SOCIETY OF ACTUARIES

EXAM MLC Models for Life Contingencies

EXAM MLC SAMPLE SOLUTIONS

Some of the questions in this study note are taken from past SOA examinations.

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Question #1
Answer: E

$$_{2|}q_{\overline{30:34}} = {}_2p_{\overline{30:34}} - {}_3p_{\overline{30:34}}$$

$$_2p_{30} = (0.9)(0.8) = 0.72$$

$$_2p_{34} = (0.5)(0.4) = 0.20$$

$$_2p_{30:34} = (0.72)(0.20) = 0.144$$

$$_2p_{\overline{30:34}} = 0.72 + 0.20 - 0.144 = 0.776$$

$$_3p_{30} = (0.72)(0.7) = 0.504$$

$$_3p_{34} = (0.20)(0.3) = 0.06$$

$$_3p_{30:34} = (0.504)(0.06) = 0.03024$$

$$_3p_{\overline{30:34}} = 0.504 + 0.06 - 0.03024$$
$$= 0.53376$$

$$_{2|}q_{\overline{30:34}} = 0.776 - 0.53376$$
$$= 0.24224$$

Alternatively,

$$_{2|}q_{\overline{30:34}} = {}_{2|}q_{30} + {}_{2|}q_{34} - {}_{2|}q_{30:34}$$
$$= {}_2p_{30}q_{32} + {}_2p_{34}q_{36} - {}_2p_{30:34}(1 - p_{32:36})$$

= (0.9)(0.8)(0.3) + (0.5)(0.4)(0.7) – (0.9)(0.8)(0.5)(0.4) [1-(0.7)(0.3)]
= 0.216 + 0.140 – 0.144(0.79)
= 0.24224

Alternatively,

$$_{2|}q_{\overline{30:34}} = {}_3q_{30} \times {}_3q_{34} - {}_2q_{30} \times {}_2q_{34}$$
$$= (1 - {}_3p_{30})(1 - {}_3p_{34}) - (1 - {}_2p_{30})(1 - {}_2p_{34})$$
$$= (1 - 0.504)(1 - 0.06) - (1 - 0.72)(1 - 0.20)$$
$$= 0.24224$$

(see first solution for $_2p_{30}$, $_2p_{34}$, $_3p_{30}$, $_3p_{34}$)

Question #2
Answer: E

$$
\begin{aligned}
1000\bar{A}_x &= 1000\left[\bar{A}^{1}_{x:\overline{10|}} + {}_{10|}\bar{A}_x\right] \\
&= 1000\left[\int_0^{10} e^{-0.04t}e^{-0.06t}(0.06)dt + e^{-0.4}e^{-0.6}\int_0^{\infty} e^{-0.05t}e^{-0.07t}(0.07)dt\right] \\
&= 1000\left[0.06\int_0^{10} e^{-0.1t}dt + e^{-1}(0.07)\int_0^{\infty} e^{-0.12t}dt\right] \\
&= 1000\left[0.06\left[\frac{-e^{-0.10t}}{0.10}\right]_0^{10} + e^{-1}(0.07)\left[\frac{-e^{-0.12t}}{0.12}\right]_0^{\infty}\right] \\
&= 1000\left[\frac{0.06}{0.10}\left[1-e^{-1}\right] + \frac{0.07}{0.12}e^{-1}\left[1-e^{-1.2}\right]\right] \\
&= 1000\left(0.37927 + 0.21460\right) = 593.87
\end{aligned}
$$

Because this is a timed exam, many candidates will know common results for constant force and constant interest without integration.

$$
\begin{aligned}
\text{For example}\quad \bar{A}^{1}_{x:\overline{10|}} &= \frac{\mu}{\mu+\delta}\left(1-{}_{10}E_x\right) \\
{}_{10}E_x &= e^{-10(\mu+\delta)} \\
\bar{A}_x &= \frac{\mu}{\mu+\delta}
\end{aligned}
$$

With those relationships, the solution becomes

$$
\begin{aligned}
1000\bar{A}_x &= 1000\left[\bar{A}^{1}_{x:\overline{10|}} + {}_{10}E_x\ A_{x+10}\right] \\
&= 1000\left[\left(\frac{0.06}{0.06+0.04}\right)\left(1-e^{-(0.06+0.04)10}\right) + e^{-(0.06+0.04)10}\left(\frac{0.07}{0.07+0.05}\right)\right] \\
&= 1000\left[(0.60)\left(1-e^{-1}\right) + 0.5833e^{-1}\right] \\
&= 593.86
\end{aligned}
$$

Question #3
Answer: D

$$E[Z] = \int_0^\infty b_t v^t \, {}_t p_x \, \mu_{x+t} \, dt = \int_0^\infty e^{0.06\,t} e^{-0.08\,t} e^{-0.05\,t} \frac{1}{20} dt$$

$$= \frac{1}{20}\left(\frac{100}{7}\right)\left[-e^{-0.07\,t}\right]_0^\infty = \frac{5}{7}$$

$$E\left[Z^2\right] = \int_0^\infty \left(b_t v^t\right)^2 {}_t p_x \mu_{x+t} dt = \int_0^\infty e^{0.12t} e^{-0.16t} e^{-0.05t} \frac{1}{20} dt = \frac{1}{20}\int_0^\infty e^{-0.09t} dt$$

$$= \frac{1}{20}\left(\frac{100}{9}\right)\left[e^{-0.09t}\right]_0^\infty = \frac{5}{9}$$

$$Var[Z] = \frac{5}{9} - \left(\frac{5}{7}\right)^2 = 0.04535$$

Question #4
Answer: C

Let *ns* = nonsmoker and *s* = smoker

k =	$q_{x+k}^{(ns)}$	$p_{x+k}^{(ns)}$	$q_{x+k}^{(s)}$	$p_{x+k}^{(s)}$
0	.05	0.95	0.10	0.90
1	.10	0.90	0.20	0.80
2	.15	0.85	0.30	0.70

$$A_{x:\overline{2}|}^{1(ns)} = \quad v \quad q_x^{(ns)} + \quad v^2 \quad p_x^{(ns)} \quad q_{x+1}^{(ns)}$$

$$\frac{1}{1.02}(0.05) \quad \frac{1}{1.02^2} \quad 0.95 \times 0.10 = 0.1403$$

$$A_{x:\overline{2}|}^{1(s)} \quad v \quad q_x^{(s)} \quad + \quad v^2 \quad p_x^{(s)} \quad q_{x+1}^{(s)}$$

$$\frac{1}{1.02} \quad (0.10) + \frac{1}{(1.02)^2} \; 0.90 \times 0.20 = 0.2710$$

$A_{x:\overline{2}|}^{1} =$ weighted average = (0.75)(0.1403) + (0.25)(0.2710)
= 0.1730

Question #5
Answer: B

$$\mu_x^{(\tau)} = \mu_x^{(1)} + \mu_x^{(2)} + \mu_x^{(3)} = 0.0001045$$
$$_t p_x^{(\tau)} = e^{-0.0001045t}$$

$$\text{APV Benefits} = \int_0^\infty e^{-\delta t} 1{,}000{,}000 \, _t p_x^{(\tau)} \mu_x^{(1)} dt$$
$$+\int_0^\infty e^{-\delta t} 500{,}000 \, _t p_x^{(\tau)} \mu_x^{(2)} dt$$
$$+\int_0^\infty e^{-\delta \tau} 200{,}000 \, _t p_x^{(\tau)} \mu_x^{(3)} dt$$
$$= \frac{1{,}000{,}000}{2{,}000{,}000} \int_0^\infty e^{-0.0601045t} dt + \frac{500{,}000}{250{,}000} \int_0^\infty e^{-0.0601045t} dt + \frac{250{,}000}{10{,}000} \int_0^\infty e^{-0.0601045t} dt$$
$$= 27.5(16.6377) = 457.54$$

Question #6
Answer: B

$$EPV \text{ Benefits} = 1000 A^1_{40:\overline{20}|} + \sum_{k=20}^{\infty} {_k E_{40}} 1000 v q_{40+k}$$
$$EPV \text{ Premiums} = \pi \ddot{a}_{40:\overline{20}|} + \sum_{k=20}^{\infty} {_k E_{40}} 1000 v q_{40+k}$$
$$\text{Benefit premiums } \Rightarrow \text{Equivalence principle } \Rightarrow$$
$$1000 A^1_{40:\overline{20}|} + \sum_{k=20}^{\infty} {_k E_{40}} 1000 v q_{40+k} = \pi \ddot{a}_{40:\overline{20}|} + \sum_{20}^{\infty} {_k E_{40}} 1000 v q_{40+k}$$
$$\pi = 1000 A^1_{40:\overline{20}|} / \ddot{a}_{40:\overline{20}|}$$
$$= \frac{161.32 - (0.27414)(369.13)}{14.8166 - (0.27414)(11.1454)}$$
$$= 5.11$$

While this solution above recognized that $\pi = 1000 P^1_{40:\overline{20}|}$ and was structured to take advantage of that, it wasn't necessary, nor would it save much time. Instead, you could do:

