SOCIETY OF ACTUARIES/CASUALTY ACTUARIAL SOCIETY

EXAM C CONSTRUCTION AND EVALUATION OF ACTUARIAL MODELS

EXAM C SAMPLE SOLUTIONS

Some of the questions in this study note are taken from past SOA/CAS examinations.

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Question #1
Key: E

The 40th percentile is the .4(12) = 4.8th smallest observation. By interpolation it is .2(86) +.8(90) = 89.2. The 80th percentile is the .8(12) = 9.6th smallest observation. By interpolation it is .4(200) +.6(210) = 206.

The equations to solve are

$$.4 = \frac{(89.2/\theta)^\gamma}{1+(89.2/\theta)^\gamma} \text{ and } .8 = \frac{(206/\theta)^\gamma}{1+(206/\theta)^\gamma}.$$

Solving each for the parenthetical expression gives $\frac{2}{3} = (89.2/\theta)^\gamma$ and $4 = (206/\theta)^\gamma$.

Taking the ratio of the second equation to the first gives $6 = (206/89.2)^\gamma$ which leads to $\gamma = \ln(6)/\ln(206/89.2) = 2.1407$. Then $4^{1/2.1407} = 206/\theta$ for $\theta = 107.8$.

Question #2
Key: E

The standard for full credibility is $\left(\frac{1.645}{.02}\right)^2\left(1+\frac{Var(X)}{E(X)^2}\right)$ where X is the claim size variable. For the Pareto variable, $E(X) = .5/5 = .1$ and $Var(X) = \frac{2(.5)^2}{5(4)} - (.1)^2 = .015$.

Then the standard is $\left(\frac{1.645}{.02}\right)^2\left(1+\frac{.015}{.1^2}\right) = 16{,}913$ claims.

Question #3
Key: B

The kernel is a triangle with a base of 4 and a height at the middle of 0.5 (so the area is 1). The length of the base is twice the bandwidth. Any observation within 2 of 2.5 will contribute to the estimate. For the observation at 2, when the triangle is centered at 2, the height of the triangle at 2.5 is .375 (it is one-quarter the way from 2 to the end of the triangle at 4 and so the height is one-quarter the way from 0.5 to 0). Similarly the points at 3 are also 0.5 away and so the height of the associated triangle is also .375. Each triangle height is weighted by the empirical probability at the associated point. So the estimate at 2.5 is (1/5)(3/8) + (3/5)(3/8) + (1/5)(0) = 12/40.

Question #4
Key: A

The distribution function is $F(x)=\int_1^x \alpha t^{-\alpha-1}dt = -t^{-\alpha}\Big|_1^x = 1-x^{-\alpha}$. The likelihood function is

$$\begin{aligned}L &= f(3)f(6)f(14)[1-F(25)]^2\\ &= \alpha 3^{-\alpha-1}\alpha 6^{-\alpha-1}\alpha 14^{-\alpha-1}(25^{-\alpha})^2\\ &\propto \alpha^3[3(6)(14)(625)]^{-\alpha}.\end{aligned}$$

Taking logs, differentiating, setting equal to zero, and solving:

$$\ln L = 3\ln\alpha - \alpha\ln 157{,}500 \text{ plus a constant}$$

$$(\ln L)' = 3\alpha^{-1} - \ln 157{,}500 = 0$$

$$\hat{\alpha} = 3/\ln 157{,}500 = .2507.$$

Question #5
Key: C

$$\pi(q\,|\,1,1) \propto p(1\,|\,q)p(1\,|\,q)\pi(q) = 2q(1-q)2q(1-q)4q^3 \propto q^5(1-q)^2$$

$$\int_0^1 q^5(1-q)^2\,dq = 1/168, \quad \pi(q\,|\,1,1) = 168q^5(1-q)^2.$$

The expected number of claims in a year is $E(X\,|\,q)=2q$ and so the Bayesian estimate is

$$E(2q\,|\,1,1) = \int_0^1 2q(168)q^5(1-q)^2\,dq = 4/3.$$

The answer can be obtained without integrals by recognizing that the posterior distribution of *q* is beta with *a* = 6 and *b* = 3. The posterior mean is $E(q\,|\,1,1) = a/(a+b) = 6/9 = 2/3$. The posterior mean of 2*q* is then 4/3.

Question #6
Key: D

For the method of moments estimate,

$$386 = e^{\mu+.5\sigma^2}, \quad 457{,}480.2 = e^{2\mu+2\sigma^2}$$

$$5.9558 = \mu + .5\sigma^2, \quad 13.0335 = 2\mu + 2\sigma^2$$

$$\hat{\mu} = 5.3949, \quad \hat{\sigma}^2 = 1.1218.$$

Then

$$E(X \wedge 500) = e^{5.3949+.5(1.1218)}\Phi\left(\frac{\ln 500 - 5.3949 - 1.1218}{\sqrt{1.1218}}\right) + 500\left[1 - \Phi\left(\frac{\ln 500 - 5.3949}{\sqrt{1.1218}}\right)\right]$$
$$= 386\Phi(-.2853) + 500[1 - \Phi(.7739)]$$
$$= 386(.3877) + 500(.2195) = 259.$$

Note-these calculations use exact normal probabilities. Rounding and using the normal table that accompanies the exam will produce a different numerical answer but the same letter answer.

Question #7
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Question #8
Key: C

Let N be the Poisson claim count variable, let X be the claim size variable, and let S be the aggregate loss variable.

$$\mu(\theta) = E(S \mid \theta) = E(N \mid \theta)E(X \mid \theta) = \theta 10\theta = 10\theta^2$$
$$v(\theta) = Var(S \mid \theta) = E(N \mid \theta)E(X^2 \mid \theta) = \theta 200\theta^2 = 200\theta^3$$
$$\mu = E(10\theta^2) = \int_1^\infty 10\theta^2 (5\theta^{-6})d\theta = 50/3$$
$$EPV = E(200\theta^3) = \int_1^\infty 200\theta^3 (5\theta^{-6})d\theta = 500$$
$$VHM = Var(10\theta^2) = \int_1^\infty (10\theta^2)^2 (5\theta^{-6})d\theta - (50/3)^2 = 222.22$$
$$k = 500/222.22 = 2.25.$$

Question #9
Key: A

$c = \exp(.71(1) + .20(1)) = 2.4843$. Then $\hat{S}(t_0; \mathbf{z}) = \hat{S}_0(t_0)^c = .65^{2.4843} = .343$.

