Notation, Terminology, and Conventions Used on Exam ALTAM
Version: Dec 21 2022

Introduction
Actuarial notation and terminology varies by country, by application, and by source. The purpose of this study note is to present the notation, terminology, and conventions that will be used in ALTAM examinations, in particular where these differ from terms used in practice, or from the notation and terminology used in *Actuarial Mathematics for Life Contingent Risks* (3rd edition) (*AMLCR*). For notation and terminology not discussed here, the meaning in *AMLCR* (including the sections of *AMLCR* which are on the Fundamentals of Actuarial Mathematics [FAM] syllabus) will apply.

Conventions
Unless specified otherwise within the examination question, the following assumptions should be made:

(i) The force of interest is constant and is greater than 0.

(ii) Future lifetimes are independent.

(iii) All lives in a question follow the same mortality table.

(iv) Expenses are payable at the start of each period.

(v) Premiums are payable throughout the term of the contract.

(vi) Expenses (including commissions) that are expressed as a percent of premium are payable when the corresponding premium is payable, and end when the premiums are no longer payable.

(vii) Confidence intervals are 2-sided, and are based on the normal approximation.

Notation and Terminology

Both the **survival function, salary rate function and salary scale** may be represented by s or s. On the examination, the symbols $\overline{s}_x$ and s_x will indicate the salary rate function and salary scale respectively. When the survival function is used on the examination, the definition will be clear from the context of the question, or the question will define the symbol.

Select and ultimate tables given in the examination will follow one of the following two conventions for values applying during the select period.

(i) Each row corresponds to a single age at selection, or

(ii) Each row corresponds to a single attained age, at different durations since selection.

The table format can be inferred from the table headers.

The **present value of future loss random variable** may be represented by ${}_0L$ or L_0 for loss at issue and ${}_tL$ or L_t for loss from t years after issue (given that the policy is in force). When the general symbol L is used to represent the present value of future losses random variable the symbol, including any subscripts or superscripts, will be defined in the text of the question.

Actuarial present value and **expected present value** are terms used for the expected value of a present value random variable. Either term may be used on the examination.

Fully discrete insurance is an insurance where both the premiums and the benefits are paid only at discrete time points. **Semi-continuous** insurance is an insurance where the premiums are paid at discrete time points and the death benefits are paid at the moment of death. **Fully continuous** insurance is an insurance where the premiums are paid continuously, and the death benefits are paid at the moment of death. Unless stated otherwise in the text of the question discrete time points are the beginnings of years for premium payments and the ends of years for death benefit payments.

Special insurance is an insurance that has either non-level benefits, non-level premiums, or both. The non-level aspects of the insurance will be described in the text of the question. If an insurance is not defined as "special" then premiums and benefits are assumed to be level unless there is explicit information in the text of the question to the contrary.

Net premiums are determined using the equivalence principle and assuming no expenses.

Gross Premiums are determined taking expenses (if any) into account, and need not use the equivalence principle. Any question requiring the calculation of a gross premium will specify the premium principle to be used (for example, setting a specified expected profit, or using the portfolio percentile premium approach). If expenses are specified in the text of a question, then the expenses need to be considered in the solution to the question. If no expenses are specified, then expenses should be assumed to be zero.

The terms **certain period** and **guarantee period** are synonymous terms. Either term may be used on the exam.

A **policy value** is the expected value of the future loss random variable. This is consistent with the usage in *AMLCR*. LTAM and MLC exams used the term *reserve* for that expected value, and that term is still regularly encountered in literature and practice. On the ALTAM examination, **policy value** will be used.

In practice, the financial statements of an insurance company will include a liability amount in respect of future outgo on a policy in force, and this amount is called the **reserve**. It may or may not be determined from policy values. In the examination, reserves may be used in the context of profit testing, or when calculating modified net premium reserves.

The policy value or reserve at time t will be represented by ${}_tV$ on the examination.

The **net premium policy value** for a policy inforce at duration $t \geq 0$ is the expected value at that time of the net future loss random variable (i.e. assuming no expenses). It uses net premiums calculated on the policy value basis, using the equivalence principle, not the actual premiums payable nor the net premiums calculated on the original premium basis. In MLC and LTAM exams these were called benefit reserves or net premium reserves.

The **gross premium policy value** for a policy inforce at duration $t \geq 0$ is the expected value at that time of the gross future loss random variable, based on the policy's actual gross premium. The mortality, interest and expense assumptions for the policy value would not necessarily be the same as those used in that gross premium calculation. In MLC and LTAM exams the gross premium policy value was called the gross premium reserve.

A **modified net premium reserve** is a net premium policy value where the net premiums are adjusted to allow implicitly for initial expenses. The **Full Preliminary Term** reserve is an example of a modified net premium reserve, in which premiums are assumed to be level from the second year to the end of the premium payment period. Any modified net premium reserve questions on the examination other than full preliminary term will specify the modification basis in the question.

In a **multiple decrement model** ${}_tq_x^{(j)}$ is the probability a life age x fails or exits in the next t years due to decrement (j). This is also called the dependent probability of exit from decrement (j). The t-year probability of failure due to all decrements is ${}_tq_x^{(\tau)}$.
The associated single decrement t-year probability of failure at age x due to decrement (j) is denoted ${}_tq_x'^{(j)}$. This is also called the independent probability of exit from decrement (j).
A multiple decrement model is a special case of a multiple state model. Any of the *AMLCR* multiple state model notation may be used with a multiple decrement model on the examination.

