



SOCIETY OF
ACTUARIES®

Long-Term Actuarial Mathematics

Exam LTAM

Date: Tuesday, April 27, 2021

Recognized by the Canadian Institute of Actuaries.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. Write your candidate number here _____. Your name must not appear.
2. Do not break the seal of this book until the supervisor tells you to do so.
3. Tables for this examination will be distributed by the Supervisor.
4. This examination has a total of 96 points. It consists of:

Section A: 20 multiple-choice questions, each worth 2 points for a total of 40 points, and

Section B: 6 written-answer questions, worth a total of 56 points. The point value for each written-answer question is indicated at the beginning of the question.

You may divide your time between the two sections of the examination (written-answer, and multiple-choice) as you choose. You should keep in mind the relative weight of the two sections.

Your written-answer paper will be graded only if your multiple-choice score is at or above a threshold set after the examination is administered.

5. Failure to stop writing or coding after time is called will result in the disqualification of your answers or further disciplinary action.
6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Multiple-Choice Instructions

1. A separate answer sheet for the multiple-choice questions is inside the front cover of this book. During the time allotted for this examination, record all your answers on the back of the answer sheet. **NO ADDITIONAL TIME WILL BE ALLOWED FOR THIS PURPOSE.**

No credit will be given for anything indicated in the examination book but not transferred to the answer sheet.

2. On the front of the answer sheet, space is provided to write and code candidate information. Complete the information requested by printing in the squares and blackening the circles (one in each column) corresponding to the letters or numbers printed. For each empty box blacken the small circle immediately above the "A" circle. Fill out the boxes titled:
 - (a) Name
(include last name, first name and middle initial)
 - (b) Candidate Number
(Candidate/Eligibility Number, use leading zeros if needed to make it a five digit number)
 - (c) Test Site Code
(The supervisor will supply the number.)
 - (d) Examination Part
(Code the examination that you are taking by blackening the circle to the left of "Exam LTAM.")
 - (e) Booklet Number
(The booklet number can be found in the upper right-hand corner of this examination book. Use leading zeros if needed to make it a four digit number.)

In the box titled "Complete this section only if instructed to do so," fill in the circle to indicate if you are using a calculator and write in the make and model number.

In the box titled "Signature and Date" sign your name and write today's date. **If the answer sheet is not signed, it will not be graded.**

Leave the boxes titled "Test Code" and "Form Code" blank.

On the back of the answer sheet fill in the Booklet Number in the space provided.

INSTRUCTIONS CONTINUED ON NEXT PAGE

3. Your score will be based on the number of questions which you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers: hence, you should answer all questions even those for which you have to guess.
4. Five answer choices are given with each multiple-choice question; each answer choice is identified by a key letter (A to E). Answer choices for some questions have been rounded. For each question, blacken the circle on the answer sheet which corresponds to the key letter of the answer choice that you select.
5. Use a soft-lead pencil to mark the answer sheet. To facilitate correct mechanical scoring, be sure that, for each question, your pencil mark is dark and completely fills only the intended circle. Make no stray marks on the answer sheet. If you have to erase, do so completely.
6. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
7. Clearly indicated answer choices in the test book can be an aid in grading examinations in the unlikely event of a lost answer sheet.
8. After the examination, the supervisor will collect this book and the answer sheet separately. **DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOK OR IN THE ESSAY ANSWER ENVELOPE.** All books and answer sheets must be returned. **THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.**

Written-Answer Instructions

1. **Write your candidate number at the top of each sheet. Your name must not appear.**
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert your written-answer sheets that you want graded into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam LTAM.
6. **Sign your essay answer envelope. If it is not signed, your examination will not be graded.**
7. For all parts of all problems, to maximize the credit earned, candidates should show as much work as possible, considering the time allotted for the question. Answers lacking justification will receive no credit. Answers should be organized so that the methods, logic, and formulas used are readily apparent. Candidates should not round their answers excessively; enough precision should be provided so that their answers can be accurately graded.