Question #10
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Question #11
Key: D

$$\Pr(\theta=1\,|\,X=5)=\frac{f(5\,|\,\theta=1)\Pr(\theta=1)}{f(5\,|\,\theta=1)\Pr(\theta=1)+f(5\,|\,\theta=3)\Pr(\theta=3)}$$

$$=\frac{(1/36)(1/2)}{(1/36)(1/2)+(3/64)(1/2)}=16/43$$

$$\Pr(X_2>8\,|\,X_1=5)=\Pr(X_2>8\,|\,\theta=1)\Pr(\theta=1\,|\,X_1=5)+\Pr(X_2>8\,|\,\theta=3)\Pr(\theta=3\,|\,X_1=5)$$

$$=(1/9)(16/43)+(3/11)(27/43)=.2126.$$

For the last line, $\Pr(X>8\,|\,\theta)=\int_8^\infty \theta(x+\theta)^{-2}dx=\theta(8+\theta)^{-1}$ is used.

Question #12
Key: C

The sample mean for X is 720 and for Y is 670. The mean of all 8 observations is 695.

$$\hat{v}=\frac{\begin{array}{c}(730-720)^2+(800-720)^2+(650-720)^2+(700-720)^2\\+(655-670)^2+(650-670)^2+(625-670)^2+(750-670)^2\end{array}}{2(4-1)}=3475$$

$$\hat{a}=\frac{(720-695)^2+(670-695)^2}{2-1}-\frac{3475}{4}=381.25$$

$$\hat{k}=3475/381.25=9.1148$$

$$\hat{Z}=\frac{4}{4+9.1148}=.305$$

$$P_c=.305(670)+.695(695)=687.4.$$

Question #13
Key: B

There are 430 observations. The expected counts are 430(.2744) = 117.99, 430(.3512) =151.02, 430(.3744) = 160.99. The test statistic is

$$\frac{(112-117.99)^2}{117.99}+\frac{(180-151.02)^2}{151.02}+\frac{(138-160.99)^2}{160.99}=9.15.$$

Question #14
Key: B

From the information, the asymptotic variance of $\hat{\theta}$ is $1/4n$. Then

$Var(2\hat{\theta}) = 4Var(\hat{\theta}) = 4(1/4n) = 1/n.$

Note that the delta method is not needed for this problem, although using it leads to the same answer.

Question #15
Key: A

The posterior probability density is

$$\pi(p\,|\,1,1,1,1,1,1,1,1) \propto \Pr(1,1,1,1,1,1,1,1\,|\,p)\pi(p) \propto p^8(2) \propto p^8.$$

$$\pi(p\,|\,1,1,1,1,1,1,1,1) = \frac{p^8}{\int_0^{.5} p^8 dp} = \frac{p^8}{(.5^9)/9} = 9(.5^{-9})p^8.$$

$$\Pr(X_9 = 1\,|\,1,1,1,1,1,1,1,1) = \int_0^{.5} \Pr(X_9 = 1\,|\,p)\pi(p\,|\,1,1,1,1,1,1,1,1)dp$$

$$= \int_0^{.5} p9(.5^{-9})p^8 dp = 9(.5^{-9})(.5^{10})/10 = .45.$$

Question #16
Key: A

$${}_3\hat{p}_1 = \frac{18}{27}\frac{26}{32}\frac{20}{25} = \frac{13}{30}.$$ Greenwood's approximation is

$$\left(\frac{13}{30}\right)^2 \left(\frac{9}{18(27)} + \frac{6}{26(32)} + \frac{5}{20(25)}\right) = .0067.$$

Question #17
Key: D

$\hat{H}(3)$ = 5/30 + 9/27 + 6/32 = 0.6875

$V\hat{a}r(\hat{H}(3))$ = $5/(30)^2 + 9/(27)^2 + 6/(32)^2$ = 0.02376

The 95% log-transformed confidence interval is:

$\hat{H}(3)U$, where $U = \exp\left(\pm\frac{1.96\sqrt{.02376}}{.6875}\right) = \exp(\pm 0.43945)$

The confidence interval is:

[0.6875 exp(–0.43945), 0.6875 exp(0.43945)] = [0.443, 1.067].

Question #18
Key: D

The means are .5(250) + .3(2,500) + .2(60,000) = 12,875 and .7(250) + .2(2,500) + .1(60,000) = 6,675 for risks 1 and 2 respectively.
The variances are $.5(250)^2 + .3(2{,}500)^2 + .2(60{,}000)^2 - 12{,}875^2 = 556{,}140{,}625$ and $.7(250)^2 + .2(2{,}500)^2 + .1(60{,}000)^2 - 6{,}675^2 = 316{,}738{,}125$ respectively.

The overall mean is (2/3)(12,875) + (1/3)(6,675) = 10,808.33 and so
EPV = (2/3)(556,140,625) + (1/3)(316,738,125) = 476,339,792 and
VHM = $(2/3)(12{,}875)^2 + (1/3)(6{,}675)^2 - 10{,}808.33^2 = 8{,}542{,}222$. Then,
k = 476,339,792/8,542,222 = 55.763 and Z = 1/(1 + 55.763) = .017617.
The credibility estimate is .017617(250) + .982383(10,808.33) = 10,622.

Question #19
Key: D

The first two sample moments are 15 and 500, and the first two population moments are
$E(X) = .5(\theta + \sigma)$ and $E(X^2) = .5(2\theta^2 + 2\sigma^2) = \theta^2 + \sigma^2$. These can be obtained either through integration or by recognizing the density function as a two-point mixture of exponential densities. The equations to solve are $30 = \theta + \sigma$ and $500 = \theta^2 + \sigma^2$. From the first equation, $\sigma = 30 - \theta$ and substituting into the second equation gives $500 = \theta^2 + (30 - \theta)^2 = 2\theta^2 - 60\theta + 900$. The quadratic equation has two solutions, 10 and 20. Because $\theta > \sigma$ the answer is 20.

Question #20
Key: D

There are four possible samples, (5,5), (5,9), (9,5), and (9,9). For each, the estimator g must be calculated. The values are 0, 4, 4, and 0 respectively. Assuming a population in which the values 5 and 9 each occur with probability .5, the population variance is
$.5(5-7)^2 + .5(9-7)^2 = 4$. The mean square error is approximated as
$.25[(0-4)^2 + (4-4)^2 + (4-4)^2 + (0-4)^2] = 8$.