EPV Benefits $= 1000A_{40} = 161.32$

$$EPV \text{ Premiums } = \pi \ddot{a}_{40:\overline{20|}} + {}_{20}E_{40} \sum_{k=0}^{\infty} {}_kE_{60} 1000vq_{60+k}$$
$$= \pi \ddot{a}_{40:\overline{20|}} + {}_{20}E_{40} 1000A_{60}$$
$$= \pi\left[14.8166 - (0.27414)(11.1454)\right] + (0.27414)(369.13)$$
$$= 11.7612\pi + 101.19$$

$$11.7612\pi + 101.19 = 161.32$$

$$\pi = \frac{161.32 - 101.19}{11.7612} = 5.11$$

Question #7
Answer: C

$$A_{70} = \frac{\delta}{i}\bar{A}_{70} = \frac{\ln(1.06)}{0.06}(0.53) = 0.5147$$

$$\ddot{a}_{70} = \frac{1 - A_{70}}{d} = \frac{1 - 0.5147}{0.06/1.06} = 8.5736$$

$$\ddot{a}_{69} = 1 + vp_{69}\ddot{a}_{70} = 1 + \left(\frac{0.97}{1.06}\right)(8.5736) = 8.8457$$

$$\ddot{a}_{69}^{(2)} = \alpha(2)\ddot{a}_{69} - \beta(2) = (1.00021)(8.8457) - 0.25739$$
$$= 8.5902$$

Note that the approximation $\ddot{a}_x^{(m)} \cong \ddot{a}_x - \frac{(m-1)}{2m}$ works well (is closest to the exact answer, only off by less than 0.01). Since *m* = 2, this estimate becomes $8.8457 - \frac{1}{4} = 8.5957$

Question #8 - Removed

Question #9 - Removed

Question #10
Answer: E

$d = 0.05 \rightarrow v = 0.95$

At issue

$$A_{40} = \sum_{k=0}^{49} v^{k+1} {}_{k|}q_{40} = 0.02\left(v^1 + ... + v^{50}\right) = 0.02v\left(1 - v^{50}\right)/d = 0.35076$$

$$\text{and } \ddot{a}_{40} = \left(1 - A_{40}\right)/d = \left(1 - 0.35076\right)/0.05 = 12.9848$$

$$\text{so } P_{40} = \frac{1000A_{40}}{\ddot{a}_{40}} = \frac{350.76}{12.9848} = 27.013$$

$$E\left({}_{10}L \middle| K_{40} \geq 10\right) = 1000A_{50}^{\text{Revised}} - P_{40}\ddot{a}_{50}^{\text{Revised}} = 549.18 - (27.013)(9.0164) = 305.62$$

where

$$A_{50}^{\text{Revised}} = \sum_{k=0}^{24} v^{k+1} {}_{k|}q_{50}^{\text{Revised}} = 0.04\left(v^1 + ... + v^{25}\right) = 0.04v\left(1 - v^{25}\right)/d = 0.54918$$

$$\text{and } \ddot{a}_{50}^{\text{Revised}} = \left(1 - A_{50}^{\text{Revised}}\right)/d = \left(1 - 0.54918\right)/0.05 = 9.0164$$

Question #11
Answer: E

Let NS denote non-smokers and S denote smokers.

The shortest solution is based on the conditional variance formula

$$\text{Var}(X) = E\left(\text{Var}(X|Y)\right) + \text{Var}\left(E(X|Y)\right)$$

Let Y = 1 if smoker; Y = 0 if non-smoker

$$E\left(\bar{a}_{\overline{T|}} \middle| Y = 1\right) = \bar{a}_x^S = \frac{1 - \bar{A}_x^S}{\delta}$$

$$= \frac{1 - 0.444}{0.1} = 5.56$$

Similarly $E\left(\bar{a}_{\overline{T|}} \middle| Y = 0\right) = \dfrac{1 - 0.286}{0.1} = 7.14$

$$E\left(E\left(\bar{a}_{\overline{T|}} | Y\right)\right) = E\left(E\left(\bar{a}_{\overline{T|}} | 0\right)\right) \times \text{Prob}(\text{Y=0}) + E\left(E\left(\bar{a}_{\overline{T|}} | 1\right)\right) \times \text{Prob}(\text{Y=1})$$

$$= (7.14)(0.70) + (5.56)(0.30)$$

$$= 6.67$$

$$E\left[\left(E\left(\bar{a}_{\overline{T}|}|Y\right)\right)^2\right]=\left(7.14^2\right)(0.70)+\left(5.56^2\right)(0.30)$$
$$=44.96$$
$$\text{Var}\left(E\left(\bar{a}_{\overline{T}|}|Y\right)\right)=44.96-6.67^2=0.47$$
$$E\left(\text{Var}\left(\bar{a}_{\overline{T}|}|Y\right)\right)=(8.503)(0.70)+(8.818)(0.30)$$
$$=8.60$$
$$\text{Var}\left(\bar{a}_{\overline{T}|}\right)=8.60+0.47=9.07$$

Alternatively, here is a solution based on
$\text{Var}(Y)=E\left(Y^2\right)-\left[E(Y)\right]^2$, a formula for the variance of any random variable.
This can be
transformed into $E\left(Y^2\right)=\text{Var}(Y)+\left[E(Y)\right]^2$ which we will use in its conditional form

$$E\left(\left(\bar{a}_{\overline{T}|}\right)^2|\text{NS}\right)=\text{Var}\left(\bar{a}_{\overline{T}|}|\text{NS}\right)+\left[E\left(\bar{a}_{\overline{T}|}|\text{NS}\right)\right]^2$$

$$\text{Var}\left[\bar{a}_{\overline{T}|}\right]=E\left[\left(\bar{a}_{\overline{T}|}\right)^2\right]-\left(E\left[\bar{a}_{\overline{T}|}\right]\right)^2$$

$$E\left[\bar{a}_{\overline{T}|}\right]=E\left[\bar{a}_{\overline{T}|}|\text{S}\right]\times\text{Prob}[\text{S}]+E\left[\bar{a}_{\overline{T}|}|\text{NS}\right]\times\text{Prob}[\text{NS}]$$
$$=0.30\bar{a}_x^{\text{S}}+0.70\bar{a}_x^{\text{NS}}$$
$$=\frac{0.30\left(1-\bar{A}_x^{\text{S}}\right)}{0.1}+\frac{0.70\left(1-\bar{A}_x^{\text{NS}}\right)}{0.1}$$
$$=\frac{0.30(1-0.444)+0.70(1-0.286)}{0.1}=(0.30)(5.56)+(0.70)(7.14)$$
$$=1.67+5.00=6.67$$

$$E\left[\left(\bar{a}_{\overline{T}|}\right)^2\right]=E\left[\bar{a}_{\overline{T}|}^{\,2}|\text{S}\right]\times\text{Prob}[\text{S}]+E\left[\bar{a}_{\overline{T}|}^{\,2}|\text{NS}\right]\times\text{Prob}[\text{NS}]$$
$$=0.30\left(\text{Var}\left(\bar{a}_{\overline{T}|}|\text{S}\right)+\left(E\left[\bar{a}_{\overline{T}|}|\text{S}\right]\right)^2\right)$$
$$+0.70\left(\text{Var}\left(\bar{a}_{\overline{T}|}|\text{NS}\right)+E\left(\bar{a}_{\overline{T}|}|\text{NS}\right)^2\right)$$
$$=0.30\left[8.818+(5.56)^2\right]+0.70\left[8.503+(7.14)^2\right]$$
11.919 + 41.638 = 53.557

$$\text{Var}\left[\bar{a}_{\overline{T}|}\right]=53.557-(6.67)^2=9.1$$

Alternatively, here is a solution based on $\bar{a}_{\overline{T|}} = \dfrac{1-v^T}{\delta}$

$$
\begin{aligned}
\operatorname{Var}\left(\bar{a}_{\overline{T|}}\right) &= \operatorname{Var}\left(\frac{1}{\delta}-\frac{v^T}{\delta}\right) \\
&= \operatorname{Var}\left(\frac{-v^T}{\delta}\right) \text{ since } \operatorname{Var}(X+\text{constant}) = \operatorname{Var}(X) \\
&= \frac{\operatorname{Var}\left(v^T\right)}{\delta^2} \text{ since } \operatorname{Var}(\text{constant}\times X) = \text{constant}^2\times\operatorname{Var}(X) \\
&= \frac{{}^2\bar{A}_x-\left(\bar{A}_x\right)^2}{\delta^2} \text{ which is Bowers formula 5.2.9}
\end{aligned}
$$

This could be transformed into ${}^2A_x = \delta^2 \operatorname{Var}\left(\bar{a}_{\overline{T|}}\right) + \bar{A}_x^2$, which we will use to get ${}^2A_x{}^{\text{NS}}$ and ${}^2A_x{}^{\text{S}}$.