In some cases, candidates are asked to show that a calculation results in a particular number. Typically the answer given will be rounded; candidates should provide a greater level of accuracy than the number given in the question. This structure of question is intended to assist the candidate by giving an indication when the calculation has been done incorrectly, providing an opportunity to explore an alternative approach. It also allows a candidate who cannot obtain the correct answer to use the answer given to proceed with subsequent parts of the problem. (Candidates who are able to solve the problem should use their exact answer for subsequent parts.)

For questions requiring candidates to derive or write down a formula or equation, the resulting expression should be simplified as far as possible, and where numerical values are provided in the problem, they should be used.

Exam LTAM

SECTION A – Multiple-Choice

****BEGINNING OF EXAMINATION****

- 1.** Which one of the following statements about the cash value of a traditional whole life policy is true?
- (A) Cash values at early durations may be negative to allow the insurer to recover the acquisition costs.
 - (B) Non-forfeiture laws require insurers to pay specified cash values, or equivalent, for traditional whole life insurance.
 - (C) Cash values for whole life policies tend to decrease at later durations.
 - (D) Whole life policies with cash values can be terminated early at the discretion of the insurer.
 - (E) The cash value may become greater than the sum insured in the later durations of a policy.

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2. In the first year of operations, each machine in a factory is subjected to a constant force of failure, μ . The probability that any given machine fails within the year is 0.1.

In the second year, the force of failure will increase linearly over the year from μ to 3μ .

In the third year, the force of failure will be constant at 3μ .

At the start of the first year the factory has 20,000 machines, all new. Calculate the expected number of machines still functioning at the end of year 3.

- (A) 10,630
- (B) 10,930
- (C) 11,230
- (D) 11,530
- (E) 11,830

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3. The probability density function of the future lifetime of (x) , evaluated at t , is denoted $f_x(t)$.

You are given:

$$f_{70}(15) = 0.05; \quad f_{80}(5) = 0.08$$

Calculate ${}_{10}q_{70}$.

- (A) 0.300
- (B) 0.325
- (C) 0.350
- (D) 0.375
- (E) 0.400

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4. Initially, ten lives are included in a four-year observation study of times to death. Subsequently:

- One life dies at time 0.7 and another at time 1.6.
- At time 2, N additional lives are added to the study group.
- A death occurs at time 3.1 and another at time 3.4.
- There are no other exits or deaths.

The Kaplan-Meier product limit estimator of the four-year survival probability is 72%.

Calculate N .

- (A) 11
- (B) 12
- (C) 13
- (D) 14
- (E) 15

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5. Consider the Lee-Carter model:

$$lm(x, t) = \alpha_x + \beta_x K_t; \quad K_t = K_{t-1} + c + \sigma_k Z_t$$

You are given that $K_0 = 1$, $c = -0.5$, $\sigma_k = 0.2$, $\alpha_{80} = -3.26$, and $\beta_{80} = 0.3$.

Calculate the 90th percentile of m_{80} when $t = 1$.

- (A) 0.046
- (B) 0.048
- (C) 0.050
- (D) 0.052
- (E) 0.054

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6. A parrot owner purchases a 3-year term life insurance of 1000 on his 5-year old parrot.

You are given:

- (i) Benefits are paid immediately on death.
- (ii) Mortality follows the following life table excerpt:

l_5	d_5	d_6	d_7
488	24	37	43

- (iii) Deaths are uniformly distributed between integer ages.
- (iv) $i = 0.04$

Calculate the standard deviation of the present value of the benefit.

- (A) 360
- (B) 366
- (C) 372
- (D) 378
- (E) 384

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7. An insurer issues a whole life annuity-due of 1 per year, payable annually, deferred for 30 years to (30).

You are given:

- (i) Mortality follows the Standard Ultimate Life Table.
- (ii) $i = 0.05$

Calculate the single net premium for this annuity.

