INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 120 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 60 points).
   a) The morning session consists of 9 questions numbered 1 through 9.
   b) The afternoon session consists of 9 questions numbered 10 through 18.

The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam FETE.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d’examen pour la version française.
1. (6 points) The Chianti Insurance Company currently has a market value capital structure of 15% debt to total assets. The company’s CFO believes more debt can be taken on, up to a limit of 30%, without losing the firm’s ability to borrow at 6%, the prime rate. The firm has a marginal tax rate of 50%. The expected return on the market next year is estimated to be 15%, and the systematic risk of the company’s equity, \( \beta_L \), is estimated to be 0.5.

(a) (3 points) Calculate:

(i) The company’s current weighted average cost of capital.

(ii) The current cost of equity.

The company decides to change its capital structure to 30% debt to total assets, from 15%.

(b) (2 points) Calculate the new weighted average cost of capital.

The company can invest in a project that generates an 8.5% expected rate of return. The systematic risk, \( \beta_L \), of the project is equivalent to that of the firm.

(c) (1 point) Recommend whether or not the company should invest in the project, and justify your recommendation.
2. (7 points) You recently graduated from college with a major in actuarial science. You and two of your college graduate friends decide to form an organization in the U.S. that provides actuarial services. You have managed to get capital funding from four older family members, in exchange for some ownership in the venture. Since you and your college graduate friends will not be contributing any capital, you have agreed to run the day to day operations of the organization. The profits will be shared equally among all 7 associates; therefore, you are all concerned with after-tax profits. If the venture is successful, you all plan on an IPO. If it is not, you agree to equally share in the business liabilities, with no risk of loss to personal property.

You are considering the following forms:

(i) Partnership

(ii) Limited Partnership

(iii) S-corporation

(iv) C-corporation (that is not treated as an S-corporation)

(a) (4 points) For this venture that you plan on undertaking:

(i) List the advantages and disadvantages of choosing each of the forms

(ii) Recommend and justify which form you should choose.

(b) (3 points) You have seen your infant organization flourish into Syrah Co., a thriving public corporation. Your IPO was successful, and the four older family members each own 15% of the company. They are not involved in the day to day operations of the business. You and your two college friends only own 4% each, but will be compensated for working on the day-to-day operations of the business.

(i) Describe potential conflicts of interest with your older family members as shareholders of the company that you and your college friends may have from investing the assets of the firm.

(ii) List and explain ways in which compensation for you and your college friends may be designed to better align your interests with the interests of your older family members as shareholders of the company.

(iii) Explain other external mechanisms that can be used as a means of corporate governance for the organization.
3. \((6 \text{ points})\) Sangiovese Life currently sells one-year group life contracts with no underwriting.

Sangiovese analyzes profit on its past sales using the three categories shown below:

<table>
<thead>
<tr>
<th>Category</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit from Contract at End of Year (millions)</td>
<td>5</td>
<td>1</td>
<td>−3</td>
</tr>
<tr>
<td>Probability that any new contract falls into this category</td>
<td>7/24</td>
<td>10/24</td>
<td>7/24</td>
</tr>
</tbody>
</table>

Each contract has a capital requirement of 5 million. Sangiovese Life requires a minimum before-tax return on capital of 15%.

The company is evaluating new underwriting software that is advertised to provide predictive information about the risk of a new group, prior to sale.

Sangiovese used the software to perform back-testing on its existing contracts, and obtained the following results:

<table>
<thead>
<tr>
<th>Risk Prediction</th>
<th>Frequency</th>
<th>Distribution by Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bad Risk</td>
<td>25%</td>
<td>A: 0, B: 1/2, C: 1/2</td>
</tr>
<tr>
<td>Good Risk</td>
<td>25%</td>
<td>A: 1/2, B: 1/2, C: 0</td>
</tr>
<tr>
<td>Uncertain</td>
<td>50%</td>
<td>A: 1/3, B: 1/3, C: 1/3</td>
</tr>
</tbody>
</table>

(a) \((4 \text{ points})\) Calculate the value of the information provided by the software on each contract underwritten.