$$
\begin{aligned}
{}^2A_x &= E\left[v^{2T}\right] \\
&= E\left[v^{2T}\,|\,\text{NS}\right]\times\text{Prob}(\text{NS}) + E\left[v^{2T}\,|\,\text{S}\right]\times\text{Prob}(\text{S}) \\
&= \left[\delta^2\operatorname{Var}\left(\bar{a}_{\overline{T|}}\,|\,\text{NS}\right)+\left(\bar{A}_x{}^{\text{NS}}\right)^2\right]\times\text{Prob}(\text{NS}) \\
&+\left[\delta^2\operatorname{Var}\left(\bar{a}_{\overline{T|}}\,|\,\text{S}\right)+\left(\bar{A}_x{}^{\text{S}}\right)^2\right]\times\text{Prob}(\text{S}) \\
&= \left[(0.01)(8.503)+0.286^2\right]\times 0.70 \\
&+\left[(0.01)(8.818)+0.444^2\right]\times 0.30 \\
&= (0.16683)(0.70)+(0.28532)(0.30) \\
&= 0.20238
\end{aligned}
$$

$$
\begin{aligned}
\bar{A}_x &= E\left[v^T\right] \\
&= E\left[v^T\,|\,\text{NS}\right]\times\text{Prob}(\text{NS}) + E\left[v^T\,|\,\text{S}\right]\times\text{Prob}(\text{S}) \\
&= (0.286)(0.70)+(0.444)(0.30) \\
&= 0.3334
\end{aligned}
$$

$$\text{Var}\left(\bar{a}_{\overline{T}|}\right) = \frac{{}^2\bar{A}_x - \left(\bar{A}_x\right)^2}{\delta^2}$$

$$= \frac{0.20238 - 0.3334^2}{0.01} = 9.12$$

Question #12 - Removed

Question #13
Answer: D

Let NS denote non-smokers, S denote smokers.

$$\text{Prob}(T < t) = \text{Prob}(T < t|\text{NS}) \times \text{Prob}(\text{NS}) + \text{Prob}(T < t|\text{S}) \times \text{Prob}(\text{S})$$

$$= \left(1 - e^{-0.1t}\right) \times 0.7 + \left(1 - e^{-0.2t}\right) \times 0.3$$

$$= 1 - 0.7e^{-0.1t} - 0.3e^{-0.2t}$$

$S_0(t) = 0.3e^{-0.2t} + 0.7e^{-0.1t}$

Want $\hat{t}$ such that $0.75 = 1 - S_0\left(\hat{t}\right)$ or $0.25 = S_0\left(\hat{t}\right)$

$0.25 = 0.3e^{-2\hat{t}} + 0.7e^{-0.1\hat{t}} = 0.3\left(e^{-0.1\hat{t}}\right)^2 + 0.7e^{-0.1\hat{t}}$

Substitute: let $x = e^{-0.1\hat{t}}$

$0.3x^2 + 0.7x - 0.25 = 0$

This is quadratic, so $x = \dfrac{-0.7 \pm \sqrt{0.49 + (0.3)(0.25)4}}{2(0.3)}$

$x = 0.3147$

$e^{-0.1\hat{t}} = 0.3147$ so $\hat{t} = 11.56$

Question #14
Answer: A

At a constant force of mortality, the benefit premium equals the force of mortality and so $\mu = 0.03$.

$$^2\bar{A}_x = 0.20 = \frac{\mu}{2\delta + \mu} = \frac{0.03}{2\delta + 0.03}$$

$$\Rightarrow \delta = 0.06$$

$$\text{Var}\left({}_0L\right) = \frac{^2\bar{A}_x - \left(\bar{A}_x\right)^2}{\left(\delta\,\bar{a}\right)^2} = \frac{0.20 - \left(\frac{1}{3}\right)^2}{\left(\frac{0.06}{0.09}\right)^2} = 0.20$$

$$\text{where} \quad A = \frac{\mu}{\mu + \delta} = \frac{0.03}{0.09} = \frac{1}{3} \qquad \bar{a} = \frac{1}{\mu + \delta} = \frac{1}{0.09}$$

Question #15 - Removed

Question #16
Answer: A

$$1000P_{40} = \frac{A_{40}}{\ddot{a}_{40}} = \frac{161.32}{14.8166} = 10.89$$

$$1000\,{}_{20}V_{40} = 1000\left(1 - \frac{\ddot{a}_{60}}{\ddot{a}_{40}}\right) = 1000\left(1 - \frac{11.1454}{14.8166}\right) = 247.78$$

$${}_{21}V = \frac{\left({}_{20}V + 5000P_{40}\right)(1+i) - 5000q_{60}}{P_{60}}$$

$$= \frac{\left(247.78 + (5)(10.89)\right) \times 1.06 - 5000\left(0.01376\right)}{1 - 0.01376} = 255$$

[Note: For this insurance, ${}_{20}V = 1000\,{}_{20}V_{40}$ because retrospectively, this is identical to whole life]

Though it would have taken <u>much</u> longer, you can do this as a prospective reserve. The prospective solution is included for educational purposes, not to suggest it would be suitable under exam time constraints.

$1000P_{40} = 10.89$ as above

$$1000A_{40} + 4000\,{}_{20}E_{40}A^{1}_{60:\overline{5|}} = 1000P_{40} + 5000P_{40} \times {}_{20}E_{40}\,\ddot{a}_{60:\overline{5|}} + \pi\,{}_{20}E_{40} \times {}_{5}E_{60}\,\ddot{a}_{65}$$

where $A^{1}_{60:\overline{5}|} = A_{60} - {}_5E_{60}\ A_{65} = 0.06674$

$$\ddot{a}_{40:\overline{20}|} = \ddot{a}_{40} - {}_{20}E_{40}\ \ddot{a}_{60} = 11.7612$$

$$\ddot{a}_{60:\overline{5}|} = \ddot{a}_{60} - {}_5E_{60}\ \ddot{a}_{65} = 4.3407$$

$$1000(0.16132) + (4000)(0.27414)(0.06674) =$$

$$= (10.89)(11.7612) + (5)(10.89)(0.27414)(4.3407) + \pi(0.27414)(0.68756)(9.8969)$$

$$\pi = \frac{161.32 + 73.18 - 128.08 - 64.79}{1.86544}$$
$$= 22.32$$

Having struggled to solve for π, you could calculate ${}_{20}V$ prospectively then (as above)
calculate ${}_{21}V$ recursively.

$$\begin{aligned}{}_{20}V &= 4000A^{1}_{60:\overline{5}|} + 1000A_{60} - 5000P_{40}\ \ddot{a}_{60:\overline{5}|} - \pi\ {}_5E_{60}\ \ddot{a}_{65}\\ &= (4000)(0.06674) + 369.13 - (5000)(0.01089)(4.3407) - (22.32)(0.68756)(9.8969)\\ &= 247.86 \text{ (minor rounding difference from } 1000\,{}_{20}V_{40})\end{aligned}$$

Or we can continue to ${}_{21}V$ prospectively

$${}_{21}V = 5000A^{1}_{61:\overline{4}|} + 1000\ {}_4E_{61}\ A_{65} - 5000P_{40}\ \ddot{a}_{61:\overline{4}|} - \pi\ {}_4E_{61}\ \ddot{a}_{65}$$

where ${}_4E_{61} = \dfrac{l_{65}}{l_{61}} v^4 = \left(\dfrac{7,533,964}{8,075,403}\right)(0.79209) = 0.73898$

$$\begin{aligned}A^{1}_{61:\overline{4}|} &= A_{61} - {}_4E_{61}\ A_{65} = 0.38279 - 0.73898 \times 0.43980\\ &= 0.05779\end{aligned}$$

$$\begin{aligned}\ddot{a}_{61:\overline{4}|} &= \ddot{a}_{61} - {}_4E_{61}\ \ddot{a}_{65} = 10.9041 - 0.73898 \times 9.8969\\ &= 3.5905\end{aligned}$$

$$\begin{aligned}{}_{21}V &= (5000)(0.05779) + (1000)(0.73898)(0.43980)\\ &\quad - (5)(10.89)(3.5905) - 22.32(0.73898)(9.8969)\\ &= 255\end{aligned}$$

Finally. A moral victory. Under exam conditions since prospective benefit reserves must equal retrospective benefit reserves, calculate whichever is simpler.