- (A) 2.94
- (B) 3.04
- (C) 3.14
- (D) 3.24
- (E) 3.34

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8. You are given:

- (i) Mortality and sickness follow the Standard Sickness-Death model.
- (ii) ${}_{20}p_{50}^{10} = 0.04525$
- (iii) $i = 0.05$

Calculate $\bar{a}_{50:\overline{20}|}^{11}$.

- (A) 10.59
- (B) 10.64
- (C) 10.69
- (D) 10.74
- (E) 10.79

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9. Jamal and Janaya, both age 55, receive a last survivor annuity-due of 2400 per year, payable monthly for 15 years.

You are given:

- (i) Jamal and Janaya have independent future lifetimes.
- (ii) $i = 0.045$
- (iii) ${}_{15}p_{55} = 0.9310$
- (iv) $\ddot{a}_{55}^{(12)} = 16.54$; $\ddot{a}_{70}^{(12)} = 12.03$; $\ddot{a}_{55:55}^{(12)} = 14.93$; $\ddot{a}_{70:70}^{(12)} = 9.84$

Calculate the expected present value of this annuity.

- (A) 25,760
- (B) 25,960
- (C) 26,160
- (D) 26,360
- (E) 26,560

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10. For a special fully discrete 20-payment whole life insurance on (40), you are given the following information:

- (i) The death benefit is 1000 for the first 10 years and 4000 thereafter.
- (ii) The annual net premium payable during the first 10 years is 25% of the annual net premium payable during the remaining 10 years.
- (iii) Mortality follows the Standard Ultimate Life Table.
- (iv) $i = 0.05$

Calculate the annual net premium payable during the first 10 years.

- (A) 17
- (B) 27
- (C) 37
- (D) 47
- (E) 57

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- 11.** A pet store offers an insurance policy when customers buy goldfish. The store sells 100 newborn goldfish, and all customers purchase the insurance.

You are given:

- (i) The insurance pays 14 immediately on the death of the goldfish.
- (ii) A single premium of 10 is paid at the time of purchase.
- (iii) Under the goldfish mortality model, $\bar{A}_0 = 0.66667$ and ${}^2\bar{A}_0 = 0.49383$.
- (iv) The goldfish have independent future lifetimes.

Using the normal approximation to the aggregate loss, calculate the probability that the portfolio will be profitable.

- (A) 94.4%
- (B) 95.4%
- (C) 96.4%
- (D) 97.4%
- (E) 98.4%

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- 12.** A life insurer issues a fully discrete whole life insurance with a face amount of 100,000 to (45).

You are given the following information:

- (i) Initial expenses are 100 and 50% of the first premium.
- (ii) Renewal expenses are 5% of the premium.
- (iii) Claim expenses of 1,000 are incurred when the benefit is paid.
- (iv) The annual gross premium is 1,050.
- (v) Mortality follows the Standard Ultimate Life Table.
- (vi) $i = 0.05$

Calculate the standard deviation of the gross future loss at issue for this policy.

- (A) 13,160
- (B) 13,260
- (C) 13,360
- (D) 13,460
- (E) 13,560

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- 13.** A continuing care retirement community (CCRC) determines its fees using a 4-state multiple state model, where State 0 is the Independent Living Unit (ILU), State 1 is the Assisted Living Unit (ALU), State 2 is the Specialized Nursing Facility (SNF), and State 3 is Dead.

The community's actuary uses the following information for determining the fee schedule for a new resident entering the ILU, age 70. You are given:

- (i) $i = 0.05$
- (ii) The following annuity values:

$\ddot{a}_{70}^{(12)00}$	$\ddot{a}_{70}^{(12)01}$	$\ddot{a}_{70}^{(12)02}$
10.0554	0.74720	0.30944

- (iii) The CCRC monthly costs, incurred at the start of each month, are:
- ILU (State 0): 2,600
 - ALU (State 1): 5,000
 - SNF (State 2): 14,000
- (iv) An initial fee of 100,000 is paid on entry; it is not refunded on death.
- (v) The monthly fee, paid at the start of each month, is X while the resident is in ILU or ALU, and $1.3X$ while the resident is in SNF.
- (vi) Fees are determined using the equivalence principle.

