

SOCIETY OF ACTUARIES

EXAM FM FINANCIAL MATHEMATICS

EXAM FM SAMPLE SOLUTIONS

Financial Economics

June 2014 changes

Questions 1-30 are from the prior version of this document. They have been edited to conform more closely to current question writing style, but are unchanged in content.

Question 31 is the former Question 58 from the interest theory question set.

Questions 32- 34 are new.

Some of the questions in this study note are taken from past SOA/CAS examinations.

These questions are representative of the types of questions that might be asked of candidates sitting for the Financial Mathematics (FM) Exam. These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on future exams.

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1. Solution: D

If the call is at-the-money, the put option with the same cost will have a higher strike price. A purchased collar requires that the put have a lower strike price.

2. Solution: C

$$66.59 - 18.64 = 500 - K \exp(-0.06) \text{ and so } K = (500 - 66.59 + 18.64) / \exp(-0.06) = 480.$$

3. Solution: D

The accumulated cost of the hedge is $(84.30 - 74.80) \exp(0.06) = 10.09$.

Let x be the market price in one year.

If $x < 0.12$ the put is in the money and the payoff is $10,000(0.12 - x) = 1,200 - 10,000x$. The sale of the jalapenos has a payoff of $10,000x - 1,000$ for a profit of $1,200 - 10,000x + 10,000x - 1,000 - 10.09 = 190$.

From 0.12 to 0.14 neither option has a payoff and the profit is $10,000x - 1,000 - 10.09 = 10,000x - 1,010$. The range is 190 to 390.

If $x > 0.14$ the call is in the money and the payoff is $-10,000(x - 0.14) = 1,400 - 10,000x$. The profit is $1,400 - 10,000x + 10,000x - 1,000 - 10.09 = 390$.

The range is 190 to 390.

4. Solution: B

The present value of the forward prices is

$1000[10(3.89)/1.06 + 15(4.11)/1.065^2 + 20(4.16)/1.07^3] = 158,968$. Any sequence of payments with that present value is acceptable. All but B have that value.

5. Solution: E

Consider buying the put and selling the call. Let x be the index price in one year. If $x > 1025$, the payoff is $1025 - x$. After buying the index for x you have $1,025 - 2x$ which is not the goal. It is not necessary to check buying the call and selling the put as that is the only other option. But as a check, if $x > 1025$, the payoff is $x - 1025$ and after buying the stock you have spent 1025. If $x < 1025$, the payoff is again $x - 1025$.

One way to get the cost is to note that the forward price is $1,000(1.05) = 1,050$. You want to pay 25 less and so must spend $25/1.05 = 23.81$ today.

6. Solution: E

In general, an investor should be compensated for time and risk. A forward contract has no investment, so the extra 5 represents the risk premium. Those who buy the stock expect to earn both the risk premium and the time value of their purchase and thus the expected stock value is greater than $100 + 5 = 105$.

7. Solution: C

All four of answers A-D are methods of acquiring the stock. Only the prepaid forward has the payment at time 0 and the delivery at time T .

8. Solution: B

Only straddles use at-the-money options and buying is correct for this speculation.

9. Solution: D

To see that D does not produce the desired outcome, begin with the case where the stock price is S and is below 90. The payoff is $S + 0 + (110 - S) - 2(100 - S) = 2S - 90$ which is not constant and so cannot produce the given diagram. On the other hand, for example, answer E has a payoff of $S + (90 - S) + 0 - 2(0) = 90$. The cost is $100 + 0.24 + 2.17 - 2(6.80) = 88.81$. With interest it is 93.36. The profit is $90 - 93.36 = -3.36$ which matches the diagram.

10. Solution: D

Answer A is true because forward contracts have no initial premium.

Answer B is true because both payoffs and profits of long forwards are opposite to short forwards.

Answer C is true because to invest in the stock, one must borrow 100 at $t = 0$, and then pay back $110 = 100(1 + 0.1)$ at $t = 1$, which is like buying a forward at $t = 1$ for 110.

Answer D is false because repeating the calculation shown for Answer C, but with 10% as a continuously compounded rate, the stock investor must now pay back $100\exp(0.1) = 110.52$ at $t = 1$; this is more expensive than buying a forward at $t = 1$ for 110.00.

Answer E is true because the calculation would be the same as shown above for Answer C but now the stock investor gets an additional dividend of 3.00 at $t = 0.5$, which the forward investor does not receive (due to not owning the stock until $t = 1$).

11. Solution: C

The future value of the cost of the options is $9.12(1.08) = 9.85$, $6.22(1.08) = 6.72$, and $4.08(1.08) = 4.41$ respectively.