Question #17
Answer: C

$$\text{Var}(Z) = {}^2A_{41} - (A_{41})^2$$

$$A_{41} - A_{40} = 0.00822 = A_{41} - (vq_{40} + vp_{40}A_{41})$$
$$= A_{41} - (0.0028/1.05 + (0.9972/1.05)A_{41})$$
$$\Rightarrow A_{41} = 0.21650$$

$${}^2A_{41} - {}^2A_{40} = 0.00433 = {}^2A_{41} - \left(v^2q_{40} + v^2p_{40}\,{}^2A_{41}\right)$$
$$= {}^2A_{41} - (0.0028/1.05^2 + \left(0.9972/1.05^2\right)^2 A_{41})$$
$${}^2A_{41} = 0.07193$$

$$\text{Var}(Z) = 0.07193 - 0.21650^2$$
$$= 0.02544$$

Question #18 - Removed

Question #19 - Removed

Question #20
Answer: D

$$\mu_x^{(\tau)} = \mu_{x+t}^{(1)} + \mu_{x+t}^{(2)}$$

$$= 0.2\mu_{x+t}^{(\tau)} + \mu_{x+t}^{(2)}$$

$$\Rightarrow \mu_{x+t}^{(2)} = 0.8\mu_{x+t}^{(\tau)}$$

$$q_x^{'(1)} = 1 - p_x^{'(1)} = 1 - e^{-\int_0^1 0.2k\, t^2\, dt} = 1 - e^{-0.2\frac{k}{3}} = 0.04$$

$$k/3 \Rightarrow \ln(1-0.04)/(-0.2) = 0.2041$$
$$k = 0.6123$$

$$_2q_x^{(2)} = \int_0^2 {}_tp_x^{(\tau)}\mu_x^{(2)}\ dt = 0.8\int_0^2\ {}_tp_x^{(\tau)}\ \ \mu_x^{(\tau)}(t)\ \ dt$$

$$= 0.8\ {}_2q_x^{(\tau)} = 0.8\left(1 - {}_2p_x^{(\tau)}\right)$$

$$_2p_x^{(\tau)} = e^{-\int_0^2 \mu_x(t)dt}$$

$$= e^{-\int_0^2 kt^2\, dt}$$

$$= e^{\frac{-8k}{3}}$$

$$= e^{\frac{-(8)(0.6123)}{3}}$$

$$= 0.19538$$

$$_2q_x^{(2)} = 0.8(1 - 0.19538) = 0.644$$

Question #21
Answer: A

k	$\min(k,3)$	$f(k)$	$f(k)\times(\min(k,3))$	$f(k)\times[\min(k,3)]^2$
0	0	0.1	0	0
1	1	(0.9)(0.2) = 0.18	0.18	0.18
2	2	(0.72)(0.3) = 0.216	0.432	0.864
3+	3	1-0.1-0.18-0.216 = 0.504	1.512	4.536
			2.124	5.580

$$E[\min(K,3)] = 2.124$$

$$E\left\{\left[\min(K,3)\right]^2\right\} = 5.580$$

$$Var[\min(K,3)] = 5.580 - 2.124^2 = 1.07$$

Note that $E[\min(K,3)]$ is the temporary curtate life expectancy, $e_{x:\overline{3}|}$ if the life is age *x*.

Question #22
Answer: B

$$S_0(60) = \frac{e^{-(0.1)(60)} + e^{-(0.08)(60)}}{2}$$
$$= 0.005354$$

$$S_0(60) = \frac{e^{-(0.1)(61)} + e^{-(0.08)(61)}}{2}$$
$$= 0.00492$$

$$q_{60} = 1 - \frac{0.00492}{0.005354} = 0.081$$

Question #23
Answer: D

Let q_{64} for Michel equal the standard q_{64} plus *c*. We need to solve for *c*.
Recursion formula for a standard insurance:

$$ {}_{20}V_{45} = \left({}_{19}V_{45} + P_{45}\right)\left(1.03\right) - q_{64}\left(1 - {}_{20}V_{45}\right) $$

Recursion formula for Michel's insurance

$$ {}_{20}V_{45} = \left({}_{19}V_{45} + P_{45} + 0.01\right)\left(1.03\right) - \left(q_{64} + c\right)(1 - {}_{20}V_{45}) $$

The values of ${}_{19}V_{45}$ and ${}_{20}V_{45}$ are the same in the two equations because we are told
Michel's benefit reserves are the same as for a standard insurance.

Subtract the second equation from the first to get:

$$ \begin{aligned} 0 &= -\left(1.03\right)(0.01) + c(1 - {}_{20}V_{45}) \\ c &= \frac{(1.03)\left(0.01\right)}{\left(1 - {}_{20}V_{45}\right)} \\ &= \frac{0.0103}{1 - 0.427} \\ &= 0.018 \end{aligned} $$

Question #24
Answer: B

K is the curtate future lifetime for one insured.
L is the loss random variable for one insurance.
L_{AGG} is the aggregate loss random variables for the individual insurances.
σ_{AGG} is the standard deviation of L_{AGG}.
M is the number of policies.

$$L = v^{K+1} - \pi \ddot{a}_{\overline{K+1|}} = \left(1 + \frac{\pi}{d}\right) v^{K+1} - \pi/d$$

$$E[L] = \left(A_x - \pi \ddot{a}_x\right) = A_x - \pi \frac{\left(1 - A_x\right)}{d}$$

$$= 0.24905 - 0.025\left(\frac{0.75095}{0.056604}\right) = -0.082618$$

$$\text{Var}[L] = \left(1 + \frac{\pi}{d}\right)^2 \left({}^2A_x - A_x^2\right) = \left(1 + \frac{0.025}{0.056604}\right)^2 \left(0.09476 - \left(0.24905\right)^2\right) = 0.068034$$

$$E[L_{AGG}] = M\,E[L] = -0.082618M$$

$$\text{Var}[L_{AGG}] = M\ \text{Var}[L] = M(0.068034) \Rightarrow \sigma_{AGG} = 0.260833\sqrt{M}$$

$$\Pr[L_{AGG} > 0] = \left[\frac{L_{AGG} - E[L_{AGG}]}{\sigma_{AGG}} > \frac{-E(L_{AGG})}{\sigma_{AGG}}\right]$$

$$\approx \Pr\left(N(0,1) > \frac{0.082618M}{\sqrt{M}\left(0.260833\right)}\right)$$

$$\Rightarrow 1.645 = \frac{0.082618\sqrt{M}}{0.260833}$$

$$\Rightarrow M = 26.97$$

$\Rightarrow$ minimum number needed = 27

Question #25
Answer: D

Annuity benefit: $Z_1 = 12,000\dfrac{1-v^{K+1}}{d}$ for $K = 0,1,2,...$

Death benefit: $Z_2 = Bv^{K+1}$ for $K = 0,1,2,...$

New benefit: $Z = Z_1 + Z_2 = 12,000\dfrac{1-v^{K+1}}{d} + Bv^{K+1}$

$$= \frac{12,000}{d} + \left(B - \frac{12,000}{d}\right)v^{K+1}$$

$$\mathrm{Var}(Z) = \left(B - \frac{12,000}{d}\right)^2 \mathrm{Var}\left(v^{K+1}\right)$$

$$\mathrm{Var}\left(Z\right) = 0 \text{ if } B = \frac{12,000}{0.08} = 150,000\,.$$

In the first formula for $\mathrm{Var}\left(Z\right)$, we used the formula, valid for any constants *a* and *b* and random variable *X*,

$$\mathrm{Var}\left(a + bX\right) = b^2\mathrm{Var}\left(X\right)$$

Question #26
Answer: A

$$\mu_{x+t:y+t} = \mu_{x+t} + \mu_{y+t} = 0.08 + 0.04 = 0.12$$
$$\bar{A}_x = \mu_{x+t} / \left(\mu_{x+t} + \delta\right) = 0.5714$$
$$\bar{A}_y = \mu_{y+t} / \left(\mu_{y+t} + \delta\right) = 0.4$$
$$\bar{A}_{xy} = \mu_{x+t:y+t} / \left(\mu_{x+t:y+t} + \delta\right) = 0.6667$$
$$\bar{a}_{xy} = 1 / \left(\mu_{x+t:y+t} + \delta\right) = 5.556$$

$$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = 0.5714 + 0.4 - 0.6667 = 0.3047$$

Premium = 0.304762/5.556 = 0.0549

Question #27
Answer: B

$$P_{40} = A_{40} / \ddot{a}_{40} = 0.16132/14.8166 = 0.0108878$$

$$P_{42} = A_{42} / \ddot{a}_{42} = 0.17636/14.5510 = 0.0121201$$

$$a_{45} = \ddot{a}_{45} - 1 = 13.1121$$

$$\begin{aligned} E\left[{}_{3}L \middle| K_{42} \geq 3\right] &= 1000A_{45} - 1000P_{40} - 1000P_{42}\, a_{45} \\ &= 201.20 - 10.89 - (12.12)(13.1121) \\ &= 31.39 \end{aligned}$$

Many similar formulas would work equally well. One possibility would be $1000\,{}_{3}V_{42} + (1000P_{42} - 1000P_{40})$, because prospectively after duration 3, this differs from the normal benefit reserve in that in the next year you collect $1000P_{40}$ instead of $1000P_{42}$.

