CP 351 ALM Assessment Formula Package

2026 - 2027

Exam booklets will include a formula package identical to the one attached to this study note. The exam committee believes that by providing many key formulas candidates will be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula package was developed by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not in the formula package.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. For example, there are several versions of the Black- Scholes-Merton option pricing formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aid that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes. In sources where some equations are numbered and others are not, the page number is provided instead.

Chapter 6 Copulas

P. 198

$$F_j(x_j) = \Pr\lceil \lfloor X_j \le x_j \rceil \rfloor$$

$$C(F_1(x_1),....,F_m(x_m)) = \Pr\left[X_1 \le x_1,.....,X_m \le x_m\right] \text{ for j=1, ….., m}$$

P. 211

$$C(u_1,u_2,...,u_m) = u_1 * u_2 * ... * u_m$$

$$C(u_1,u_2,...,u_m) = \min(u_1,u_2,...,u_m)$$

$$C(u,v) = \max(u+v-1,0)$$

$$C(u_1,u_2,...,u_m) = (u_1^{-\theta} + u_2^{-\theta} + ... + u_m^{-\theta} - (m-1))^{-\frac{1}{\theta}}, \theta > 0$$

$$C(u_1,u_2,...,u_m) = Exp\{-((-Ln(u_1))^{\theta} + (-Ln(u_2))^{\theta} + ... + (-Ln(u_m))^{\theta})^{\frac{1}{\theta}}\} \ , \ \theta > 0$$

$$C(u_1,u_2,...,u_m) = \Phi_P(z_{u_1}, z_{u_2}, ..., z_{u_m}) \ , \ -1 \le \rho \le 1, z_\alpha = \Phi^{-1}(\alpha)$$

P. 212

$$1 = u + v - C(u,v) + \overline{C}(u,v)$$

$$\overline{C}(u,v) = \Pr[F_x(X) > u, F_y(Y) > v] \ , \ \hat{C}(u,v) = \overline{C}(1-u,1-v)$$

P. 214

$$\rho(X,Y) = \frac{Cov[X,Y]}{\sqrt{Var[X]Var[Y]}} = \frac{E[XY] - E[X]E[Y]}{\sqrt{Var[X]Var[Y]}}$$

P. 216

$$\rho_S(X,Y) = \rho(U,V) = \frac{E[UV] - E[U]E[V]}{\sqrt{Var[U]Var[V]}}$$

$$r = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{r_{x_i} - \overline{r}_x}{s_{r_x}}\right)\left(\frac{r_{y_i} - \overline{r}_y}{s_{r_y}}\right)$$

P. 218 - 219

$$\begin{aligned} r_k &= \Pr[(X - X^*)(Y - Y^*) > 0] - \Pr[(X - X^*)(Y - Y^*) < 0] \\ &= E[sign((X - X^*)(Y - Y^*))] \end{aligned} \quad (6.14, 6.15)$$

Where $sign(z) = \{-1, z < 0, 0, z = 0, 1, z > 0\}$

$$t_k = \frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} sign((x_i - x_j)(y_i - y_j)) \quad (6.16)$$

P. 224

$$C(u,v) = \varphi^{-1}(\varphi(u) + \varphi(v))$$

P. 225

$$\phi(u) = \frac{(u^{-\theta} - 1)^{\delta}}{\theta^{\delta}}$$

Chapter 8: Market Risk Models

Pg 272 $$E_{t-1}\left[\sigma_t^2\right] = \sigma_t^2 \quad (8.4)$$

Pg 272

$$E_{t-2}\left[\sigma_t^2\right] = a_0 + (a_1 + b)\sigma_{t-1}^2 \quad (8.5)$$

Pg 273

$$\lim_{x\to\infty} a_0\left(\frac{1-(a_1+b)^{t-1}}{1-(a_1+b)}\right) + (a_1+b)^{t-1}\sigma_1^2 = \frac{a_0}{1-(a_1+b)} \quad (8.6)$$

Pg 275

TGARCH variance: $\sigma_t^2 = a_0 + (a_1 + \gamma \min(Y_{t-1}, 0))(Y_{t-1} - \mu)^2 + b\sigma_{t-1}^2$ (8.7)