Question #21
Key: B

From the Poisson distribution, $\mu(\lambda)=\lambda$ and $v(\lambda)=\lambda$. Then,
$\mu=E(\lambda)=6/100=.06, \quad EPV=E(\lambda)=.06, \quad VHM=Var(\lambda)=6/100^2=.0006$ where the various moments are evaluated from the gamma distribution. Then,
$k=.06/.0006=100$ and $Z=450/(450+100)=9/11$ where the 450 is the total number of insureds contributing experience. The credibility estimate of the expected number of claims for one insured in month 4 is (9/11)(25/450) + (2/11)(.06) = .056364. For 300 insureds the expected number of claims is 300(.056364) = 16.9.

Question #22
Key: C

The likelihood function is $L(\alpha,\theta)=\prod_{j=1}^{200}\frac{\alpha\theta^{\alpha}}{(x_j+\theta)^{\alpha+1}}$ and its logarithm is

$l(\alpha,\theta)=200\ln(\alpha)+200\alpha\ln(\theta)-(\alpha+1)\sum_{i=1}^{200}\ln(x_i+\theta)$. When evaluated at the hypothesized values of 1.5 and 7.8, the loglikelihood is –821.77. The test statistic is 2(821.77–817.92) = 7.7. With two degrees of freedom (0 free parameters in the null hypothesis versus 2 in the alternative), the test statistic falls between the 97.5th percentile (7.38) and the 99th percentile (9.21).

Question #23
Key: E

Assume that $\theta>5$. Then the expected counts for the three intervals are
$15(2/\theta)=30/\theta, 15(3/\theta)=45/\theta,$ and $15(\theta-5)/\theta=15-75/\theta$ respectively. The quantity to minimize is

$$\frac{1}{5}\left[(30\theta^{-1}-5)^2+(45\theta^{-1}-5)^2+(15-75\theta^{-1}-5)^2\right].$$

Differentiating (and ignoring the coefficient of 1/5) gives the equation
$-2(30\theta^{-1}-5)30\theta^{-2}-2(45\theta^{-1}-5)45\theta^{-2}+2(10-75\theta^{-1})75\theta^{-2}=0$. Multiplying through by θ^3 and dividing by 2 reduces the equation to
$-(30-5\theta)30-(45-5\theta)45+(10\theta-75)75=-8550+1125\theta=0$ for a solution of
$\hat{\theta}=8550/1125=7.6$.

Question #24
Key: E

$\pi(\theta|1) \propto \theta(1.5\theta^{.5}) \propto \theta^{1.5}$. The required constant is the reciprocal of $\int_0^1 \theta^{1.5} d\theta = \theta^{2.5}/2.5\Big|_0^1 = .4$ and so $\pi(\theta|1) = 2.5\theta^{1.5}$. The requested probability is
$\Pr(\theta > .6|1) = \int_{.6}^1 2.5\theta^{1.5} d\theta = \theta^{2.5}\Big|_{.6}^1 = 1 - .6^{2.5} = .721.$

Question #25
Key: A

k	kn_k / n_{k-1}
0	
1	0.81
2	0.92
3	1.75
4	2.29
5	2.50
6	3.00

Positive slope implies that the negative binomial distribution is a good choice. Alternatively, the sample mean and variance are 1.2262 and 1.9131 respectively. With the variance substantially exceeding the mean, the negative binomial model is again supported.

Question #26
Key: B

The likelihood function is $\frac{e^{-1/(2\theta)}}{2\theta} \cdot \frac{e^{-2/(2\theta)}}{2\theta} \cdot \frac{e^{-3/(2\theta)}}{2\theta} \cdot \frac{e^{-15/(3\theta)}}{3\theta} = \frac{e^{-8/\theta}}{24\theta^4}$. The loglikelihood function is $-\ln 24 - 4\ln(\theta) - 8/\theta$. Differentiating with respect to θ and setting the result equal to 0 yields $-\frac{4}{\theta} + \frac{8}{\theta^2} = 0$ which produces $\hat{\theta} = 2$.

Question #27
Key: E

The absolute difference of the credibility estimate from its expected value is to be less than or equal to $k\mu$ (with probability *P*). That is,

$$\left|[ZX_{partial} + (1-Z)M] - [Z\mu + (1-Z)M]\right| \le k\mu$$

$$-k\mu \le ZX_{partial} - Z\mu \le k\mu.$$

Adding μ to all three sides produces answer choice (E).

Question #28
DELETED

Question #29
Key: B

The probabilities are from a binomial distribution with 6 trials. Three successes were observed.

$$\Pr(3\,|\,\text{I}) = \binom{6}{3}(.1)^3(.9)^3 = .01458, \Pr(3\,|\,\text{II}) = \binom{6}{3}(.2)^3(.8)^3 = .08192,$$

$$\Pr(3\,|\,\text{III}) = \binom{6}{3}(.4)^3(.6)^3 = .27648$$

The probability of observing three successes is .7(.01458) + .2(.08192) + .1(.27648) = .054238. The three posterior probabilities are:

$$\Pr(\text{I}\,|\,3) = \frac{.7(.01458)}{.054238} = .18817,\ \Pr(\text{II}\,|\,3) = \frac{.2(.08192)}{.054238} = .30208,\ \Pr(\text{III}\,|\,3) = \frac{.1(.27648)}{.054238} = .50975.$$

The posterior probability of a claim is then
.1(.18817) + .2(.30208) + .4(.50975) = .28313.

Question #30
Key: E

$.542 = \hat{F}(n) = 1 - e^{-\hat{H}(n)}, \quad \hat{H}(n) = .78.$ The Nelson-Aalen estimate is the sum of successive *s*/*r* values. From the problem statement, *r* = 100 at all surrender times while the *s*-values follow the pattern 1, 2, 3, …. Then,

$$.78 = \frac{1}{100} + \frac{2}{100} + \cdots + \frac{n}{100} = \frac{n(n+1)}{200}$$ and the solution is *n* = 12.

Question # 31
Answer: C

$$g = [12(.45)] = [5.4] = 5; \quad h = 5.4 - 5 = 0.4.$$

$$\hat{\pi}_{.45} = .6x_{(5)} + .4x_{(6)} = .6(360) + .4(420) = 384.$$

Question # 32
Answer: D

N is distributed *Poisson*(λ)

$$\mu = E(\lambda) = \alpha\theta = 1(1.2) = 1.2.$$

$$v = E(\lambda) = 1.2; \quad a = Var(\lambda) = \alpha\theta^2 = 1(1.2)^2 = 1.44.$$

$$k = \frac{1.2}{1.44} = \frac{5}{6}; \quad Z = \frac{2}{2 + 5/6} = \frac{12}{17}.$$

Thus, the estimate for Year 3 is

$$\frac{12}{17}(1.5) + \frac{5}{17}(1.2) = 1.41.$$

Note that a Bayesian approach produces the same answer.