If $S < 35$ no call is in the money and the profits are -9.85 , -6.72 , and -4.41 . The condition is not met.

If $35 < S < 40$ the 35-strike call returns $S - 35$ and the profit is $S - 44.85$. For the 45-strike call to have a lower profit than the 35-strike call, we need $-4.41 < S - 44.85$ or $S > 40.44$. This is inconsistent with the assumption.

If $40 < S < 45$ the same condition applies for comparing the 35- and 45-strike calls and so $S > 40.44$ is needed. The 40-strike call has a profit of $S - 40 - 6.72 = S - 46.72$. For the 45-strike to exceed the 40-strike, we need $-4.41 > S - 46.72$ or $S < 42.31$.

There is no need to consider $S > 45$.

12. Solution: B

Let S be the price of the index in six months.

The put premium has future value (at $t = 0.5$) of $74.20[1 + (0.04/2)] = 75.68$.

The 6-month profit on a long put position is $\max(1,000 - S, 0) - 75.68$.

The 6-month profit on a short put position is $75.68 - \max(1,000 - S, 0)$.

$$0 = 75.68 - \max(1,000 - S, 0).$$

$$75.68 = \max(1,000 - S, 0).$$

$$75.68 = 1,000 - S. S = 924.32.$$

13. Solution: D

Buying a call in conjunction with a short position is a form of insurance called a cap. Answers (A) and (B) are incorrect because a floor is the purchase of a put to insure against a long position. Answer (E) is incorrect because writing a covered call is the sale of a call along with a long position in the stock, so that the investor is selling rather than buying insurance.

The profit is the payoff at time 2 less the future value of the initial cost. The stock payoff is -75 and the option payoff is $75 - 60 = 15$ for a total of -60 . The future value of the initial cost is $(-50 + 10)(1.03)(1.03) = -42.44$. the profit is $-60 - (-42.44) = -17.56$.

14. Solution: A

Let C be the price for the 40-strike call option. Then, $C + 3.35$ is the price for the 35-strike call option. Similarly, let P be the price for the 40-strike put option. Then, $P - x$ is the price for the 35-strike put option, where x is the desired quantity. Using put-call parity, the equations for the 35-strike and 40-strike options are, respectively,

$$(C + 3.35) + 35e^{-0.02} - 40 = P - x$$

$$C + 40e^{-0.02} - 40 = P.$$

Subtracting the first equation from the second, $5e^{-0.02} - 3.35 = x$, $x = 1.55$.

15. Solution: C

The initial cost to establish this position is $5(2.78) - 3(6.13) = -4.49$. Thus, you are receiving 4.49 up front. This grows to $4.49\exp[0.25(0.08)] = 4.58$ after 3 months. Then, if S is the value of the stock at time 0.25, the profit is

$5\max(S - 40, 0) - 3\max(S - 35, 0) + 4.58$. The following cases are relevant:

$S < 35$: Profit = $0 - 0 + 4.58 = 4.58$.

$35 < S < 40$: Profit = $0 - 3(S - 35) + 4.58 = -3S + 109.58$. Minimum of -10.42 is at $S = 40$ and maximum of 4.58 is at $S = 35$.

$S > 40$: Profit is $5(S - 40) - 3(S - 35) + 4.58 = 2S - 90.42$. Minimum of -10.42 is at $S = 40$ and maximum if infinity.

Thus the minimum profit is -10.42 for a maximum loss of 10.42 and the maximum profit is infinity.

16. Solution: D

The straddle consists of buying a 40-strike call and buying a 40-strike put. This costs $2.78 + 1.99 = 4.77$ and grows to $4.77\exp(0.02) = 4.87$ at three months. The strangle consists of buying a 35-strike put and a 45-strike call. This costs $0.44 + 0.97 = 1.41$ and grows to $1.41\exp(0.02) = 1.44$ at three months. Let S be the stock price in three months.

For $S < 40$, the straddle has a profit of $40 - S - 4.87 = 35.13 - S$.

For $S > 40$, the straddle has a profit of $S - 40 - 4.87 = S - 44.87$.

For $S < 35$, the strangle has a profit of $35 - S - 1.44 = 33.56 - S$.

For $35 < S < 45$, the strangle has a profit of -1.44 .

For $S > 45$, the strangle has a profit of $S - 45 - 1.44 = S - 46.44$.

For $S < 35$ the strangle underperforms the straddle.

For $35 < S < 40$, the strangle outperforms the straddle if $-1.44 > 35.13 - S$ or $S > 36.57$. At this point only Answer D can be correct.

As a check, for $40 < S < 45$, the strangle outperforms the straddle if $-1.44 > S - 44.87$ or $S < 43.43$.