Calculate X .

- (A) 2270
- (B) 2290
- (C) 2310
- (D) 2330
- (E) 2350

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14. For a whole life insurance on (70), you are given:

- (i) Mortality follows the Standard Ultimate Life Table.
- (ii) $i = 0.05$
- (iii) ${}_tV$ is the net premium reserve at time t .
- (iv) ${}_tV(1+i) = q_{70+t}(2500) + p_{70+t}({}_{t+1}V)$ for $t = 1, 2, \dots$

Calculate ${}_5V$.

- (A) 1238
- (B) 1255
- (C) 1272
- (D) 1288
- (E) 1305

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15. For a fully discrete whole life insurance of 1000 on (70), you are given:

- (i) $i = 0.05$
- (ii) Mortality follows the Standard Ultimate Life Table.
- (iii) The net premium is 35.66.
- (iv) Deaths are uniformly distributed between integer ages.

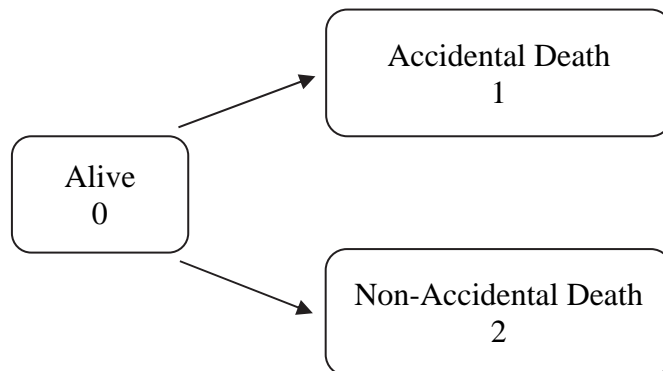
Calculate the net premium reserve at duration 10.33.

- (A) 298
- (B) 319
- (C) 322
- (D) 324
- (E) 329

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16. An insurer issues a special whole life insurance to (45), using the following accidental death model:



You are given:

- (i) The policy pays 10,000 for Accidental Deaths and 5,000 for Non-Accidental Deaths.
- (ii) The death benefit is payable at the moment of death.
- (iii) Premiums are payable continuously while the policyholder is Alive.
- (iv) $i = 0.04$
- (v) The following table of actuarial functions, evaluated at 4%

x	\bar{a}_x^{00}	$1000 \bar{A}_x^{01}$	$1000 \bar{A}_x^{02}$
45	17.0915	1.7086	327.9484
55	14.2540	1.4835	439.5236

Calculate the net premium reserve at the end of 10 years.

- (A) 730
- (B) 750
- (C) 770
- (D) 800
- (E) 830

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17. A whole life sickness-death benefit policy is issued to a Healthy life age 50.

You are given:

- (i) The policy pays 1,000 per year, continuously, while the policyholder is Sick.
- (ii) The policy pays 10,000 at the moment of the policyholder's death.
- (iii) The policyholder pays a premium of 450 continuously while the policyholder is Healthy.
- (iv) Transitions follow the Standard Sickness-Death Model.
- (v) $i = 0.05$

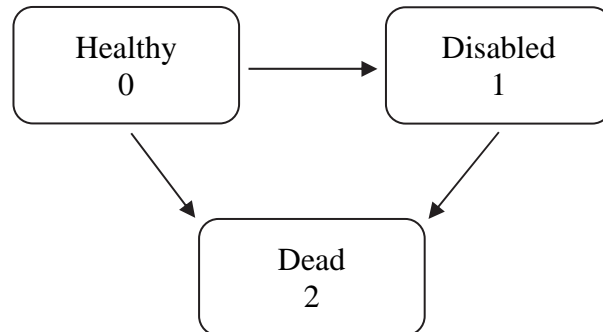
Calculate the gross premium reserve at time 15, given that the policyholder is Healthy at that time.