Question # 33
Answer: C

At the time of the second failure,

$$\hat{H}(t) = \frac{1}{n} + \frac{1}{n-1} = \frac{23}{132} \Rightarrow n = 12.$$

At the time of the fourth failure,

$$\hat{H}(t) = \frac{1}{12} + \frac{1}{11} + \frac{1}{10} + \frac{1}{9} = .3854.$$

Question # 34
Answer: B

The likelihood is:

$$L = \prod_{j=1}^{n} \frac{r(r+1)\cdots(r+x_j-1)\beta^{x_j}}{x_j!(1+\beta)^{r+x_j}} \propto \prod_{j=1}^{n} \beta^{x_j}(1+\beta)^{-r-x_j}.$$

The loglikelihood is:

$$l=\sum_{j=1}^{n}\left[x_j\ln\beta-(r+x_j)\ln(1+\beta)\right]$$

$$l'=\sum_{j=1}^{n}\left[\frac{x_j}{\beta}-\frac{r+x_j}{1+\beta}\right]=0$$

$$0=\sum_{j=1}^{n}\left[x_j(1+\beta)-(r+x_j)\beta\right]=\sum_{j=1}^{n}x_j-rn\beta$$

$$0=n\bar{x}-rn\beta;\quad \hat{\beta}=\bar{x}/r.$$

Question # 35
Answer: C

The Bühlmann credibility estimate is $Zx+(1-Z)\mu$ where x is the first observation. The Bühlmann estimate is the least squares approximation to the Bayesian estimate. Therefore, Z and μ must be selected to minimize

$$\frac{1}{3}[Z+(1-Z)\mu-1.5]^2+\frac{1}{3}[2Z+(1-Z)\mu-1.5]^2+\frac{1}{3}[3Z+(1-Z)\mu-3]^2.$$

Setting partial derivatives equal to zero will give the values. However, it should be clear that μ is the average of the Bayesian estimates, that is,

$$\mu=\frac{1}{3}(1.5+1.5+3)=2.$$

The derivative with respect to Z is (deleting the coefficients of 1/3):

$$2(-Z+.5)(-1)+2(.5)(0)+2(Z-1)(1)=0$$
$$Z=.75.$$

The answer is

$$.75(1)+.25(2)=1.25.$$

Question # 36
Answer: E

The confidence interval is $(\hat{S}(t_0)^{1/\theta},\hat{S}(t_0)^{\theta}).$

Taking logarithms of both endpoints gives the two equations

$$\ln .695 = -.36384 = \frac{1}{\theta}\ln \hat{S}(t_0)$$
$$\ln .843 = -.17079 = \theta \ln \hat{S}(t_0).$$

Multiplying the two equations gives

$$.06214 = [\ln \hat{S}(t_0)]^2$$
$$\ln \hat{S}(t_0) = -.24928$$
$$\hat{S}(t_0) = .77936.$$

The negative square root is required in order to make the answer fall in the interval (0,1).

Question # 37
Answer: B

The likelihood is:

$$L = \frac{\alpha 150^{\alpha}}{(150+225)^{\alpha+1}} \frac{\alpha 150^{\alpha}}{(150+525)^{\alpha+1}} \frac{\alpha 150^{\alpha}}{(150+950)^{\alpha+1}}$$
$$= \frac{\alpha^3 150^{3\alpha}}{(375 \bullet 675 \bullet 1100)^{\alpha+1}}.$$

The loglikelihood is:

$$l = 3\ln\alpha + 3\alpha \ln 150 - (\alpha+1)\ln(375 \bullet 675 \bullet 1100)$$
$$l' = \frac{3}{\alpha} + 3\ln 150 - \ln(375 \bullet 675 \bullet 1100) = \frac{3}{\alpha} - 4.4128$$
$$\hat{\alpha} = 3/4.4128 = .6798.$$

Question # 38
Answer: D

For this problem, r = 4 and n = 7. Then,

$\hat{v} = \frac{33.60}{4(7-1)} = 1.4$ and $\hat{a} = \frac{3.3}{4-1} - \frac{1.4}{7} = .9.$

Then,

$$k = \frac{1.4}{.9} = \frac{14}{9}; \quad Z = \frac{7}{7 + (14/9)} = \frac{63}{77} = .82.$$

Question # 39
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Question # 40
Answer: E

X	$F_n(x)$	$F_n(x^-)$	$F_0(x)$	$\lvert F_n(x) - F_0(x)\rvert$	$\lvert F_n(x^-) - F_0(x)\rvert$
29	0.2	0	0.252	0.052	0.252
64	0.4	0.2	0.473	0.073	0.273
90	0.6	0.4	0.593	0.007	0.193
135	0.8	0.6	0.741	0.059	0.141
182	1.00	0.8	0.838	0.162	0.038

where:
$\hat{\theta} = \bar{x} = 100$ and $F_0(x) = 1 - e^{-x/100}$.
The maximum value from the last two columns is 0.273.

Question # 41
Answer: E

$$\mu = E(\lambda) = 1; \quad v = E(\sigma^2) = 1.25; \quad a = Var(\lambda) = 1/12.$$
$$k = v/a = 15; \quad Z = \frac{1}{1+15} = \frac{1}{16}.$$

Thus, the estimate for Year 2 is
$$\frac{1}{16}(0) + \frac{15}{16}(1) = .9375.$$

Question # 42
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Question # 43
Answer: E

The posterior density, given an observation of 3 is:

$$\pi(\theta \mid 3) = \frac{f(3 \mid \theta)\pi(\theta)}{\int_1^\infty f(3 \mid \theta)\pi(\theta)d\theta} = \frac{\dfrac{2\theta^2}{(3+\theta)^3}\dfrac{1}{\theta^2}}{\int_1^\infty 2(3+\theta)^{-3}d\theta}$$

$$= \frac{2(3+\theta)^{-3}}{-(3+\theta)^{-2}\Big|_1^\infty} = 32(3+\theta)^{-3}, \quad \theta > 1.$$

Then,

$$\Pr(\Theta > 2) = \int_2^\infty 32(3+\theta)^{-3}d\theta = -16(3+\theta)^{-2}\Big|_2^\infty = \frac{16}{25} = .64.$$