For $S > 45$, the strangle outperforms the straddle if $S - 46.44 > S - 44.87$, which is not possible.

17. Solution: B

Strategy I – Yes. It is a bear spread using calls, and bear spreads perform better when the prices of the underlying asset goes down.

Strategy II – Yes. It is also a bear spread – it just uses puts instead of calls.

Strategy III – No. It is a box spread, which has no price risk; thus, the payoff is the same ($1,000 - 950 = 50$), no matter the price of the underlying asset.

18. Solution: B

First calculate the expected one-year profit without using the forward. This is $0.2(700 + 150 - 750) + 0.5(700 + 170 - 850) + 0.3(700 + 190 - 950) = 20 + 10 - 18 = 12$. Next, calculate the expected one-year profit when buying the 1-year forward for 850. This is $1(700 + 170 - 850) = 20$. Thus, the expected profit increases by $20 - 12 = 8$ as a result of using the forward.

19. Solution: D

There are 3 cases, one for each row in the probability table.

For all 3 cases, the future value of the put premium is $100\exp(0.06) = 106.18$.

In Case 1, the 1-year profit is $750 - 800 - 106.18 + \max(900 - 750, 0) = -6.18$.

In Case 2, the 1-year profit is $850 - 800 - 106.18 + \max(900 - 850, 0) = -6.18$.

In Case 3, the 1-year profit is $950 - 800 - 106.18 + \max(900 - 950, 0) = 43.82$.

Thus, the expected 1-year profit = $0.7(-6.18) + 0.3(43.82) = -4.326 + 13.146 = 8.82$.

20. Solution: B

We need the future value of the current stock price minus the future value of each of the 12 dividends, where the valuation date is time 3. Thus, the forward price is

$$\begin{aligned} & 200e^{0.04(3)} - 1.50[e^{0.04(2.75)} + e^{0.04(2.5)}1.01 + \dots + e^{0.04(0.25)}1.01^{10} + 1.01^{11}] \\ &= 200e^{0.12} - 1.50e^{0.11}[1 + (e^{-0.01}1.01) + \dots + (e^{-0.01}1.01)^{10} + (e^{-0.01}1.01)^{11}] \\ &= 200e^{0.12} - 1.50e^{0.11} \frac{1 - (e^{-0.01}1.01)^{12}}{1 - e^{-0.01}1.01} = 225.50 - 1.67442(11.99672) = 205.41. \end{aligned}$$

21. Solution: E

The fair value of the forward contract is given by $S_0e^{(r-d)T} = 110e^{(0.05-0.02)0.5} = 111.66$.

This is 0.34 less than the observed price. Thus, one could exploit this arbitrage opportunity by selling the observed forward at 112 and buying a synthetic forward at 111.66, making $112 - 111.66 = 0.34$ profit.

22. Solution: B

First, determine the present value of the forward contracts. On a per ton basis, it is $1,600/1.05 + 1,700/1.055^2 + 1,800/1.06^3 = 4,562.49$.

Then, solve for the level swap price, x , as follows:

$$4,562.49 = x/1.05 + x/1.055^2 + x/1.06^3 = 2.69045x.$$

Thus, $x = 4,562.49 / 2.69045 = 1,695.81$.

Thus, the amount he would receive each year is $50(1,695.81) = 84,790.38$.

23. Solution: E

The notional amount and the future 1-year LIBOR rates (not given) do not factor into the calculation of the swap's fixed rate.

Required quantities are

(1) Zero-coupon bond prices:

$$1.04^{-1} = 0.96154, 1.045^{-2} = 0.91573, 1.0525^{-3} = 0.85770, 1.0625^{-4} = 0.78466, 1.075^{-5} = 0.69656.$$

(2) 1-year implied forward rates:

$$0.04, 1.045^2 / 1.04 - 1 = 0.05002, 1.0525^3 / 1.045^2 = 0.06766,$$

$$1.0625^4 / 1.0525^3 - 1 = 0.09307, 1.075^5 / 1.0625^4 - 1 = 0.12649.$$

The fixed swap rate is:

$$\frac{0.96154(0.04) + 0.9173(0.05002) + 0.85770(0.06766) + 0.78466(0.09307) + 0.69656(0.12649)}{0.96154 + 0.91573 + 0.85770 + 0.78466 + 0.69656} = 0.07197.$$

The calculation can be done without the implied forward rates as the numerator is $1 - 0.69656 = 0.30344$.

24. Solution: D

(A) is a reason because hedging reduces the risk of loss, which is a primary function of derivatives.

(B) is a reason because derivatives can be used to hedge some risks that could result in bankruptcy.

(C) is a reason because derivatives can provide a lower-cost way to effect a financial transaction.