- (A) 5066
- (B) 5096
- (C) 5146
- (D) 5256
- (E) 5326

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18. You are conducting a profit test on a permanent disability insurance policy issued to (x) , based on the following model:



You are given:

- (i) ${}_4p_x^{00} = 0.65$; ${}_5p_x^{00} = 0.60$; ${}_4p_x^{01} = 0.20$; ${}_5p_x^{01} = 0.23$
- (ii) $\text{Pr}_5^{(0)} = 70.0$; $\text{Pr}_5^{(1)} = 10.0$
- (iii) The risk discount rate is 10% per year.
- (iv) $NPV(4) = 25.4$

Calculate $NPV(5)$.

- (A) 50
- (B) 51
- (C) 53
- (D) 55
- (E) 57

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19. For a defined benefit pension plan, you are given:

- (i) An employee who is currently age 55 has 20 years of service.
- (ii) Her salary in the year following valuation will be 70,000.
- (iii) Salaries increase by 1.5% per year.
- (iv) The retirement benefit is based on the average of the final two years' salary.
- (v) The retirement benefit is paid as an annual annuity due starting at age 65.
- (vi) The accrual rate is 2.0% per year of service.
- (vii) There are no exits from the plan other than death and retirement.
- (viii) No benefits are payable upon death before retirement.
- (ix) Mortality follows the Standard Ultimate Life Table.
- (x) $i = 0.05$

Calculate the normal cost for the year for the employee under the projected unit credit method.

- (A) 12,676
- (B) 12,776
- (C) 12,876
- (D) 12,976
- (E) 13,076

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20. An employee aged 55 is eligible for post-retirement health care coverage.

You are given the following valuation information:

- (i) The benefit premium increases each year with a rate of health inflation of $j = 0.02$.
- (ii) The benefit premium increases for each year of age by a factor of $c = 1.04$.
- (iii) Mortality is the only decrement, and is assumed to follow the Standard Ultimate Life Table.
- (iv) Retirement occurs only at age 65.
- (v) At $i^* = -1.018\%$, $\ddot{a}_{65} = 26.607$.
- (vi) The current health care benefit premium at age 65 is $B(65, 0) = 2338$.
- (vii) $i = 0.05$

Calculate the expected present value of the post-retirement health care benefits for this employee.

- (A) 43,000
- (B) 44,000
- (C) 45,000
- (D) 46,000
- (E) 47,000

Exam LTAM

SECTION B – Written-Answer

1. (10 points) A life insurer issues a 5-year term insurance, with premiums payable half-yearly, to Korina, who is age 60. The death benefit of 100,000 is paid at the end of the half-year of death.

You are given:

- (i) Expenses are 10% of each semi-annual premium.
- (ii) $i = 0.05$
- (iii) Mortality follows the Standard Ultimate Life Table; the force of mortality of the Standard Ultimate Life Table is:

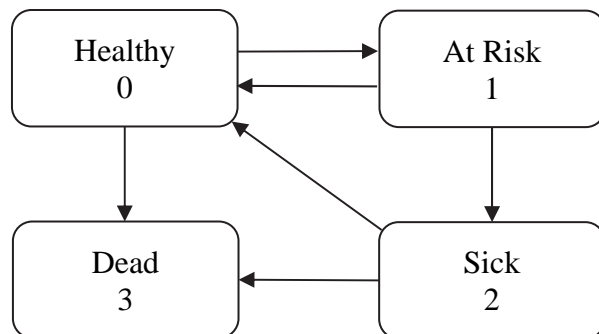
$$\mu_{x+t} = A + Bc^{x+t}, \text{ where } A = 2.2 \times 10^{-4}, B = 2.7 \times 10^{-6}, c = 1.124$$

- (iv) The gross premium is 270 per half-year.