Question # 44
Answer: B

$$L = F(1000)^7[F(2000) - F(1000)]^6[1 - F(2000)]^7$$
$$= (1 - e^{-1000/\theta})^7(e^{-1000/\theta} - e^{-2000/\theta})^6(e^{-2000/\theta})^7$$
$$= (1-p)^7(p - p^2)^6(p^2)^7$$
$$= p^{20}(1-p)^{13}$$

where $p = e^{-1000/\theta}$. The maximum occurs at p = 20/33 and so $\hat{\theta} = -1000/\ln(20/33) = 1996.90.$

Question # 45
Answer: A

$$E(X \mid \theta) = \theta / 2.$$

$$E(X_3 \mid 400, 600) = \int_{600}^\infty E(X \mid \theta)f(\theta \mid 400, 600)d\theta = \int_{600}^\infty \frac{\theta}{2}3\frac{600^3}{\theta^4}d\theta = \frac{3(600^3)}{2}\frac{\theta^{-2}}{-2}\Big|_{600}^\infty$$

$$= \frac{3(600^3)(600^{-2})}{4} = 450.$$

Question # 46
Answer: D

The data may be organized as follows:

t	*Y*	*d*	$\hat{S}(t)$
2	10	1	(9/10) = .9
3	9	2	.9(7/9) = .7
5	7	1	.7(6/7) = .6
6	5	1	.6(4/5) = .48
7	4	1	.48(3/4) = .36
9	2	1	.36(1/2) = .18

Because the product-limit estimate is constant between observations, the value of $\hat{S}(8)$ is found from $\hat{S}(7) = .36$.

Question # 47
Answer: C

The maximum likelihood estimate for the Poisson distribution is the sample mean:

$$\hat{\lambda} = \bar{x} = \frac{50(0) + 122(1) + 101(2) + 92(3)}{365} = 1.6438.$$

The table for the chi-square test is:

Number of days	Probability	Expected*	Chi-square
0	$e^{-1.6438} = .19324$	70.53	5.98
1	$1.6438e^{-1.6438} = .31765$	115.94	0.32
2	$\frac{1.6438^2 e^{-1.6438}}{2} = .26108$	95.30	0.34
3+	.22803**	83.23	0.92

*365x(Probability) **obtained by subtracting the other probabilities from 1

The sum of the last column is the test statistic of 7.56. Using 2 degrees of freedom (4 rows less 1 estimated parameter less 1) the model is rejected at the 2.5% significance level but not at the 1% significance level.

Question # 48
Answer: D

$$\mu(0) = \frac{.4(0) + .1(1) + .1(2)}{.6} = .5; \quad \mu(1) = \frac{.1(0) + .2(1) + .1(2)}{.4} = 1$$

$$\mu = .6(.5) + .4(1) = .7$$

$$a = .6(.5^2) + .4(1^2) - .7^2 = .06$$

$$v(0) = \frac{.4(0) + .1(1) + .1(4)}{.6} - .5^2 = \frac{7}{12}; \quad v(1) = \frac{.1(0) + .2(1) + .1(4)}{.4} - 1^2 = .5$$

$$v = .6(7/12) + .4(.5) = 11/20$$

$$k = v/a = 55/6; \quad Z = \frac{10}{10 + 55/6} = \frac{60}{115}$$

Bühlmann credibility premium = $\frac{60}{115}\frac{10}{10} + \frac{55}{115}(.7) = .8565$.

Question # 49
Answer: C

$$\mu = .5(0) + .3(1) + .1(2) + .1(3) = .8$$

$$\sigma^2 = .5(0) + .3(1) + .1(4) + .1(9) - .64 = .96$$

$$E(S_n^2) = \frac{n-1}{n}\sigma^2 = \frac{3}{4}(.96) = .72$$

$$bias = .72 - .96 = -.24.$$

Question # 50
Answer: C

The four classes have means .1, .2, .5, and .9 respectively and variances .09, .16, .25, and .09 respectively.

Then,

$\mu = .25(.1 + .2 + .5 + .9) = .425$

$v = .25(.09 + .16 + .25 + .09) = .1475$

$a = .25(.01 + .04 + .25 + .81) - .425^2 = .096875$

$k = .1475 / .096875 = 1.52258$

$$Z = \frac{4}{4 + 1.52258} = .7243$$

The estimate is $[.7243(2/4) + .2757(.425)] \cdot 5 = 2.40$.

Question # 51
DELETED

Question # 52
Answer: A

The distribution used for simulation is given by the observed values.

Question # 53
Answer: B

First obtain the distribution of aggregate losses:

Value	Probability
0	1/5
25	(3/5)(1/3) = 1/5
100	(1/5)(2/3)(2/3) = 4/45
150	(3/5)(2/3) = 2/5
250	(1/5)(2)(2/3)(1/3) = 4/45
400	(1/5)(1/3)(1/3) = 1/45

$$\mu = (1/5)(0) + (1/5)(25) + (4/45)(100) + (2/5)(150) + (4/45)(250) + (1/45)(400) = 105$$

$$\sigma^2 = (1/5)(0^2) + (1/5)(25^2) + (4/45)(100^2) + (2/5)(150^2) + (4/45)(250^2) + (1/45)(400^2) - 105^2 = 8,100.$$

Question # 54
Answer: A

Loss Range	Cum. Prob.
0 – 100	0.320
100 – 200	0.530
200 – 400	0.800
400 – 750	0.960
750 – 1000	0.980
1000 – 1500	1.000

At 400, *F*(*x*) = 0.8 = $1-e^{-\frac{400}{\theta}}$; solving gives $\theta = 248.53$.

Question # 55
Answer: B

$$\Pr(class1\,|\,1) = \frac{(1/2)(1/3)}{(1/2)(1/3)+(1/3)(1/6)+(1/6)(0)} = \frac{3}{4}$$
$$\Pr(class2\,|\,1) = \frac{(1/3)(1/6)}{(1/2)(1/3)+(1/3)(1/6)+(1/6)(0)} = \frac{1}{4}$$
$$\Pr(class3\,|\,1) = \frac{(1/6)(0)}{(1/2)(1/3)+(1/3)(1/6)+(1/6)(0)} = 0$$

because the prior probabilities for the three classes are 1/2, 1/3, and 1/6 respectively.