Transition probabilities for a multiple state model may be presented in matrix form. For a model with m+1 states, labelled $0,1,2,\cdots,m$, the 1-year transition probability matrix is

$$\boldsymbol{P}_x = \begin{pmatrix} p_x^{00} & p_x^{01} & \cdots & p_x^{0m} \\ p_x^{10} & p_x^{11} & \cdots & p_x^{1m} \\ \vdots & \vdots & \ddots & \vdots \\ p_x^{m0} & p_x^{m1} & \cdots & p_x^{mm} \end{pmatrix}$$

Retirement benefits based on years of service include any fractional years, unless explicitly stated otherwise. For example, a pension plan that stipulates that annual retirement benefits are calculated by the participant's final salary x 0.02 x the number of years of service would provide an annual benefit of 8200 for a participant with a final salary of 20,000 and 20.5 years of service ($20{,}000 \times 0.02 \times 20.5$).

In the examination, the term **Normal Cost** is synonymous with the term **Normal Contribution**. Either term may be used on the examination.

Actuarial liability and **actuarial accrued liability** are synonymous terms and either term may appear on the exam.

Pension reduction factors are used in the context of benefit calculations for individuals who retire earlier than the normal retirement age. These factors are expressed as <u>simple</u> rates per unit time, rather than compound rates per unit time, unless explicitly stated otherwise. For example, a pension plan may stipulate a 5% reduction for each year prior to normal retirement age, 65. For a participant retiring at age 60, this factor would result in a 25% reduction to the normal retirement benefit.

Profit and Gain

For a time period where all cash flows occur only at the beginning and end of the time period, **Profit** for the time period occurs at the end of the time period and is (a) minus (b) where:

(a) is the accumulated value of the sum of the reserve at the end of the previous period and the cash flows that occur at the beginning of the period; and

(b) is the sum of the value of the reserve at the end of the period and the cash flows that occur at the end of the period.

Expenses at the inception of a contract may be classified as negative profit at time 0 or may be part of period 1 cash flow and included in the profit calculation for period 1. Any initial expenses that are not part of the period 1 cash flow will be identified in the question as **pre-contract expenses**. If a reserve is to be established at time 0, before the first premium is received, it would be part of the time 0 profit. Any such reserve will be identified in the question.

Expected profit is the profit calculated using the gross premium, expected cash flows at the beginning and end of the period, and accumulating beginning of year values at the assumed interest rate. The assumptions used to calculate the expected cash flows may or may not be the same as those used to calculate the reserve and/or premium.

Actual profit is calculated using the gross premium, actual cash flows at the beginning and end of the period, and accumulating beginning of year values using the actual rate of interest earned during the period.

Gain is the actual profit minus the expected profit for the period. **Gain by source** is the gain calculated where the effect of the difference between the observed values and the expected values in the profit calculations from one source is reflected, while the differences for the other sources are not. Typical sources are expenses, interest, mortality, and withdrawal. Often, gains from multiple sources are calculated sequentially. For example, the gain from mortality might be calculated first, reflecting the difference between the observed mortality and the assumed mortality, and the gain from interest calculated second, reflecting the difference between the observed and assumed interest, while using only the observed mortality.

The examination will only include questions asking for gain by source where the reserves are gross premium reserves and expected profits are based on the reserve assumptions. Under those conditions, the expected profit is 0 and the sum of the gains by source is equal to the actual profit.

Universal Life

The following terminology and conventions will be used in examination questions. These are all consistent with *AMLCR*, but may differ from terminology and conventions used in practice in some cases.

(i) Account Values (AV) are calculated at discrete, regular intervals. The question will indicate the calculation period.

(ii) Premiums are paid at the start of each time interval, and benefits are paid at the end of each time interval.

(iii) Premiums are level and paid throughout the term of the policy, unless the question explicitly states otherwise.

(iv) The Cost of Insurance rate is the mortality rate used to determine the cost of insurance.

(v) The rate of interest used to discount the mortality charge may be assumed to be equal to the credited rate unless otherwise specified.

(vi) Expense charges, credited interest rates, and other account value factors are assumed to be constant throughout the term of the policy, unless the question explicitly states otherwise.

(vii) The Death Benefit at t , DB_t , is the amount of benefit that would be payable at t in the event that the insured dies in the time interval that ends at t.

(viii) The Additional Death Benefit at t , ADB_t , is the difference between the Death Benefit and the Account Value at t, i.e., $ADB_t = DB_t - AV_t$

(ix) Corridor factors set an age-dependent minimum Death Benefit at t, in terms of the Account Value at t. Specifically, given a corridor factor c, the Death Benefit at t must be at least $c \times$ the Account Value at t.

(x) If no corridor factors are stated in the question, assume the policy has none.

(xi) Type A UL policies have a fixed Death Benefit, unless the corridor factor minimum death benefit applies.

(xii) Type B UL policies have a fixed Additional Death Benefit, so the Death Benefit is the sum of the pre-specified Additional Death Benefit and the Account Value, unless the corridor factor minimum applies.

(xiii) The Cash Value of the policy is the Account Value minus a Surrender Penalty if applicable. Cash Surrender Value is an equivalent term to Cash Value and may also be used on the exam.

(xiv) Reserves are equal to Account Values unless a no-lapse guarantee applies, or unless otherwise specified.