(a) (3 points)

- (i) Show that $100,000 \cdot {}_{0.5}q_{64} = 260$ to the nearest 10. You should calculate the value to the nearest 1.
- (ii) Show that the gross premium reserve at time 4 is 30 to the nearest 10. You should calculate the reserve to the nearest 0.1.

In the final year of the policy, the insurer decides to re-evaluate the gross premium reserve due to a virus that is spreading through the community. The insurer uses the following multiple state model.



1. Continued

A Healthy person will move to State 1, At Risk, if exposed to the virus. Every person moving into the At Risk state will be tested for the virus. If the result of the test is positive, the person moves to State 2, Sick; otherwise, the person will move back to the Healthy state.

The transition intensities are:

$$\mu_x^{01} = 0.06$$

μ_x^{03} is the force of mortality of the Standard Ultimate Life Table, as in (iii) above.

$$\mu_x^{10} = 20.0; \quad \mu_x^{12} = 80.0$$

$$\mu_x^{20} = 4.0; \quad \mu_x^{23} = 5\mu_x^{03}$$

You are also given that Korina is Healthy at time 4, and that premiums are payable conditional on being in State 0, 1, or 2.

- (b) (2 points) Calculate the probability that Korina will remain in the Healthy state throughout the final year of the policy, ${}_1P_{64}^{\overline{00}}$.
- (c) (3 points) The insurer re-evaluates the gross premium reserve at time 4 using the model above, with the same interest rate and expense assumptions as in part (a).

You are given the following probabilities:

x	${}_{0.5}P_x^{00}$	${}_{0.5}P_x^{01}$	${}_{0.5}P_x^{02}$	${}_{0.5}P_x^{03}$	${}_{0.5}P_x^{13}$	${}_{0.5}P_x^{23}$
64.0	0.98661	0.00059	0.01016	0.00264	0.00605	0.00700
64.5	0.98646	0.00059	0.01016	0.00279	0.00640	0.00740

Calculate the revised gross premium reserve for Korina's policy at time 4.

- (d) (2 points) Calculate the expected time, in days, of each sojourn in the At Risk state.

2. (10 points) An insurer issues a 20-year term insurance policy to a life age 50. The sum insured is 1,000,000 for the first 10 years, and 360,000 for the remainder of the term.

Level premiums are paid annually in advance throughout the term of the policy. Death benefits are paid at the end of the year of death.

You are given the following information:

- (i) Initial Expenses: 200 plus 50% of the first premium.
 - (ii) Renewal Expenses: 5% of the second and subsequent premiums, plus 10 per year at the start of the second and subsequent years.
 - (iii) Mortality follows the Standard Ultimate Life Table.
 - (iv) $i = 0.05$
 - (v) Gross premiums are calculated using the equivalence principle.
- (a) (3 points) Show that the gross premium is 2,055 to the nearest 5. You should calculate the premium to the nearest 0.1.
- (b) (3 points)
- (i) Show that the gross premium reserve at time 10 is negative.
 - (ii) Explain why a negative gross premium reserve indicates a problem with the policy.
- (c) (3 points) The insurer decides to change the reserve method to Full Preliminary Term.
- Calculate the revised reserve at time 10.
- (d) (1 point) Explain whether changing the reserving method has solved the problem identified in (b)(ii).

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3. (9 points) NED Life issues fully discrete 20-year endowment insurances of 100,000 on both a broker-issued basis and a direct-marketed basis.