Pg 275

$$\begin{aligned} Y_t \mid F_{t-1} &= r - \tfrac{\sigma_t^2}{2} + \sigma_t \varepsilon_t \\ \sigma_t^2 &= a_0 + a_1 (Y_{t-1} - (r - \frac{\sigma_t^2}{2}))^2 + b\sigma_{t-1}^2 \end{aligned} \quad (8.8)$$

Pg 281

$$\pi_1 = \frac{p_{21}}{p_{12} + p_{21}} \quad \text{and} \quad \pi_2 = \frac{p_{12}}{p_{12} + p_{21}} \quad (8.12)$$

Pg 285

$$l(\theta) = \sum_{i=1}^{n} \ln f(x_i \mid x_1, ..., x_{i-1}; \theta) \quad (8.13)$$

Pg 288

$$\begin{aligned} f(y_2 \mid y_1) &= f(y_2, \rho_1 = 1, \rho_2 = 1 \mid y_1) + f(y_2, \rho_1 = 1, \rho_2 = 2 \mid y_1) \\ &+ f(y_2, \rho_1 = 2, \rho_2 = 1 \mid y_1) + f(y_2, \rho_1 = 2, \rho_2 = 2 \mid y_1) \end{aligned} \quad (8.14)$$

Pg 288

$$f(y_2, \rho_1 = 1, \rho_2 = 1 \mid y_1) = f(y_2 \mid y_1, \rho_1 = 1, \rho_2 = 1) * p(\rho_2 = 1 \mid y_1, \rho_1 = 1) * p(\rho_1 = 1 \mid y_1) \quad (8.15)$$

Chapter 15: Risk mitigation using options and derivatives

Pg 423 K, priced at P_0, with delta of $-\Phi(-d_1)$. Let ω_1 denote the number of units of stock purchased, and ω_2 denote the number of put options. Then we can find a delta-neutral portfolio by solving the following equations for ω_1 and ω_2 :

$V(0) = \omega_1 S(0) + \omega_2 \mathrm{P}_0$ (portfolio value),

$0 = \omega_1 - \omega_2 \Phi(-d_1)$ (portfolio delta).

Pg 428

$$V(0) = \omega_1 S(0) + \omega_2 \mathrm{P}_0 - \omega_3 c_0$$
$$0 = \omega_1 - \omega_2 \Phi(-d_1^{\mathrm{P}}) - \omega_3 \Phi(d_1^c)$$
$$0 = \omega_2 \left(\frac{\phi(d_1^{\mathrm{P}})}{S(0)\sigma\sqrt{T^{\mathrm{P}}}}\right) - \omega_3 \left(\frac{\phi(d_1^c)}{S(0)\sigma\sqrt{T^c}}\right)$$

Pg 434 market value at time 0 of 1 due at t is $v(t)$. The swap rate c is

$$c = \frac{1 - v(n)}{\sum_{k=1}^{n} v(k)}$$

Pg 437

$$x = \frac{v(1) + v(2) + \ldots\ldots + v(n)}{F_1 v(1) + F_2 v(2) + \ldots\ldots + F_n v(n)}$$

$$x = \frac{F_0 - v(n)}{F_1 v(1) + F_2 v(2) + \ldots\ldots + F_n v(n)}$$

Black-Scholes formula for the price of a T-year put option, on a non-dividend paying stock with price S_t at time t, and with strike price K:

$$p_0 = Ke^{-rT}\Phi(-d_2(0,T)) - S_0\Phi(-d_1(0,T))$$
$$p_t = Ke^{-r(T-t)}\Phi(-d_2(t,T)) - S_t\Phi(-d_1(t,T))$$
$$d_1(t_1,t_2) = \frac{\log\left(\frac{S_{t_1}}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(t_2 - t_1)}{\sigma\sqrt{t_2 - t_1}}$$
$$d_2(t_1,t_2) = d_1(t_1,t_2) - \sigma\sqrt{t_2 - t_1}$$

$$0 \le t_1 < t_2, 0 < t < T$$

Chapter 16: Risk Transfer

Pg 461

Probability of first loss (PFL)

Probability of last loss (PLL)

The expected (proportionate) loss (EL)