Question #28
Answer: E

$$E\left[\min(T, 40)\right] = 40 - 0.005(40)^2 = 32$$

$$\begin{aligned} 32 &= \int_0^{40} t\, f(t)\, dt + \int_{40}^{w} 40 f(t)\, dt \\ &= \int_0^{w} t\, f(t)\, dt - \int_{40}^{w} t\, f(t)\, dt + 40(.6) \\ &= 86 - \int_{40}^{w} t f(t)\, dt \end{aligned}$$

$$\int_{40}^{w} t f(t) dt = 54$$

$$\mathring{e}_{40} = \frac{\int_{40}^{w} (t-40)\, f(t)\, dt}{s(40)} = \frac{54 - 40(.6)}{.6} = 50$$

Question #29
Answer: B

$d = 0.05 \Rightarrow v = 0.95$

Step 1 Determine p_x from Kevin's work:

$$608 + 350 v p_x = 1000 v q_x + 1000 v^2 p_x \left(p_{x+1} + q_{x+1} \right)$$
$$608 + 350(0.95) p_x = 1000(0.95)(1 - p_x) + 1000(0.9025) p_x (1)$$
$$608 + 332.5 p_x = 950(1 - p_x) + 902.5 p_x$$
$$p_x = 342/380 = 0.9$$

Step 2 Calculate $1000 P_{x:\overline{2}|}$, as Kira did:

$$608 + 350(0.95)(0.9) = 1000 P_{x:\overline{2}|}\left[1 + (0.95)(0.9)\right]$$
$$1000 P_{x:\overline{2}|} = \frac{[299.25 + 608]}{1.855} = 489.08$$

The first line of Kira's solution is that the expected present value of Kevin's benefit premiums is equal to the expected present value of Kira's, since each must equal the expected present value of benefits. The expected present value of benefits would also have been easy to calculate as
$(1000)(0.95)(0.1) + (1000)\left(0.95^2\right)(0.9) = 907.25$

Question #30
Answer: E

Because no premiums are paid after year 10 for (*x*), ${}_{11}V_x = A_{x+11}$

One of the recursive reserve formulas is ${}_{h+1}V = \dfrac{\left({}_hV + \pi_h\right)(1+i) - b_{h+1} q_{x+h}}{p_{x+h}}$

$${}_{10}V = \frac{(32{,}535 + 2{,}078) \times (1.05) - 100{,}000 \times 0.011}{0.989} = 35{,}635.642$$
$${}_{11}V = \frac{(35{,}635.642 + 0) \times (1.05) - 100{,}000 \times 0.012}{0.988} = 36{,}657.31 = A_{x+11}$$

Question #31
Answer: B

The survival function is $S_0(t) = \left(1 - \frac{t}{\omega}\right)$:

Then,

$$\overset{\circ}{e}_x = \frac{\omega - x}{2} \ and \ {}_t p_x = \left(1 - \frac{t}{\omega - x}\right)$$

$$\overset{\circ}{e}_{45} = \frac{105-45}{2} = 30$$

$$\overset{\circ}{e}_{65} = \frac{105-65}{2} = 20$$

$$\overset{\circ}{e}_{45:65} = \int_0^{40} {}_t p_{45:65} dt = \int_0^{40} \frac{60-t}{60} \times \frac{40-t}{40} dt$$

$$= \frac{1}{60 \times 40}\left(60 \times 40 \times t - \frac{60+40}{2} t^2 + \frac{1}{3} t^3\right)\Big|_0^{40}$$

$$= 15.56$$

$$\overset{\circ}{e}_{\overline{45:65}} = \overset{\circ}{e}_{45} + \overset{\circ}{e}_{65} - \overset{\circ}{e}_{45:65}$$
$$= 30 + 20 - 15.56 = 34$$

In the integral for $\overset{\circ}{e}_{45:65}$, the upper limit is 40 since 65 (and thus the joint status also)
can survive a maximum of 40 years.

Question #32
Answer: E

$$\mu_4 = -S_0{}'(4) / S_0(4)$$

$$= \frac{-\left(-e^4/100\right)}{1-e^4/100}$$

$$= \frac{e^4/100}{1-e^4/100}$$

$$= \frac{e^4}{100-e^4} = 1.202553$$

Question # 33
Answer: A

$$q_x^{(i)} = q_x^{(\tau)}\left[\frac{\ln\ p_x'^{(i)}}{\ln\ p_x^{(\tau)}}\right] = q_x^{(\tau)}\left[\frac{\ln\ e^{-\mu^{(i)}}}{\ln\ e^{-\mu^{(\tau)}}}\right]$$

$$= q_x^{(\tau)} \times \frac{\mu^{(i)}}{\mu^{(\tau)}}$$

$$\mu_x{}^{(\tau)} = \mu_x{}^{(1)} + \mu_x{}^{(2)} + \mu_x{}^{(3)} = 1.5$$

$$q_x^{(\tau)} = 1 - e^{-\mu(\tau)} = 1 - e^{-1.5}$$

=0.7769

$$q_x^{(2)} = \frac{(0.7769)\mu^{(2)}}{\mu^{(\tau)}} = \frac{(0.5)(0.7769)}{1.5}$$

$$= 0.2590$$

Question # 34
Answer: D

$$_{2|2}A_{[60]} = v^3 \quad \times \quad {}_2p_{[60]} \quad \times \quad q_{[60]+2} \quad +$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

pay at end of year 3	live 2 years	then die in year 3

$$+v^4 \quad \times \quad {}_3p_{[60]} \quad \times \quad q_{60+3}$$

pay at end of year 4	live 3 years	then die in year 4

$$= \frac{1}{(1.03)^3}(1-0.09)(1-0.11)(0.13) + \frac{1}{(1.03)^4}(1-0.09)(1-0.11)(1-0.13)(0.15)$$
$$= 0.19$$

Question # 35
Answer: B

$$\bar{a}_x = \bar{a}_{x:\overline{5}|} + {}_5E_x\ \bar{a}_{x+5}$$

$$\bar{a}_{x:\overline{5}|} = \frac{1-e^{-0.07(5)}}{0.07} = 4.219$$, where $0.07 = \mu + \delta$ for $t < 5$

$$_5E_x = e^{-0.07(5)} = 0.705$$

$$\bar{a}_{x+5} = \frac{1}{0.08} = 12.5$$, where $0.08 = \mu + \delta$ for $t \geq 5$

$$\therefore \bar{a}_x = 4.219 + (0.705)(12.5) = 13.03$$

Question #36
Answer: D

$$p_x^{(\tau)} = p'^{(1)}_x p'^{(2)}_x = 0.8(0.7) = 0.56$$

$$q_x^{(1)} = \left[\frac{\ln\left(p'^{(1)}_x\right)}{\ln\left(p_x^{(\tau)}\right)}\right] q_x^{(\tau)}$$ since UDD in double decrement table

$$= \left[\frac{\ln(0.8)}{\ln(0.56)}\right] 0.44$$

$$= 0.1693$$

$$_{0.3}q_{x+0.1}^{(1)} = \frac{0.3q_x^{(1)}}{1-0.1q_x^{(\tau)}} = 0.053$$

To elaborate on the last step:

$$_{0.3}q_{x+0.1}^{(1)} = \frac{\left(\begin{array}{l}\text{Number dying from cause}\\ 1 \text{ between } x+0.1 \text{ and } x+0.4\end{array}\right)}{\text{Number alive at } x+0.1}$$

Since UDD in double decrement,

$$= \frac{l_x^{(\tau)}(0.3)q_x^{(1)}}{l_x^{(\tau)}\left(1-0.1q_x^{(\tau)}\right)}$$

Question #37
Answer: E

The benefit premium is $\frac{1}{\bar{a}_x} - \delta = \frac{1}{12} - 0.04 = 0.04333$

$$_oL = v^T - (0.04333 + 0.0066)\bar{a}_{\overline{T|}} + 0.02 + 0.003\bar{a}_{\overline{T|}}$$

$$= v^T - 0.04693\left(\frac{1-v^T}{\delta}\right) + 0.02$$

$$= v^T\left(1 + \frac{0.04693}{\delta}\right) - \frac{0.04693}{\delta} + 0.02$$

$$Var\left({}_oL\right) = Var\left(v^T\right)\left(1 + \frac{0.04693}{\delta}\right)^2 = 0.1(4.7230) = 0.4723$$

Question #38 - Removed

Question # 39 - Removed

Question # 40
Answer: D

Use Mod to designate values unique to this insured.

$$\ddot{a}_{60} = (1 - A_{60})/d = (1 - 0.36933)/\left[(0.06)/(1.06)\right] = 11.1418$$

$$1000P_{60} = 1000A_{60}/\ddot{a}_{60} = 1000(0.36933/11.1418) = 33.15$$

$$A_{60}^{Mod} = v\left(q_{60}^{Mod} + p_{60}^{Mod}A_{61}\right) = \frac{1}{1.06}\left[0.1376 + (0.8624)(0.383)\right] = 0.44141$$

$$\ddot{a}^{Mod} = \left(1 - A_{60}^{Mod}\right)/d = (1 - 0.44141)/\left[0.06/1.06\right] = 9.8684$$

$$E\left[{}_0L^{Mod}\right] = 1000\left(A_{60}^{Mod} - P_{60}\ddot{a}_{60}^{Mod}\right)$$

$$= 1000\left[0.44141 - 0.03315(9.8684)\right]$$

$$= 114.27$$

Question # 41
Answer: D

The prospective reserve at age 60 per 1 of insurance is A_{60}, since there will be no future premiums. Equating that to the retrospective reserve per 1 of coverage, we have:

$$A_{60} = P_{40} \frac{\ddot{s}_{40:\overline{10}|}}{{}_{10}E_{50}} + P_{50}^{Mod}\ \ddot{s}_{50:\overline{10}|} - {}_{20}k_{40}$$

$$A_{60} = \frac{A_{40}}{\ddot{a}_{40}} \times \frac{\ddot{a}_{40:\overline{10}|}}{{}_{10}E_{40}\ {}_{10}E_{50}} + P_{50}^{Mod} \frac{\ddot{a}_{50:\overline{10}|}}{{}_{10}E_{50}} - \frac{A^1_{40:\overline{20}|}}{{}_{20}E_{40}}$$