You are given:

- (i) For direct-marketed policies, mortality follows the Standard Ultimate Life Table.
 - (ii) For broker-issued policies, mortality follows a 1-year select and ultimate model, where ultimate mortality follows the Standard Ultimate Life Table, and where $q_{[x]} = 0.6 \times q_x$.
 - (iii) $i = 0.05$
 - (iv) ${}^2A_{50:\overline{20}|} = 0.15415$
- (a) (1 point) Explain why the first year mortality is expected to be lower for the broker-issued policies.
- (b) (2 points) You are given that $10,000 \times {}^2A_{[50]:\overline{20}|} = 1,538$ to the nearest 1. Calculate the value to the nearest 0.1.

You are also given that, for the broker-issued policies:

- (i) $A_{[50]:\overline{20}|} = 0.38817$
 - (ii) $K_{[50]}$ denotes the curtate future lifetime of a select life age 50.
 - (iii) ${}_0L$ denotes the future loss at issue random variable for a broker-issued policy on a select life age 50.
 - (iv) NED Life sets the premium P such that $E[{}_0L] = 0$.
- (c) (3 points)
- (i) Write an expression for ${}_0L$ in terms of $K_{[50]}$, the premium P , and interest rate functions.
 - (ii) You are given that P is 3,020 to the nearest 10. Calculate the premium to the nearest 1.

3. Continued

(d) (3 points)

- (i) You are given that the standard deviation of ${}_0L$ is 9,100 to the nearest 100. Calculate the standard deviation to the nearest 1.
- (ii) NED Life issues 1,000 of these broker-issued policies to lives with independent future lifetimes.

Using the normal approximation, calculate the probability that the aggregate loss at issue on the portfolio exceeds 200,000.

4. (10 points) For a deterministic mortality improvement scale, you are given the following information.

- (i) Improvement factors up to 2020 have been evaluated from the industry database.
- (ii) Long term improvement factors are assumed to apply from 2040, and are constant for each age.
- (iii) The improvement factors for 2021 – 2039, inclusive, are determined using a combination of age-based and cohort-based cubic splines, with 50% of weight attached to the age-based spline and 50% to the cohort-based spline.
- (iv) Selected short and long term values of improvement factors are given in the following table.

Age, x	Short Term		Long Term
	$\varphi(x, 2019)$	$\varphi(x, 2020)$	$\varphi(x, 2040 + k), k = 0, 1, 2, \dots$
44	0.020	0.021	0.010
45	0.021	0.022	0.010
46	0.022	0.023	0.010
47	0.022	0.023	0.010
\vdots	\vdots	\vdots	\vdots
50	0.024	0.025	0.010

- (a) (2 points) Explain how cohort and age effects apply in longevity modelling.
- (b) (2 points) You are given that the cohort-based cubic spline for a life age 46 in 2020, and for $t \geq 0$, is $C_c(46+t, 2020+t) = \tilde{a}t^3 + \tilde{b}t^2 + \tilde{c}t + \tilde{d}$, where:

$$\tilde{a} = 8.25 \times 10^{-6}; \quad \tilde{b} = -2.98 \times 10^{-4}.$$

Calculate \tilde{c} and \tilde{d} .

- (c) (3 points) The age-based cubic spline for age 50, and for $t \geq 0$, is

$$C_a(50, 2020+t) = at^3 + bt^2 + ct + d, \text{ where } a = 6.25 \times 10^{-6}.$$

Calculate b , c , and d .

4. Continued

(d) (3 points) The age-based cubic spline for age 48 has the following parameters:

$$a = 6.0 \times 10^{-6}; \quad b = -2.05 \times 10^{-4}; \quad c = 0.001; \quad d = 0.024.$$

- (i) Calculate $C_a(48, 2022)$.
- (ii) Calculate $C_c(48, 2022)$.
- (iii) You are given that $q(48, 2021) = 0.00511$. Calculate $q(48, 2022)$.

5. (8 points) You are given the following information about Devon, who is a member of a Career Average Earnings pension plan.