(D) is not a reason because derivatives are often used to avoid these types of restrictions.

(E) is a reason because an insurance contract can be thought of as a hedge against the risk of loss.

25. Solution: C

(A) is accurate because both types of individuals are involved in the risk-sharing process.

(B) is accurate because this is the primary reason reinsurance companies exist.

(C) is not accurate because reinsurance companies share risk by issuing rather than investing in catastrophe bonds. They are ceding this excess risk to the bondholder.

(D) is accurate because it is diversifiable risk that is reduced or eliminated when risks are shared.

(E) is accurate because this is a fundamental idea underlying risk management and derivatives.

26. Solution: B

I is true. The forward seller has unlimited exposure if the underlying asset's price increases.

II is true. The call issuer has unlimited exposure if the underlying asset's price rises.

III is false. The maximum loss on selling a put is $FV(\text{put premium}) - \text{strike price}$.

27. Solution: A

I is true. As the value of the house decreases due to insured damage, the policyholder will be compensated for the loss. Homeowners insurance is a put option.

II is false. Returns from equity-linked CDs are zero if prices decline, but positive if prices rise. Thus, owners of these CDs benefit from rising prices.

III is false. A synthetic forward consists of a long call and a short put, both of which benefit from rising prices (so the net position also benefits as such).

28. Solution: E

(A) is true. Derivatives are used to shift income, thereby potentially lowering taxes.

(B) is true. As with taxes, the transfer of income lowers the probability of bankruptcy.

(C) is true. Hedging can safeguard reserves, and reduce the need for external financing, which has both explicit (e.g., fees) and implicit (e.g., reputational) costs.

(D) is true. When operating internationally, hedging can reduce exchange rate risk.

(E) is false. A firm that credibly hedges will reduce the riskiness of its cash flows, and will be able to increase debt capacity, which will lead to tax savings, since interest is deductible.

29. Solution: A

The current price of the stock and the time of future settlement are not relevant, so let both be 1. Then the following payments are required:

Outright purchase, payment at time 0, amount of payment = 1.

Fully leveraged purchase, payment at time 1, amount of payment = $\exp(r)$.

Prepaid forward contract, payment at time 0, amount of payment = $\exp(-d)$.

Forward contract, payment at time 1, amount of payment = $\exp(r-d)$.

Since $r > d > 0$, $\exp(-d) < 1 < \exp(r-d) < \exp(r)$.

The correct ranking is given by (A).

30. Solution: C

(A) is a distinction. Daily marking to market is done for futures, not forwards.

(B) is a distinction. Futures are more liquid; in fact, if you use the same broker to buy and sell, your position is effectively cancelled.

(C) is not a distinction. Forwards are more customized, and futures are more standardized.

(D) is a distinction. With daily settlement, credit risk is less with futures (v. forwards).

(E) is a distinction. Futures markets, like stock exchanges, have daily price limits.

31. Solution: E

The transaction costs are 2 (1 for the forward and 1 for the stock)

The price of the forward is therefore $(50 + 2)(1.06) = 55.12$.

32. Solution: C

The notional value of this long futures position is $1500(20)(250) = 7.5$ million. Thus, the initial margin requirement is 10% of 7.5 million, or 750,000, and the maintenance margin requirement is 75% of 750,000, or 562,500. The margin account balance, 1 day later, after the S&P 500 index has fallen by 50 to 1450, is: $750,000\exp(0.05/365) - 50(20)(250) = 750,102.75 - 250,000 = 500,102.75$. Because this is below the maintenance margin level, a margin call is necessary. The amount of the margin call is $562,500 - 500,102.75 = 62,397.25$.

33: Solution: E

Option I is American-style, and thus, it can be exercised at any time during the 6-month period. Since it is a put, the payoff is greatest when the stock price is smallest (18). The payoff is $20 - 18 = 2$.

Option II is Bermuda-style, and can be exercised at any time during the 2nd 3-month period. Since it is a call, the payoff is greatest when the stock price S is largest (28). The payoff is $28 - 25 = 3$.

Option III is European-style, and thus, it can be exercised only at maturity. Since it is a 30-strike put, the payoff equation is $30 - 26 = 4$.

The ranking is $\text{III} > \text{II} > \text{I}$.

34. Solution: E

Since the 2-year forward price is higher than the 1-year forward price, the buyer, relative to the forward prices, overall pays more at the end of the first year but less at the end of the second year. So this means that the buyer pays the swap counterparty at the end of the first year but receives money back from the swap counterparty at the end of the second year. So the buyer lends to the swap counterparty at the 1-year effective forward interest rate, from the end of the first year to the end of the second year, namely 6%.