Let X denote the loss metric used to trigger the cat bond. The attachment point denoted

x_l is the threshold triggering the payments to the cedant, so

$$PFL = \Pr\left[X > x_l\right]$$

The exhaustion point, denoted

x_h is the threshold representing the maximum loss, so

$$PLL = \Pr\left[X > x_h\right]$$

Let

q denote the proportion of principal lost,

$$q = \begin{Bmatrix} 0, X \le x_l \\ \dfrac{X - x_l}{x_h - x_l}, x_l < X \le x_h \\ 1, X > x_h \end{Bmatrix}$$

f and F represent the density and distribution function of X respectively, then

$$EL = E\left[q\right] = \int_{x_l}^{x_h} \frac{x - x_l}{x_h - x_l} f(x)dx + (1 - F(x_h))$$

The sponsor's loss L is assumed to be proportionate to X and $L = kX$ for some constant k

The expected value of loss layer

$$k\left(\int_{x_l}^{x_h} (x - x_l) f(x)dx + (x_h - x_l)(1 - F(x_h))\right) = k(x_h - x_l)EL$$

Fixed Income Securities: Tools for Today's Markets, 4th Edition. Tuckman and Serrat

Chapter 4: DV01, Duration, and Convexity

Pg 108

$$DV01 = -\frac{1}{10,000}\frac{dP}{dy} \quad (4.5)$$

Pg 112

$$D = -\frac{1}{P}\frac{dP}{dy} \quad (4.11)$$

Pg 114

$$C = \frac{1}{P}\frac{d^2P}{dy^2} \quad (4.14)$$

Pg 119

$$P = \frac{c}{2}\sum_{t=1}^{2T}\frac{1}{(1+\frac{y}{2})^t} + \frac{100}{(1+\frac{y}{2})^{2T}} \quad (4.20)$$

$$P = \frac{c}{y}\left(1 - \frac{1}{(1+\frac{y}{2})^{2T}}\right) + \frac{100}{(1+\frac{y}{2})^{2T}} \quad (4.21)$$

Pg 120

$$DV01 = \frac{1}{10,000}\frac{1}{1+\frac{y}{2}}\left[\frac{c}{2}\sum_{t=1}^{2T}\frac{t}{2}\frac{1}{(1+\frac{y}{2})^t} + T\frac{100}{(1+\frac{y}{2})^{2T}}\right] \quad (4.22)$$

$$DV01 = \frac{1}{10,000}\left[\frac{c}{y^2}\left(1 - \frac{1}{(1+\frac{y}{2})^{2T}}\right) + T(1-\frac{c}{100y})\frac{100}{(1+\frac{y}{2})^{2T+1}}\right] \quad (4.23)$$

$$D = \frac{1}{P}\frac{1}{1+\frac{y}{2}}\left[\frac{c}{2}\sum_{t=1}^{2T}\frac{t}{2}\frac{1}{(1+\frac{y}{2})^t} + T\frac{100}{(1+\frac{y}{2})^{2T}}\right] \quad (4.24)$$

$$D = \frac{1}{P}\left[\frac{c}{y^2}\left(1 - \frac{1}{(1+\frac{y}{2})^{2T}}\right) + T(1-\frac{c}{100y})\frac{100}{(1+\frac{y}{2})^{2T+1}}\right] \quad (4.25)$$

Pg 122

$$DV01_{c=0} = \frac{T}{100(1+\frac{y}{2})^{2T+1}} \tag{4.27}$$

$$D_{c=0} = \frac{T}{(1+\frac{y}{2})} \tag{4.28}$$

$$DV01_{c=100y} = \frac{1}{100y}\left[1-\frac{1}{(1+\frac{y}{2})^{2T}}\right] \tag{4.29}$$

$$D_{c=100y} = \frac{1}{y}\left[1-\frac{1}{(1+\frac{y}{2})^{2T}}\right] \tag{4.30}$$

Pg 124

$$DV01 = \frac{PD}{10,000} \tag{4.31}$$

Pg 126

$$C = \frac{1}{P(1+\frac{y}{2})^2}\left[\frac{c}{2}\sum_{t=1}^{2T}\frac{(t/2)((t+0.5)/2)}{(1+\frac{y}{2})^t} + \frac{100T(T+0.5)}{(1+\frac{y}{2})^{2T}}\right] \tag{4.32}$$

$$C_{c=0} = \frac{T(T+0.5)}{(1+\frac{y}{2})^2} \tag{4.33}$$

$$C_{c=100y} = \frac{2}{y}\left[1-\frac{1}{(1+\frac{y}{2})^{2T}}\right] - \frac{2T}{y(1+\frac{y}{2})^{2T+1}} \tag{4.34}$$