- (i) On the valuation date, 1/1/2021, Devon is 63 years old.
- (ii) Devon joined the pension plan on 1/1/1998.
- (iii) On 1/1/2021 Devon's total past earnings are 2,400,000.
- (iv) Devon's salary in 2021, assuming she stays in employment for the whole year, will be 170,000.
- (v) All salaries are set on the 1st of January each year.
- (vi) Contributions are paid on the 1st of January each year.
- (vii) The accrual rate is 2%.
- (viii) The normal pension form is a single life, monthly annuity-due.
- (ix) On death in service, the benefit is a lump sum of 4 times the annual pension that the member would have received had they retired at the time of their death.
- (x) There are no other benefits.

The valuation assumptions are:

- Decrements from active service follow the Standard Service Table.
- Exits before age 65 occur half-way through the year of age.
- $i = 0.05$
- $\ddot{a}_{63.5}^{(12)} = 13.5139$; $\ddot{a}_{64.5}^{(12)} = 13.2312$; $\ddot{a}_{65}^{(12)} = 13.0870$
- The funding method is Traditional Unit Credit.

(a) (4 points)

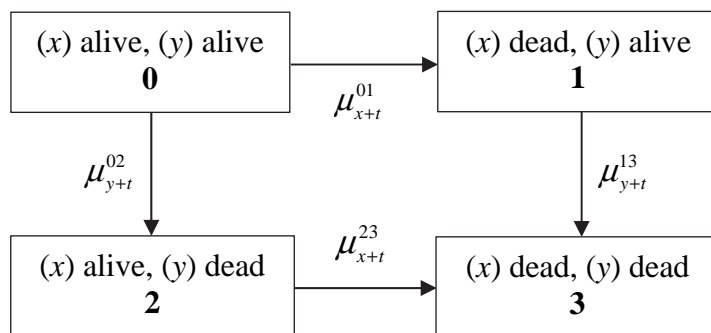
- (i) Calculate the Actuarial Liability for the age retirement pension for Devon at 1/1/2021.
- (ii) Calculate the Normal Contribution for the age retirement pension for Devon at 1/1/2021.

(b) (4 points)

- (i) Calculate the Actuarial Liability for the death in service benefit for Devon at 1/1/2021.
- (ii) Calculate the Normal Contribution for the death in service benefit for Devon at 1/1/2021.

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6. (9 points) Assume the following 4-state joint life model of mortality.



- (a) (1 point) Describe the annuity that has actuarial value \bar{a}_{xy}^{01} .
- (b) (2 points) Write down the Kolmogorov forward differential equations, with boundary conditions, for ${}_t p_{xy}^{00}$ and ${}_t p_{xy}^{01}$.
- (c) (3 points) You are given the Euler-Maclaurin-Woolhouse approximation formula

$$\int_0^{\infty} g(t) dt \approx \sum_{k=0}^{\infty} g(k) - \frac{1}{2} g(0) + \frac{1}{12} g'(0).$$

Use this approximation, with $g(t) = {}_t p_{xy}^{00} e^{-\delta t}$, to prove that

$$\bar{a}_{xy}^{00} \approx \ddot{a}_{xy}^{00} - \frac{1}{2} - \frac{1}{12} (\mu_x^{01} + \mu_y^{02} + \delta).$$

Hint: Use the Kolmogorov forward differential equations, and the chain rule.

- (d) (3 points) Zoe and Amanda are both 70 years old. They purchase a last survivor annuity, with a level benefit payable continuously.

The single net premium is 1,000,000.

You are given the following information:

- (i) The mortality of each life follows the Standard Ultimate Life Table.
- (ii) Zoe and Amanda have independent future lifetimes.

6. Continued

(iii) $i = 0.05$

(iv) $\mu_{70} = 0.009881$

(v) $\bar{a}_{70|70} = 2.0317$

Calculate the level annual rate of benefit, using the Euler-Maclaurin-Woolhouse approximation.

****END OF EXAMINATION****

USE THIS PAGE FOR YOUR SCRATCH WORK