$$0.36913 = \frac{0.16132}{14.8166} \times \frac{7.70}{(0.53667)(0.51081)} + P_{50}^{Mod} \frac{7.57}{0.51081} - \frac{0.06}{0.27414}$$

$$0.36913 = 0.30582 + 14.8196\ P_{50}^{Mod} - 0.21887$$

$$1000\ P_{50}^{Mod} = 19.04$$

Alternatively, you could equate the retrospective and prospective reserves at age 50. Your equation would be:

$$A_{50} - P_{50}^{Mod}\ \ddot{a}_{50:\overline{10}|} = \frac{A_{40}}{\ddot{a}_{40}} \times \frac{\ddot{a}_{40:\overline{10}|}}{{}_{10}E_{40}} - \frac{A^1_{40:\overline{10}|}}{{}_{10}E_{40}}$$

where $A^1_{40:\overline{10}|} = A_{40} - {}_{10}E_{40}\ A_{50}$

$$= 0.16132 - (0.53667)(0.24905)$$

$$= 0.02766$$

$$0.24905 - \left(P_{50}^{Mod}\right)(7.57) = \frac{0.16132}{14.8166} \times \frac{7.70}{0.53667} - \frac{0.02766}{0.53667}$$

$$1000 P_{50}^{Mod} = \frac{(1000)(0.14437)}{7.57} = 19.07$$

Alternatively, you could set the expected present value of benefits at age 40 to the expected present value of benefit premiums. The change at age 50 did not change the benefits, only the pattern of paying for them.

$$A_{40} = P_{40}\ \ddot{a}_{40:\overline{10}|} + P_{50}^{Mod}\ {}_{10}E_{40}\ \ddot{a}_{50:\overline{10}|}$$

$$0.16132 = \left(\frac{0.16132}{14.8166}\right)(7.70) + \left(P_{50}^{Mod}\right)(0.53667)(7.57)$$

$$1000P_{50}^{Mod} = \frac{(1000)(0.07748)}{4.0626} = 19.07$$

Question # 42
Answer: A

$$d_x^{(2)} = q_x^{(2)} \times l_x^{(\tau)} = 400$$

$$d_x^{(1)} = 0.45(400) = 180$$

$$q_x'^{(2)} = \frac{d_x^{(2)}}{l_x^{(\tau)} - d_x^{(1)}} = \frac{400}{1000 - 180} = 0.488$$

$$p_x'^{(2)} = 1 - 0.488 = 0.512$$

Note: The UDD assumption was not critical except to have all deaths during the year so that 1000 - 180 lives are subject to decrement 2.

Question #43
Answer: D

Use “age” subscripts for years completed in program. E.g., p_0 applies to a person newly hired (“age” 0).

Let decrement 1 = fail, 2 = resign, 3 = other.

Then $q_0'^{(1)} = 1/4,\ q_1'^{(1)} = 1/5,\ q_2'^{(1)} = 1/3$

$q_0'^{(2)} = 1/5,\ q_1'^{(2)} = 1/3,\ q_2'^{(2)} = 1/8$

$q_0'^{(3)} = 1/10,\ q_1'^{(3)} = 1/9,\ q_2'^{(3)} = 1/4$

This gives $p_0^{(\tau)} = (1-1/4)(1-1/5)(1-1/10) = 0.54$

$p_1^{(\tau)} = (1-1/5)(1-1/3)(1-1/9) = 0.474$

$p_2^{(\tau)} = (1-1/3)(1-1/8)(1-1/4) = 0.438$

So $l_0^{(\tau)} = 200,\ l_1^{(\tau)} = 200\,(0.54) = 108$, and $l_2^{(\tau)} = 108\,(0.474) = 51.2$

$$q_2^{(1)} = \left[\log\, p_2'^{(1)} / \log\, p_2^{(\tau)}\right] q_2^{(\tau)}$$

$$q_2^{(1)} = \left[\log\left(\tfrac{2}{3}\right) / \log(0.438)\right]\left[1-0.438\right]$$

$$= (0.405/0.826)(0.562)$$

$$= 0.276$$

$$d_2^{(1)} = l_2^{(\tau)}\, q_2^{(1)}$$
$$= (51.2)(0.276) = 14$$

Question #44 - Removed

Question #45
Answer: E

For the given life table function: $\overset{\circ}{e}_x = \frac{\omega - x}{2}$

$$
\begin{aligned}
{}_{k|}q_x &= \frac{1}{\omega - x} \\
A_x &= \sum_{k=b}^{\omega-x-1} v^{k+1}\,{}_{k|}q_x = \frac{1}{\omega - x}\sum_{k=b}^{\omega-x-1} v^{k+1} \\
A_x &= \frac{a_{\overline{\omega-x}|}}{\omega - x} \\
\ddot{a}_x &= \frac{1 - A_x}{d}
\end{aligned}
$$

$\overset{\circ}{e}_{50} = 25 \Rightarrow \omega = 100$ for typical annuitants

$\overset{\circ}{e}_y = 15 \Rightarrow y =$ Assumed age = 70

$$
\begin{aligned}
&A_{70} = \frac{a_{\overline{30}|}}{30} = 0.45883 \\
&\ddot{a}_{70} = 9.5607 \\
&500000 = b\,\ddot{a}_{70} \Rightarrow b = 52,297
\end{aligned}
$$

Question #46
Answer: B

$$
\begin{aligned}
{}_{10}E_{30:40} = {}_{10}p_{30}\,{}_{10}p_{40}\,v^{10} &= \left({}_{10}p_{30}v^{10}\right)\left({}_{10}p_{40}v^{10}\right)(1+i)^{10} \\
&= \left({}_{10}E_{30}\right)\left({}_{10}E_{40}\right)(1+i)^{10} \\
&= (0.54733)(0.53667)(1.79085) \\
&= 0.52604
\end{aligned}
$$

The above is only one of many possible ways to evaluate ${}_{10}p_{30}\ {}_{10}p_{40}\,v^{10}$, all of which should give 0.52604

$$
\begin{aligned}
a_{30:40:\overline{10}|} &= a_{30:40} - {}_{10}E_{30:40}\ a_{30+10:40+10} \\
&= (\ddot{a}_{30:40} - 1) - (0.52604)(\ddot{a}_{40:50} - 1) \\
&= (13.2068) - (0.52604)(11.4784) \\
&= 7.1687
\end{aligned}
$$

Question #47
Answer: A

Equivalence Principle, where π is annual benefit premium, gives

$$1000\left(A_{35}+(IA)_{35}\times\pi\right)=\ddot{a}_x\pi$$

$$\pi=\frac{1000A_{35}}{\left(\ddot{a}_{35}-(IA)_{35}\right)}=\frac{1000\times 0.42898}{(11.99143-6.16761)}$$
$$=\frac{428.98}{5.82382}$$
$$=73.66$$

We obtained $\ddot{a}_{35}$ from

$$\ddot{a}_{35}=\frac{1-A_{35}}{d}=\frac{1-0.42898}{0.047619}=11.99143$$

Question #48 - Removed

Question #49
Answer: C

$$\mu_{xy}=\mu_x+\mu_y=0.14$$
$$\bar{A}_x=\bar{A}_y=\frac{\mu}{\mu+\delta}=\frac{0.07}{0.07+0.05}=0.5833$$
$$\bar{A}_{xy}=\frac{\mu_{xy}}{\mu_{xy}+\delta}=\frac{0.14}{0.14+0.05}=\frac{0.14}{0.19}=0.7368 \text{ and } \bar{a}_{xy}=\frac{1}{\mu_{xy}+\delta}=\frac{1}{0.14+0.05}=5.2632$$
$$P=\frac{\bar{A}_{\overline{xy}}}{\bar{a}_{xy}}=\frac{\bar{A}_x+\bar{A}_y-\bar{A}_{xy}}{\bar{a}_{xy}}=\frac{2(0.5833)-0.7368}{5.2632}=0.0817$$

Question #50
Answer: E

$$\left({}_{20}V + P_{20}\right)\left(1+i\right) - q_{40}\left(1 - {}_{21}V\right) = {}_{21}V$$

$$\left(0.49 + 0.01\right)\left(1+i\right) - 0.022\left(1 - 0.545\right) = 0.545$$

$$\left(1+i\right) = \frac{\left(0.545\right)\left(1-0.022\right) + 0.022}{0.50}$$

$$= 1.11$$

$$\left({}_{21}V + P_{20}\right)\left(1+i\right) - q_{41}\left(1 - {}_{22}V\right) = {}_{22}V$$

$$\left(0.545{+}.01\right)\left(1.11\right) - q_{41}\left(1 - 0.605\right) = 0.605$$

$$q_{41} = \frac{0.61605 - 0.605}{0.395}$$

$$= 0.028$$

Question #51
Answer: E

$$1000\ P_{60} = 1000\ A_{60}\ /\ \ddot{a}_{60}$$

$$= 1000\ v\left(q_{60} + p_{60} A_{61}\right) / \left(1 + p_{60}\ v\ \ddot{a}_{61}\right)$$

$$= 1000\left(q_{60} + p_{60}\ \ A_{61}\right) / \left(1.06 + p_{60}\ \ddot{a}_{61}\right)$$

$$= \left(15 + \left(0.985\right)\left(382.79\right)\right) / \left(1.06 + \left(0.985\right)\left(10.9041\right)\right) = 33.22$$