Sangiovese Life will underwrite exactly 12 contracts per year. The underwriting software can be licensed for three years for an up-front fee of 1 million.

(b) \((2 \text{ points})\) Recommend whether Sangiovese Life should acquire the three-year license.
4. (7 points) Your company is working on a project which currently has no flexibility and a value of 300 million. At each stage of the project the value may increase or decrease, as shown in the diagram below:

\[ V_0 = 300 \]

\[ \rightarrow 360 \quad \rightarrow 432 \]

\[ \rightarrow 250 \quad \rightarrow 300 \quad \rightarrow 208.33 \]

An expansion option is now available and would allow the project to expand by 30%, at each future stage, but for a cost of 110 million at each future stage.

The risk-free rate is 4.8%.

(a) (2 points) Describe all assumptions that you will use to price this real option.

(b) (1 point) Calculate \( q \), the risk neutral probability, to two decimal places.

(c) (4 points) Calculate each of the following, assuming the expansion option is available at each future stage of the project:

(i) The value at each future stage

(ii) The current value of the project

(iii) The percentage change in value of the project.
5. (7 points) Rioja Insurance plans to partially reinsure a recently issued block of whole life insurance which is primarily backed by long term bonds. Viognier Re is an offshore reinsurer (not licensed in your country of domicile) who has offered to reinsure the whole life business.

(a) (2 points) List reasons for maintaining assets with the ceding company in a reinsurance structure.

(b) (2 points) Evaluate the advantages and disadvantages to Rioja of the approach in (a).

In addition to the regular allowance, Viognier Re is providing an initial one time allowance for the transaction based on the business reinsured (essentially treating the reserve as new premium). Actual investment income is as expected.

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premiums</td>
<td>$750,000</td>
</tr>
<tr>
<td>Allowance</td>
<td>9%</td>
</tr>
<tr>
<td>Expenses &amp; Commissions</td>
<td>$75,000</td>
</tr>
<tr>
<td>Reserve</td>
<td>$1,060,200</td>
</tr>
<tr>
<td>Change in Reserve for NB</td>
<td>$727,500</td>
</tr>
<tr>
<td>Change in Reserve in force</td>
<td>$74,215</td>
</tr>
<tr>
<td>Free Surplus</td>
<td>$20,000</td>
</tr>
</tbody>
</table>

(c) (2 points) Calculate the coinsurance percentage (%) required such that the free surplus set aside is adequate to support sales for one year.

(d) (1 point) Describe conditions which should be included in a treaty in order to gain acceptance by the regulators in Rioja’s country of domicile.
6. (8 points) Investment assets $U$ and $V$ follow geometric Brownian motions with volatilities $\sigma_u$ and $\sigma_v$ respectively; they are correlated with correlation coefficient $\rho$. Consider a contract which pays the greater of $U$ or $V$ at time $T$.

(a) (2 points) Derive the value of this contract at issue; define all symbols.

You are given:
- The current price of gold is $1,250 per ounce with a volatility of 20%.
- The current price of platinum is $1,600 per ounce with a volatility of 25%.
- The correlation coefficient between the two asset prices is $\rho = 0.7$.
- Storage costs are assumed to be zero.

(b) (i) (3 points) Calculate the price of a 1-year European option which pays the price of 4 ounces of gold or 3 ounces of platinum, whichever is greater.

(ii) (1 point) Describe the effect on the option price if the assets require storage at non-zero cost (without further calculation).

Assume the option given in (b)(i) is available through a trader; however you believe that the correlation coefficient between these two assets is higher than that implied by the trader’s asking price.

(c) (2 points) Describe how you could make money.
7. (7 points)

(a) (1 point) Describe and critique the Comparative Advantage argument to explain the popularity of interest rate swaps in capital financing.