Chapter 6: Regression Hedging and Principal Component Analysis

Pg 158

Let F^{JNJ}, $DV01^{JNJ}$, F^{30}, $DV01^{30}$ be the face amounts and DV01s of JNJ and 30-year Treasury bonds, respectively.

$$F^{30} = -F^{JNJ}\frac{DV01^{JNJ}}{DV01^{30}}\hat{\beta} \quad (6.8)$$

Pg 159

$$P\&L = -F^{JNJ}\frac{DV01^{JNJ}}{100}\Delta y_t^{JNJ} - F^{30}\frac{DV01^{30}}{100}\Delta y_t^{30} \quad (6.11)$$

Pg 164

$$y_t = \alpha + \beta x_t + \varepsilon_t \quad (6.22)$$

$$\Delta y_t = \beta \Delta x_t + \Delta \varepsilon_t \quad (6.24)$$

Fixed Income Securities: Valuation, Risk, and Risk Management, Veronesi, 2010

Chapter 5 Interest Rate Derivatives: Forwards and Swaps

P. 157

$$F(t,T_1,T_2) = \frac{1}{(1+\frac{f_n(t,T_1,T_2)}{n})^{n(T_2-T_1)}} \quad (5.5)$$

$$F(t,T_1,T_2) = e^{-f(t,T_1,T_2)(T_2-T_1)} \quad (5.6)$$

P. 160

$$f(0,T,T+\Delta) = r(0,T) + (T+\Delta)\frac{r(0,T+\Delta)-r(0,T)}{\Delta} \quad (5.14)$$

P. 161

$$r(0,T_n)T_n = (r(0,T_1)\Delta + f(0,T_1,T_2)\Delta + ... + f(0,T_{n-1},T_n)\Delta) \quad (5.21)$$

P. 166

Value of FRA at $t = V^{FRA}(t) = V^{fixed}(t) - V^{floating}(t)$
$= N*[\frac{Z(0,T_1)}{Z(0,T_2)}*Z(t,T_2) - Z(t,T_1)]$ (5.25)

P. 170

$$P_c^{fwd}(0,T,T^*) = \frac{c}{2}\sum_{i=1}^{n} P_z^{fwd}(0,T,T_i) + P_z^{fwd}(0,T,T_n) \quad (5.34)$$

P. 175

$$V^{swap}(T_i;c,T) = 100 - (\frac{c}{2}100\sum_{j=i+1}^{M} Z(T_i,T_j) + Z(T_i,T_m)100) \quad (5.40)$$

P. 176

$$c = n(\frac{1-Z(0,T_M)}{\sum_{j=1}^{M} Z(0,T_j)}) \quad (5.43)$$

P. 181

$$f_n^s(0,T,T^*) = n\frac{1-F(0,T,T^*)}{\sum_{j=1}^{M} F(0,T,T_j)}$$ (5.51, This is a correction to the formula in the text.)

Chapter 6 Interest Rate Derivatives: Futures and Options

P. 203

P&L from futures at t=k*contract size*[$P^{fut}(t,T) - P^{fut}(t-dt,T)$] (6.4)

Handbook of Asset and Liability Management Vol 2. Applications and Case Studies

Chapter 13

None

Chapter 18

P. 839

$r(t) = \beta_0 + (\beta_1 + \beta_2)(1-e^{-t/\tau}) / (t/\tau) - \beta_2 e^{-t/\tau}$ Nelson Siegel yield curve

$\lim_{t\to\infty} r(t) = \beta_0$ Long interest rate

$\lim_{t\to 0} r(t) = \beta_0 + \beta_1$ Short interest rate

β_2 and τ Curvature & scaling parameters

Please note that the formula on page 839 contains a couple of typos. Here we have made corrections.