Question #52 - Removed

Question #53
Answer: E

$g = -\ln(0.96) = 0.04082$

$\mu^{02}_{x+t:y+t} = 0.04082 - 0.01 = 0.03082$

$h = -\ln(0.97) = 0.03046$

$\mu^{01}_{x+t:y+t} = 0.03046 - 0.01 = 0.02046$

$${}_5p_{xy} = {}_5p^{00}_{xy} = \exp\left(-\int_0^5 \mu^{01}_{x+t:y+t} + \mu^{02}_{x+t:y+t} + \mu^{03}_{x+t:y+t} dt\right) = e^{-5(0.06128)} = 0.736$$

Question #54 - Removed

Question #55
Answer: B

$l_x = \omega - x = 105 - x$

$\Rightarrow {}_tp_{45} = l_{45+t} / l_{45} = (60 - t)/60$

Let K be the curtate future lifetime of (45). Then the sum of the payments is 0 if $K \le 19$ and is $K - 19$ if $K \ge 20$.

$${}_{20|}\ddot{a}_{45} = \sum_{K=20}^{60} 1 \times \left(\frac{60-K}{60}\right) \times 1$$

$$= \frac{(40 + 39 + \ldots + 1)}{60} = \frac{(40)(41)}{2(60)} = 13.6\overline{6}$$

Hence,

$$\text{Prob}\left(K - 19 > 13.6\overline{6}\right) = \text{Prob}\left(K > 32.6\overline{6}\right)$$

$$= \text{Prob}(K \ge 33) \text{ since } K \text{ is an integer}$$

$$= \text{Prob}(T \ge 33)$$

$$= {}_{33}p_{45} = \frac{l_{78}}{l_{45}} = \frac{27}{60}$$

$$= 0.450$$

Question #56
Answer: C

$$^2\bar{A}_x = \frac{\mu}{\mu + 2\delta} = 0.25 \rightarrow \mu = 0.04$$

$$\bar{A}_x = \frac{\mu}{\mu + \delta} = 0.4$$

$$\left(\bar{I}\bar{A}\right)_x = \int_0^\infty {}_{s|}\bar{A}_x \, ds$$

$$\int_0^\infty {}_sE_x \, \bar{A}_x \, ds$$

$$= \int_0^\infty \left(e^{-0.1s}\right)(0.4)\, ds$$

$$= (0.4)\left(\frac{-e^{-0.1s}}{0.1}\right)\Bigg|_0^\infty = \frac{0.4}{0.1} = 4$$

Alternatively, using a more fundamental formula but requiring more difficult integration.

$$\left(\bar{I}\bar{A}\right)_x = \int_0^\infty t \; {}_tp_x \, \mu_x(t) e^{-\delta t} dt$$

$$= \int_0^\infty t \, e^{-0.04t}(0.04)e^{-0.06t} dt$$

$$= 0.04\int_0^\infty t \, e^{-0.1t} dt$$

(integration by parts, not shown)

$$= 0.04\left(\frac{-t}{0.1} - \frac{1}{0.01}\right)e^{-0.1t}\Big|_0^\infty$$

$$= \frac{0.04}{0.01} = 4$$

Question #57
Answer: E

Subscripts A and B here distinguish between the tools and do not represent ages.

We have to find $\overset{o}{e}_{\overline{AB}}$

$$\overset{o}{e}_A = \int_0^{10}\left(1-\frac{t}{10}\right)dt = t-\frac{t^2}{20}\Bigg|_0^{10} = 10-5=5$$

$$\overset{o}{e}_B = \int_0^{7}\left(1-\frac{t}{7}\right)dt = t-\frac{t^2}{14}\Bigg|_0^{7} = 49-\frac{49}{14}=3.5$$

$$\overset{o}{e}_{AB} = \int_o^{7}\left(1-\frac{t}{7}\right)\left(1-\frac{t}{10}\right)dt = \int_0^{7}\left(1-\frac{t}{10}-\frac{t}{7}+\frac{t^2}{70}\right)dt$$

$$= t-\frac{t^2}{20}-\frac{t^2}{14}+\frac{t^3}{210}\Bigg|_0^{7}$$

$$= 7-\frac{49}{20}-\frac{49}{14}+\frac{343}{210}=2.683$$

$$\overset{o}{e}_{\overline{AB}} = \overset{o}{e}_A+\overset{o}{e}_B-\overset{o}{e}_{AB}$$

$$=5+3.5-2.683=5.817$$

Question #58
Answer: A

$$\mu_{x+t}^{(\tau)} = 0.100+0.004=0.104$$

$${}_t p_x^{(\tau)} = e^{-0.104t}$$

Expected present value (EPV) = EPV for cause 1 + EPV for cause 2.

$$2000\int_0^5 e^{-0.04t}e^{-0.104t}(0.100)dt+500{,}000\int_0^5 e^{-0.04t}\,e^{-0.104t}(0.400)dt$$

$$=\left(2000(0.10)+500,000(0.004)\right)\int_0^5 e^{-0.144t}dt=\frac{2200}{0.144}\left(1-e^{-0.144(5)}\right)=7841$$

Question #59
Answer: A

$R = 1 - p_x = q_x$

$S = 1 - p_x \times e^{(-k)}$ since $e^{-\int_0^1 (\mu_{x+t} + k)dt} = e^{-\int_0^1 \mu_{x+t}\,dt - \int_0^1 k\,dt}$

$$= e^{-\int_0^1 \mu_{x+t}\,dt} e^{-\int_0^1 k\,dt}$$

So $S = 0.75R \Rightarrow 1 - p_x \times e^{-k} = 0.75q_x$

$$e^{-k} = \frac{1 - 0.75q_x}{p_x}$$

$$e^{k} = \frac{p_x}{1 - 0.75q_x} = \frac{1 - q_x}{1 - 0.75q_x}$$

$$k = \ln\left[\frac{1 - q_x}{1 - 0.75q_x}\right]$$

Question #60
Answer: C

$A_{60} = 0.36913 \qquad d = 0.05660$

${}^2A_{60} = 0.17741$

and $\sqrt{{}^2A_{60} - A_{60}^2} = 0.202862$

Expected Loss on one policy is $E\left[L(\pi)\right] = \left(100,000 + \frac{\pi}{d}\right)A_{60} - \frac{\pi}{d}$

Variance on one policy is $\text{Var}\left[L(\pi)\right] = \left(100,000 + \frac{\pi}{d}\right)^2 \left({}^2A_{60} - A_{60}^2\right)$

On the 10000 lives,

$E[S] = 10,000E\left[L(\pi)\right]$ and $\text{Var}[S] = 10,000\ \text{Var}\left[L(\pi)\right]$

The π is such that $0 - E[S]/\sqrt{\text{Var}[S]} = 2.326$ since $\Phi(2.326) = 0.99$

$$\frac{10,000\left(\frac{\pi}{d} - \left(100,000 + \frac{\pi}{d}\right)A_{60}\right)}{100\left(100,000 + \frac{\pi}{d}\right)\sqrt{{}^2A_{60} - A_{60}^2}} = 2.326$$

$$\frac{100\left(\frac{\pi}{d} - \left(100,000 + \frac{\pi}{d}\right)\right)(0.36913)}{\left(100,000 + \frac{\pi}{d}\right)(0.202862)} = 2.326$$

$$\frac{0.63087\frac{\pi}{d} - 36913}{100,000 + \frac{\pi}{d}} = 0.004719$$

$$0.63087\frac{\pi}{d} - 36913 = 471.9 = 0.004719\frac{\pi}{d}$$

$$\frac{\pi}{d} = \frac{36913 + 471.9}{0.63087 - 0.004719}$$

$$= 59706$$

$$\pi = 59706 \times d = 3379$$

Question #61
Answer: C

$$ {}_1V = \left({}_0V + \pi\right)(1+i) - \left(1000 + {}_1V - {}_1V\right) \times q_{75} $$
$$ = 1.05\pi - 1000 q_{75} $$

Similarly,

$$ {}_2V = \left({}_1V + \pi\right) \times 1.05 - 1000 q_{76} $$
$$ {}_3V = \left({}_2V + \pi\right) \times 1.05 - 1000 q_{77} $$
$$ 1000 = {}_3V = \left(1.05^3\pi + 1.05^2 \cdot \pi + 1.05\pi\right) - 1000 \times q_{75} \times 1.05^2 - 1000 \times 1.05 \times q_{76} - 1000 \times q_{77} \quad * $$
$$ \pi = \frac{1000 + 1000\left(1.05^2 q_{75} + 1.05 q_{76} + q_{77}\right)}{(1.05)^3 + (1.05)^2 + 1.05} $$
$$ = \frac{1000^x\left(1 + 1.05^2 \times 0.05169 + 1.05 \times 0.05647 + 0.06168\right)}{3.310125} $$
$$ = \frac{1000 \times 1.17796}{3.310125} = 355.87 $$

* This equation is an algebraic manipulation of the three equations in three unknowns $\left({}_1V, {}_2V, \pi\right)$. One method – usually effective in problems where benefit = stated amount plus reserve, is to multiply the ${}_1V$ equation by 1.05^2, the ${}_2V$ equation by 1.05, and add those two to the ${}_3V$ equation: in the result, you can cancel out the ${}_1V$, and ${}_2V$ terms. Or you can substitute the ${}_1V$ equation into the ${}_2V$ equation, giving ${}_2V$ in terms of π, and then substitute that into the ${}_3V$ equation.