The class means are

$$\mu(1) = (1/3)(0)+(1/3)(1)+(1/3)(2) = 1$$
$$\mu(2) = (1/6)(1)+(2/3)(2)+(1/6)(3) = 2.$$

The expectation is

$$E(X_2\,|\,1) = (3/4)(1)+(1/4)(2) = 1.25.$$

Question # 56
Answer: **E**

The first, second, third, and sixth payments were observed at their actual value and each contributes *f*(*x*) to the likelihood function. The fourth and fifth payments were paid at the policy limit and each contributes 1 – *F*(*x*) to the likelihood function. This is answer (E).

Question #57
Answer is E

For an interval running from *c* to *d*, the uniform density function is $f(x) = g/[n(d-c)]$ where *g* is the number of observations in the interval and *n* is the sample size. The contribution to the second raw moment for this interval is:

$$\int_c^d x^2 \frac{g}{n(d-c)}dx = \left.\frac{gx^3}{3n(d-c)}\right|_c^d = \frac{g(d^3-c^3)}{3n(d-c)}.$$

For this problem, the second raw moment is:

$$\frac{1}{90}\left[\frac{30(25^3-0^3)}{3(25-0)} + \frac{32(50^3-25^3)}{3(50-25)} + \frac{20(100^3-50^3)}{3(100-50)} + \frac{8(200^3-100^3)}{3(200-100)}\right] = 3958.33.$$

Question #58
Answer is B

Because the Bayes and Bühlmann results must be identical, this problem can be solved either way. For the Bühlmann approach, $\mu(\lambda) = v(\lambda) = \lambda$. Then, noting that the prior distribution is a gamma distribution with parameters 50 and 1/500, we have:

$\mu = E(\lambda) = 50/500 = 0.1$

$v = E(\lambda) = 0.1$

$a = Var(\lambda) = 50/500^2 = 0.0002$

$k = v/a = 500$

$Z = 1500/(1500+500) = 0.75$

$\bar{X} = \dfrac{75+210}{600+900} = 0.19.$

The credibility estimate is $0.75(0.19) + 0.25(0.1) = 0.1675$. For 1100 policies, the expected number of claims is 1100(0.1675) = 184.25.

For the Bayes approach, the posterior density is proportional to (because in a given year the number of claims has a Poisson distribution with parameter λ times the number of policies)

$\dfrac{e^{-600\lambda}(600\lambda)^{75}}{75!}\dfrac{e^{-900\lambda}(900\lambda)^{210}}{210!}\dfrac{(500\lambda)^{50}e^{-500\lambda}}{\lambda\Gamma(50)} \propto \lambda^{335}e^{-2000\lambda}$ which is a gamma density with parameters 335 and 1/2000. The expected number of claims per policy is 335/2000 = 0.1675 and the expected number of claims in the next year is 184.25.

Question #59
Answer is E

The *q-q* plot takes the ordered values and plots the *j*th point at *j*/(*n*+1) on the horizontal axis and at $F(x_j;\theta)$ on the vertical axis. For small values, the model assigns more probability to being below that value than occurred in the sample. This indicates that the model has a heavier left tail than the data. For large values, the model again assigns more probability to being below that value (and so less probability to being above that value). This indicates that the model has a lighter right tail than the data. Of the five answer choices, only E is consistent with these observations. In addition, note that as you go from 0.4 to 0.6 on the horizontal axis (thus looking at the middle 20% of the data), the *q-q* plot increases from about 0.3 to 0.4 indicating that the model puts only about 10% of the probability in this range, thus confirming answer E.

Question #60
Answer is C

The posterior probability of having one of the coins with a 50% probability of heads is proportional to (.5)(.5)(.5)(.5)(4/6) = 0.04167. This is obtained by multiplying the probabilities of making the successive observations 1, 1, 0, and 1 with the 50% coin times the prior probability of 4/6 of selecting this coin. The posterior probability for the 25% coin is proportional to (.25)(.25)(.75)(.25)(1/6) = 0.00195 and the posterior probability for the 75% coin is proportional to (.75)(.75)(.25)(.75)(1/6) = 0.01758. These three numbers total 0.06120. Dividing by this sum gives the actual posterior probabilities of 0.68088, 0.03186, and 0.28726. The expected value for the fifth toss is then (.68088)(.5) + (.03186)(.25) + (.28726)(.75) = 0.56385.

Question #61
Answer is A

Because the exponential distribution is memoryless, the excess over the deductible is also exponential with the same parameter. So subtracting 100 from each observation yields data from an exponential distribution and noting that the maximum likelihood estimate is the sample mean gives the answer of 73.

Working from first principles,

$$L(\theta) = \frac{f(x_1)f(x_2)f(x_3)f(x_4)f(x_5)}{[1-F(100)]^5} = \frac{\theta^{-1}e^{-125/\theta}\theta^{-1}e^{-150/\theta}\theta^{-1}e^{-165/\theta}\theta^{-1}e^{-175/\theta}\theta^{-1}e^{-250/\theta}}{(e^{-100/\theta})^5}$$
$$= \theta^{-5}e^{-365/\theta}.$$

Taking logarithms and then a derivative gives

$$l(\theta) = -5\ln(\theta) - 365/\theta, l'(\theta) = -5/\theta + 365/\theta^2 = 0.$$

The solution is $\hat{\theta} = 365/5 = 73.$

Question #62
Answer is D

The number of claims for each insured has a binomial distribution with *n* = 1 and *q* unknown. We have

$$\mu(q) = q, v(q) = q(1-q)$$
$$\mu = E(q) = 0.1, \text{ given in item (iv)}$$
$$a = Var(q) = E(q^2) - E(q)^2 = E(q^2) - 0.01 = 0.01, \text{ given in item (v)}$$
$$\text{Therefore, } E(q^2) = 0.02$$
$$v = E(q - q^2) = 0.1 - 0.02 = 0.08$$
$$k = v/a = 8, Z = \frac{10}{10+8} = 5/9.$$

Then the expected number of claims in the next one year is (5/9)(0) + (4/9)(0.1) = 2/45 and the expected number of claims in the next five years is 5(2/45) = 2/9 = 0.22.

Question #63
DELETED

Question #64
Answer is E

The model distribution is $f(x|\theta) = 1/\theta, 0 < x < \theta.$ Then the posterior distribution is proportional to

$$\pi(\theta|400,600) \propto \frac{1}{\theta}\frac{1}{\theta}\frac{500}{\theta^2} \propto \theta^{-4}, \theta > 600.$$

It is important to note the range. Being a product, the posterior density function is non-zero only when all three terms are non-zero. Because one of the observations was equal to 600, the value of the parameter must be greater than 600 in order for the density function at 600 to be positive. Or, by general reasoning, posterior probability can only be assigned to possible values. Having observed the value 600 we know that parameter values less than or equal to 600 are not possible.