ALM for Life, Annuities, and Pensions, Hatfield (2024) – Section 5

P. 52

$$D_{lower} \leq Dur_A + \frac{L}{E}(Dur_A - Dur_L) \leq D_{upper}$$

$$D_{lower} + \frac{L}{E}Dur_L \leq \left(1 + \frac{L}{E}\right) Dur_A \leq D_{upper} + \frac{L}{E}Dur_L$$

$$\frac{D_{lower} + \frac{L}{E}Dur_L}{1 + \frac{L}{E}} \leq Dur_A \leq \frac{D_{upper} + \frac{L}{E}Dur_L}{1 + \frac{L}{E}}$$

$$\frac{E \cdot D_{lower} + L \cdot Dur_L}{E + L} \leq Dur_A \leq \frac{E \cdot D_{upper} + L \cdot Dur_L}{E + L}$$

$$\frac{E \cdot D_{lower} + L \cdot Dur_L}{A} \leq Dur_A \leq \frac{E \cdot D_{upper} + L \cdot Dur_L}{A}$$

$$\frac{E \cdot D_{lower} + DDur_L}{A} \leq Dur_A \leq \frac{E \cdot D_{upper} + DDur_L}{A}. \qquad (5.3.2)$$

P53

$$C_{lower} \leq Conv_A + \frac{L}{E}(Conv_A - Conv_L) \leq C_{upper}$$

$$C_{lower} + \frac{L}{E}Conv_L \leq \left(1 + \frac{L}{E}\right) Conv_A \leq C_{upper} + \frac{L}{E}Conv_L$$

$$\frac{C_{lower} + \frac{L}{E}Conv_L}{1 + \frac{L}{E}} \leq Conv_A \leq \frac{C_{upper} + \frac{L}{E}Conv_L}{1 + \frac{L}{E}}$$

$$\frac{E \cdot C_{lower} + L \cdot Conv_L}{E + L} \leq Conv_A \leq \frac{E \cdot C_{upper} + L \cdot Conv_L}{E + L}$$

$$\frac{E \cdot C_{lower} + L \cdot Conv_L}{A} \leq Conv_A \leq \frac{E \cdot C_{upper} + L \cdot Conv_L}{A}$$

$$\frac{E \cdot D_{lower} + DConv_L}{A} \leq Conv_A \leq \frac{E \cdot D_{upper} + DConv_L}{A}. \qquad (5.4.1)$$

Regulatory Capital Adequacy for Life Insurance Companies, SOA Research

Section 4: US Capital Requirements

P. 11

Overall C-2 insurance risk charge:

$$C2 = \sqrt{C2_{mortality}^2 + C2_{longevity}^2 + 2 \times C2_{mortality} \times C2_{longevity} \times CorrFactor}$$

Authorized Control Level (ACL) RBC:

$$ACL\ RBC = C0 + C4a + \sqrt{(C1o + C3a)^2 + (C1cs + C3c)^2 + C2^2 + C3b^2 + C4b^2}$$

where:
• $C0$: Asset Risk-Affiliates
• $C1cs$: Unaffiliated common stock and affiliated noninsurance common stock
• $C1o$: Asset Risk-Other (excluding common stock)
• $C2$: Insurance Risk
• $C3a$: Interest Rate Risk
• $C3b$: Health Credit Risk
• $C3c$: Market Risk
• $C4a$: Business Risk
• $C4b$: Health Administrative Expense Business Risk

Section 5: Canadian Solvency Requirements

Total Ratio:

P. 15

$$\frac{Available\ Capital + Surplus\ Allowance + Eligible\ Deposits}{Base\ Solvency\ Buffer}$$

Core Ratio:

$$\frac{Tier\ 1\ Capital + 70\%\ of\ Surplus\ Allowance + 70\%\ of\ Eligible\ Deposits}{Base\ Solvency\ Buffer}$$

Section 6: Solvency II

P. 18

$$Basic\ SCR = \sqrt{\sum_{i,j} Corr_{i,j} \times SCR_i \times SCR_j}$$

where $Corr_{i,j}$ represents the correlation factor associated with each of the five Solvency Capital Risks (Market Risk, Default Risk, and the three underwriting risks).

P. 32

Risk Margin (RM):

$$RM = CoC \times \sum_{t} \frac{SCR(t)}{(1 + r(t+1))^{t+1}}$$

Where:

- *CoC* denotes the Cost-of-Capital rate;
- *SCR(t)* denotes the Solvency Capital Requirement after t years;
- *r(t + 1)* denotes the basic risk-free interest rate for the maturity of t + 1 years.