Malbec Corp. prefers to pay a floating rate of interest. Valpolicella Corp. prefers to pay a fixed rate of interest.

Current Terms for Borrowing

<table>
<thead>
<tr>
<th></th>
<th>Fixed (semiannual compounding)</th>
<th>Floating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Malbec</td>
<td>8.0%</td>
<td>6-month LIBOR + 1.30%</td>
</tr>
<tr>
<td>Valpolicella</td>
<td>9.7%</td>
<td>6-month LIBOR + 1.91%</td>
</tr>
</tbody>
</table>

(b) (3 points) Show that a swap rate of 7.36% between Malbec and Valpolicella would lower the borrowing costs for both companies, and calculate the rate reduction for each company.

Malbec and Valpolicella entered into a 3-year interest rate swap. After one year, the current bid and offer rates in the fixed swap market are (with semiannual compounding):

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Bid</th>
<th>Offer</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8.15%</td>
<td>8.19%</td>
</tr>
</tbody>
</table>

Assume:

- The swap has a $10 million notional, and 4 future swap payments will take place
- Both sides of the swap pay semiannually
- Valpolicella has invested in a $10 million project with the proceeds from their borrowing. The present value of the project’s cash flows has decreased in value by $192,000 due to interest rate moves
- The 0.5, 1, and 1.5 year continuously compounded LIBOR zero rates are 7.9%, 8.0%, and 8.1% respectively.

(c) (3 points) Evaluate whether or not the swap has proved to be an effective asset liability management tool for Valpolicella.
8. (8 points) Suppose that $f$ and $g$ are the prices of traded securities dependent on a single source of uncertainty and that $\sigma_f$ and $\sigma_g$ are constants, $r$ is the constant risk-free rate and $dz$ is a Wiener process.

$$df = (r + \sigma_g \sigma_f) f dt + \sigma_f f dz$$
$$dg = (r + \sigma_g^2) g dt + \sigma_g g dz$$

(a) (1 point) Identify the market price of risk in the processes above.

(b) (2 points) Demonstrate the following using Itô’s lemma.

$$d \ln f = (r + \sigma_g \sigma_f - \frac{1}{2} \sigma_f^2) dt + \sigma_f dz$$
$$d \ln g = (r + \frac{1}{2} \sigma_g^2) dt + \sigma_g dz$$

(c) (1 point) Show that

$$d \left( \ln \frac{f}{g} \right) = -\frac{(\sigma_f - \sigma_g)^2}{2} dt + (\sigma_f - \sigma_g) dz$$

(d) (2 points) Demonstrate the following using Itô’s lemma given $H = \frac{f}{g}$:

$$dH = (\sigma_f - \sigma_g) H dz$$

(e) (1 point) Explain, in words, why $H = \frac{f}{g}$ is a martingale.

(f) (1 point) Calculate the expected price of $H$ in one week if the current price of $f$ is $12$ and the current price of $g$ is $4$. 
9. \hspace{1cm} (4 points) You observe the following for a 1-year cap struck at 3%, which resets every 3 months and makes a payment at the end of each period based on the rates at the beginning of that period. The initial yield curve is flat at 3%.

<table>
<thead>
<tr>
<th>Tenor (yrs)</th>
<th>Cap Price per 10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>2.946</td>
</tr>
<tr>
<td>0.75</td>
<td>6.874</td>
</tr>
<tr>
<td>1</td>
<td>11.396</td>
</tr>
</tbody>
</table>

Given:
\[
\text{Caplet} = L\delta_k P(0, t_{k+1})[F_k N(d_1) - R_k N(d_2)]
\]

(a) \hspace{0.5cm} (1 point) Calculate the time \( T = 1 \) caplet price.

(b) \hspace{0.5cm} (3 points) Calculate the implied volatility for the \( T = 1 \) caplet for Black’s model.

**END OF EXAMINATION**
MORNING SESSION
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