The constant is obtained from $\int_{600}^{\infty} \theta^{-4} d\theta = \frac{1}{3(600)^3}$ and thus the exact posterior density is

$\pi(\theta \mid 400, 600) = 3(600)^3 \theta^{-4}, \theta > 600.$ The posterior probability of an observation exceeding 550 is

$$\Pr(X_3 > 550 \mid 400, 600) = \int_{600}^{\infty} \Pr(X_3 > 550 \mid \theta)\pi(\theta \mid 400, 600) d\theta$$

$$= \int_{600}^{\infty} \frac{\theta - 550}{\theta} 3(600)^3 \theta^{-4} d\theta = 0.3125$$

where the first term in the integrand is the probability of exceeding 550 from the uniform distribution.

Question #65
Answer is C

$$E(N) = r\beta = 0.40$$
$$Var(N) = r\beta(1+\beta) = 0.48$$
$$E(Y) = \theta/(\alpha - 1) = 500$$
$$Var(Y) = \theta^2 \alpha / \left[(\alpha - 1)^2 (\alpha - 2) \right] = 750,000$$

Therefore,

$$E(X) = 0.40(500) = 200$$
$$Var(X) = 0.40(750,000) + 0.48(500)^2 = 420,000$$

The full credibility standard is $n = \left(\frac{1.645}{0.05} \right)^2 \frac{420,000}{200^2} = 11,365$ and then

$Z = \sqrt{2500 / 11,365} = 0.47.$

Question #66
Answer is E

The sample variance is $s^2 = \frac{(1-3)^2 + (2-3)^2 + (3-3)^2 + (4-3)^2 + (5-3)^2}{4} = 2.5.$ The estimator of *E*[*X*] is the sample mean and the variance of the sample mean is the variance divided by the sample size, estimated here as 2.5/*n*. Setting the standard deviation of the estimator equal to 0.05 gives the equation $\sqrt{2.5/n} = 0.05$ which yields *n* = 1000.

Question #67
Answer is E

$$\mu(r) = E(X \mid r) = E(N)E(Y) = r\beta\theta/(\alpha-1) = 100r$$
$$v(r) = Var(X \mid r) = Var(N)E(Y)^2 + E(N)Var(Y)$$
$$= r\beta(1+\beta)\theta^2/(\alpha-1)^2 + r\beta\alpha\theta^2/[(\alpha-1)^2(\alpha-2)] = 210,000r.$$

$$v = E(210,000r) = 210,000(2) = 420,000$$
$$a = Var(100r) = (100)^2(4) = 40,000$$
$$k = v/a = 10.5$$
$$Z = 100/(100+10.5) = 0.905.$$

Question #68
Answer is B

Using all participants, $S^{\mathrm{T}}(4) = \left(1-\frac{35}{300}\right)\left(1-\frac{74}{265}\right)\left(1-\frac{34}{191}\right)\left(1-\frac{32}{157}\right) = 0.41667.$

Using only Country B, $S^{\mathrm{B}}(4) = \left(1-\frac{15}{100}\right)\left(1-\frac{20}{85}\right)\left(1-\frac{20}{65}\right)\left(1-\frac{10}{45}\right) = 0.35.$

The difference is, $S^{\mathrm{T}}(4) - S^{\mathrm{B}}(4) = 0.41667 - 0.35 = 0.0667 = 0.07.$

Question #69
Answer is B

For an exponential distribution the maximum likelihood estimate of the mean is the sample mean. We have
$$E(\bar{X}) = E(X) = \theta, Var(\bar{X}) = Var(X)/n = \theta^2/n.$$
$$cv = SD(\bar{X})/E(\bar{X}) = [\theta/\sqrt{n}]/\theta = 1/\sqrt{n} = 1/\sqrt{5} = 0.447.$$

If the above facts are not known, the loglikelihood function can be used:
$$L(\theta) = \theta^{-n}e^{-\Sigma x_j/\theta}, \quad l(\theta) = -n\ln\theta - n\bar{X}/\theta, \quad l'(\theta) = -n\theta^{-1} + n\bar{X}\theta^{-2} = 0 \Rightarrow \hat{\theta} = \bar{X}.$$
$$l''(\theta) = n\theta^{-2} - 2n\bar{X}\theta^{-3}, \quad I(\theta) = E[-n\theta^{-2} + 2n\bar{X}\theta^{-3}] = n\theta^{-2}.$$
Then, $Var(\hat{\theta}) = \theta^2/n.$

Question #70
Answer is D

Because the total expected claims for business use is 1.8, it must be that 20% of business users are rural and 80% are urban. Thus the unconditional probabilities of being business-rural and business-urban are 0.1 and 0.4 respectively. Similarly the probabilities of being pleasure-rural and pleasure-urban are also 0.1 and 0.4 respectively. Then,

$$\mu = 0.1(1.0) + 0.4(2.0) + 0.1(1.5) + 0.4(2.5) = 2.05$$

$$v = 0.1(0.5) + 0.4(1.0) + 0.1(0.8) + 0.4(1.0) = 0.93$$

$$a = 0.1(1.0^2) + 0.4(2.0^2) + 0.1(1.5^2) + 0.4(2.5^2) - 2.05^2 = 0.2225$$

$$k = v/a = 4.18$$

$$Z = 1/(1+4.18) = 0.193.$$

Question #71
Answer is A

No. claims	Hypothesized	Observed	Chi-square
1	250	235	$15^2/250 = 0.90$
2	350	335	$15^2/350 = 0.64$
3	240	250	$10^2/240 = 0.42$
4	110	111	$1^2/110 = 0.01$
5	40	47	$7^2/40 = 1.23$
6+	10	22	$12^2/10 = 14.40$

The last column sums to the test statistic of 17.60 with 5 degrees of freedom (there were no estimated parameters), so from the table reject at the 0.005 significance level.

