare defined contribution and defined benefit pension plans including final salary and career average earning plans.
- b) Describe retiree health care plans.
- c) Identify and interpret the common states and decrements for pension plans, and the parametric and tabular models, including Markov chain models, associated with these decrements.
- d) Given particular participant data, plan provisions, and valuation assumptions, apply the models mentioned in learning outcome 6c to defined benefit pension plans and calculate and interpret replacement ratios, accrued benefits, gain or loss, and their expected values with adjustments such as the early retirement reduction factor.
- e) Given particular participant data, plan provisions, and valuation assumptions, calculate and interpret the actuarial accrued liability and the normal cost for a defined benefit plan under the projected unit credit (PUC) cost method and the traditional unit credit (TUC) cost method.
- f) Identify and interpret the assumptions and methods for retiree health care plans. Given particular participant data, plan provisions, and valuation assumptions, calculate and interpret the expected present value of future benefits, accumulated postretirement benefit obligation (APBO), and the normal cost or service cost for retiree health care plans.
- g) Calculate and interpret the effect of changes in underlying valuation assumptions such as mortality, discrete salary increase changes, other decrements and interest on the quantities mentioned in learning outcomes 6d, 6e, and 6f.
- h) Apply appropriate approximation methods such as uniform distribution of deaths, constant force, Woolhouse, and Euler.

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**Resources**

- *Actuarial Mathematics for Life Contingent Risks, Third Edition* Dickson, C.M.D., Hardy, M.R., Waters, H.R. (2020), Cambridge University Press ISBN: 978-1-108-47808-3. Exercises are considered part of the required readings.
  - Chapters 1 – 11, Chapter 13 and Chapters 18 and 19 excluding 1.12, 2.7, 3.13, 4.8, 5.14, 6.8, 7.2.5, 7.5, 7.6, 7.9, 7.10, 8.11, 10.8, 11.13, 13.8, 13.10, 18.6, 18.7, and 19.5
  - The student should also exclude information on De Moivre's Law on Page 42.
  - The student should also exclude parametric maximum likelihood estimators in Chapter 18.
- [Notation and Terminology used on Exam LTAM](#)
- [Exam LTAM Tables](#)  
[Excel Workbook for Exam LTAM Tables](#) (These spreadsheets were used to develop the tables used for the LTAM exam and is provided for educational purposes only. The workbook will not be available at the LTAM exam.)

Note: The text and any study notes will not be available with the examination booklet. A copy of the Tables will be available.