Question #72
Answer is C

In part (ii) you are given that $\mu = 20$. In part (iii) you are given that $a = 40$. In part (iv) you are given that $v = 8,000$. Therefore, $k = v/a = 200$. Then,

$$\bar{X} = \frac{800(15) + 600(10) + 400(5)}{1800} = \frac{100}{9}$$

$$Z = \frac{1800}{1800 + 200} = 0.9$$

$$P_c = 0.9(100/9) + 0.1(20) = 12.$$

Question #73
Answer is C

$$\Pr(X > 30,000) = S(30,000) = \left(1 - \frac{1}{10 - 2/2}\right)\left(1 - \frac{1}{7 - 2/2}\right) = 20/27 = 0.741.$$

Question #74
DELETED

Question #75
Answer is D

$$E(X) = \int_{\delta}^{\infty} \frac{x}{\theta} e^{-(x-\delta)/\theta} dx = \int_0^{\infty} \frac{y+\delta}{\theta} e^{-y/\theta} dx = \theta + \delta$$

$$E(X^2) = \int_{\delta}^{\infty} \frac{x^2}{\theta} e^{-(x-\delta)/\theta} dx = \int_0^{\infty} \frac{y^2 + 2y\delta + \delta^2}{\theta} e^{-y/\theta} dx = 2\theta^2 + 2\theta\delta + \delta^2.$$

Both derivations use the substitution $y = x - \delta$ and then recognize that the various integrals are requesting moments from an ordinary exponential distribution. The method of moments solves the two equations

$\theta + \delta = 10$

$2\theta^2 + 2\theta\delta + \delta^2 = 130.6$

producing $\hat{\delta} = 4.468.$

It is faster to do the problem if it is noted that $X = Y + \delta$ where *Y* has an ordinary exponential distribution. Then $E(X) = E(Y) + \delta = \theta + \delta$ and $Var(X) = Var(Y) = \theta^2.$

Question #76
Answer is D

The posterior density is proportional to the product of the probability of the observed value and the prior density. Thus, $\pi(\theta \mid N > 0) \propto \Pr(N > 0 \mid \theta)\pi(\theta) = (1 - e^{-\theta})\theta e^{-\theta}.$

The constant of proportionality is obtained from $\int_0^{\infty} \theta e^{-\theta} - \theta e^{-2\theta} d\theta = \frac{1}{1^2} - \frac{1}{2^2} = 0.75.$

The posterior density is $\pi(\theta \mid N > 0) = (4/3)(\theta e^{-\theta} - \theta e^{-2\theta}).$

Then,

$$\Pr(N_2 > 0 \mid N_1 > 0) = \int_0^\infty \Pr(N_2 > 0 \mid \theta)\pi(\theta \mid N_1 > 0)d\theta = \int_0^\infty (1-e^{-\theta})(4/3)(\theta e^{-\theta} - \theta e^{-2\theta})d\theta$$

$$= \frac{4}{3}\int_0^\infty \theta e^{-\theta} - 2\theta e^{-2\theta} + \theta e^{-3\theta} d\theta = \frac{4}{3}\left(\frac{1}{1^2} - \frac{2}{2^2} + \frac{1}{3^2}\right) = 0.8148.$$

Question #77
Answer is E

The interval is centered at 2.09 and the plus/minus term is 0.46 which must equal $1.96\hat{\sigma}$ and so $\hat{\sigma} = 0.2347$. For the log-transformed interval we need $\phi = e^{1.96(0.2347)/2.09} = 1.2462$. The lower limit is 2.09/1.2462 = 1.68 and the upper limit is 2.09(1.2462) = 2.60.

Question #78
Answer is B

From item (ii), $\mu = 1000$ and *a* = 50. From item (i), *v* = 500. Therefore, *k* = *v*/*a* = 10 and
Z = 3/(3+10) = 3/13. Also, $\bar{X} = (750 + 1075 + 2000)/3 = 1275$. Then
$P_c = (3/13)(1275) + (10/13)(1000) = 1063.46.$

Question #79
Answer is C

$$f(x) = p\frac{1}{100}e^{-x/100} + (1-p)\frac{1}{10{,}000}e^{-x/10{,}000}$$

$$L(100, 200) = f(100)f(2000)$$

$$= \left(\frac{pe^{-1}}{100} + \frac{(1-p)e^{-0.01}}{10{,}000}\right)\left(\frac{pe^{-20}}{100} + \frac{(1-p)e^{-0.2}}{10{,}000}\right)$$

Question #80
Key: C

Model Solution:

For a binomial random variable with $n = 100$ and $p = q_{70} = 0.03318$, simulate number of deaths:

$i = 0: (1-p)^{100} = 0.03424 = f(0) = F(0)$

Since $0.18 > F(0)$, continue

$$i = 1: f(1) = f(0)(n)(p)/(1-p)$$
$$= (0.03424)(100)(0.03318)/(0.96682)$$
$$= 0.11751$$

$F(1) = F(0) + f(1) = 0.03424 + 0.11751 = 0.15175$

Since $0.18 > F(1)$, continue

$$i = 2: f(2) = f(1)[(n-1)/2](p)/(1-p)$$
$$= (0.11751)(99/2)(0.03318/0.96682)$$
$$= 0.19962$$

$F(2) = F(1) + f(2) = 0.15175 + 0.19962 = 0.35137$

Since $0.18 < F(2)$, number of claims $= 2$, so claim amount $= 20$.

Question # 81
Answer: C

Which distribution is it from?

0.25 < 0.30, so it is from the exponential.

Given that Y is from the exponential, we want

$$\Pr(Y \le y) = F(y) = 0.69$$
$$1 - e^{-y/\theta} = 0.69$$
$$1 - e^{-y/0.5} = 0.69 \text{ since mean = 0.5}$$
$$\frac{-y}{0.5} = \ln(1-0.69) = -1.171$$
$$y = 0.5855$$

Question #82
Key: B

If you happen to remember this distribution from the Simulation text (example 4d in third edition), you could use:

$$n = Int\left(\frac{\log(1-u)}{\log q}\right)+1 = Int\frac{\log 0.95}{\log 0.1}+1 = 0+1 = 1$$

For mere mortals, you get the simulated value of *N* from the definition of the inverse transformation method:

$f(1) = F(1) = 0.9$

$0.05 \leq 0.9$ so $n = 1$

$$x_1 = \frac{1}{\lambda}\log^{(1-v_1)} = -\frac{1}{0.01}\log 0.7 = 35.67$$

The amount of total claims during the year $= 35.67$

Question #83
Key: B

$F(0) = 0.8$
$F(t) = 0.8 + 0.00025(t\text{-}1000)$, $\quad 1000 \leq t \leq 5000$

$0.75 \Rightarrow$ 0 found since $F(0) \geq 0.75$

$0.85 \Rightarrow$ 2000 found since $F(2000) = 0.85$

Average of those two outcomes is 1000.