Question #62
Answer: D

$$ \bar{A}^{\,1}_{28:\overline{2}|} = \int_0^2 e^{-\delta t}\, \tfrac{1}{72}\, dt $$
$$ = \frac{1}{72\delta}\left(1 - e^{-2\delta}\right) = 0.02622 \text{ since } \delta = \ln(1.06) = 0.05827 $$
$$ \ddot{a}_{28:\overline{2}|} = 1 + v\left(\frac{71}{72}\right) = 1.9303 $$
$$ {}_3V = 500{,}000\,\bar{A}^{\,1}_{28:\overline{2}|} - 6643\,\ddot{a}_{28:\overline{2}|} $$

= 287

Question #63
Answer: D

Let $\overline{A}_x$ and $\overline{a}_x$ be calculated with μ_{x+t} and $\delta = 0.06$
Let $\overline{A}_x^{\,*}$ and $\overline{a}_x^{\,*}$ be the corresponding values with μ_{x+t} increased by 0.03 and δ decreased by 0.03

$$\overline{a}_x = \frac{1-\overline{A}_x}{\delta} = \frac{0.4}{0.06} = 6.667$$

$$\overline{a}_x^{\,*} = \overline{a}_x$$

$$\left[\text{Proof: } \overline{a}_x^{\,*} = \int_0^\infty e^{-\int_0^t (\mu_{x+s}+0.03)ds} e^{-0.03t} dt\right.$$

$$= \int_0^\infty e^{-\int_0^t \mu_{x+s}\, ds} e^{-0.03t} e^{-0.03t} dt$$

$$= \int_0^\infty e^{-\int_0^t \mu_{x+s}\, ds} e^{-0.06t} dt$$

$$\left. = \overline{a}_x \right]$$

$$\begin{aligned} \overline{A}_x^{\,*} = 1 - 0.03\overline{a}_x^{\,*} &= 1 - 0.03\overline{a}_x \\ &= 1-(0.03)(6.667) \\ &= 0.8 \end{aligned}$$

Question #64
Answer: A

	bulb ages				
Year	0	1	2	3	# replaced
0	10000	0	0	0	-
1	1000	9000	0	0	1000
2	100+2700	900	6300	0	2800
3	280+270+3150				3700

The diagonals represent bulbs that don't burn out.
E.g., of the initial 10,000, (10,000) (1-0.1) = 9000 reach year 1.
(9000) (1-0.3) = 6300 of those reach year 2.

Replacement bulbs are new, so they start at age 0.
At the end of year 1, that's (10,000) (0.1) = 1000
At the end of 2, it's (9000) (0.3) + (1000) (0.1) = 2700 + 100
At the end of 3, it's (2800) (0.1) + (900) (0.3) + (6300) (0.5) = 3700

Expected present value $= \frac{1000}{1.05} + \frac{2800}{1.05^2} + \frac{3700}{1.05^3}$
$= 6688$

Question #65
Answer: E

$$\overset{\circ}{e}_{25:\overline{25}|} = \int_0^{15} {}_t p_{25} dt + {}_{15}p_{25} \int_0^{10} {}_t p_{40} dt$$

$$= \int_0^{15} e^{-.04t} dt + \left(e^{-\int_0^{15} .04\, ds} \right) \int_0^{10} e^{-.05t} dt$$

$$= \frac{1}{.04}\left(1 - e^{-.60}\right) + e^{-.60}\left[\frac{1}{.05}\left(1 - e^{-.50}\right)\right]$$

$$= 11.2797 + 4.3187$$

$$= 15.60$$

Question #66
Answer: C

$$_5 p_{[60]+1} =$$
$$\left(1-q_{[60]+1}\right)\left(1-q_{[60]+2}\right)\left(1-q_{63}\right)\left(1-q_{64}\right)\left(1-q_{65}\right)$$
$$=(0.89)(0.87)(0.85)(0.84)(0.83)$$
$$=0.4589$$

Question # 67
Answer: E

$$12.50 = \bar{a}_x = \frac{1}{\mu+\delta} \Rightarrow \mu+\delta = 0.08 \Rightarrow \mu = \delta = 0.04$$

$$\bar{A}_x = \frac{\mu}{\mu+\delta} = 0.5$$
$$^2\bar{A}_x = \frac{\mu}{\mu+2\delta} = \frac{1}{3}$$

$$\operatorname{Var}\left(\bar{a}_{\overline{T}|}\right) = \frac{^2\bar{A}_x - \bar{A}_x^2}{\delta^2}$$
$$= \frac{\frac{1}{3}-\frac{1}{4}}{0.0016} = 52.083$$

S.D. $= \sqrt{52.083} = 7.217$

Question # 68
Answer: D

$$v = 0.90 \Rightarrow d = 0.10$$
$$A_x = 1 - d\ddot{a}_x = 1 - (0.10)(5) = 0.5$$

Benefit premium $\pi = \dfrac{5000A_x - 5000vq_x}{\ddot{a}_x}$

$$= \frac{(5000)(0.5) - 5000(0.90)(0.05)}{5} = 455$$

$$\text{10th benefit reserve for fully discrete whole life} = 1 - \frac{\ddot{a}_{x+10}}{\ddot{a}_x}$$

$$0.2 = 1 - \frac{\ddot{a}_{x+10}}{5} \Rightarrow \ddot{a}_{x+10} = 4$$

$$A_{x+10} = 1 - d\ddot{a}_{x+10} = 1 - (0.10)(4) = 0.6$$
$${}_{10}V = 5000A_{x+10} - \pi\ddot{a}_{x+10} = (5000)(0.6) - (455)(4) = 1180$$

Question #69
Answer: D

v is the lowest premium to ensure a zero % chance of loss in year 1 (The present value of the payment upon death is *v*, so you must collect at least *v* to avoid a loss should death occur).
Thus *v* = 0.95.

$$\text{E}(\text{Z}) = vq_x + v^2 p_x q_{x+1} = 0.95 \times 0.25 + (0.95)^2 \times 0.75 \times 0.2$$
$$= 0.3729$$
$$\text{E}(\text{Z}^2) = v^2 q_x + v^4 p_x\, q_{x+1} = (0.95)^2 \times 0.25 + (0.95)^4 \times 0.75 \times 0.2$$
$$= 0.3478$$

$$\text{Var}(\text{Z}) = \text{E}(\text{Z}^2) - (\text{E}(\text{Z}))^2 = 0.3478 - (0.3729)^2 = 0.21$$

Question #70
Answer: D

Expected present value (EPV) of future benefits =

$$= (0.005\times 2000+0.04\times 1000)/1.06+(1-0.005-0.04)(0.008\times 2000+0.06\times 1000)/1.06^2$$
$$= 47.17+64.60$$
$$= 111.77$$

EPV of future premiums $=\left[1+(1-0.005-0.04)/1.06\right]50$

$$= (1.9009)(50)$$
$$= 95.05$$

$$E\left[{}_1L \middle| K_{55}\geq 1\right] = 111.77-95.05 = 16.72$$

Question #71 - Removed

Question #72
Answer: A

Let Z be the present value random variable for one life.
Let S be the present value random variable for the 100 lives.

$$E(Z) = 10\int_5^{\infty} e^{\delta t}e^{\mu t}\mu\, dt$$
$$= 10\frac{\mu}{\delta+\mu}e^{-(\delta+\mu)5}$$
$$= 2.426$$
$$E(Z^2) = 10^2\left(\frac{\mu}{2\delta+\mu}\right)e^{-(2\delta+\mu)5}$$
$$= 10^2\left(\frac{0.04}{0.16}\right)\left(e^{-0.8}\right) = 11.233$$
$$Var(Z) = E(Z^2)-\left(E(Z)\right)^2$$
$$= 11.233-2.426^2$$
$$= 5.348$$

$$\mathrm{E(S)} = 100\,\mathrm{E(Z)} = 242.6$$
$$\mathrm{Var(S)} = 100\ \mathrm{Var(Z)} = 534.8$$
$$\frac{F-242.6}{\sqrt{534.8}} = 1.645 \rightarrow F = 281$$

Question #73
Answer: D

Prob{only 1 survives} = 1-Prob{both survive}-Prob{neither survives}

$$= 1 - {}_3p_{50} \times {}_3p_{[50]} - \left(1 - {}_3p_{50}\right)\left(1 - {}_3p_{[50]}\right)$$
$$= 1 - \underbrace{(0.9713)(0.9698)(0.9682)}_{=0.912012}\underbrace{(0.9849)(0.9819)(0.9682)}_{0.936320} - (1 - 0.912012)(1 - 0.93632)$$
$$= 0.140461$$

Question # 74 - Removed

Question #75 - Removed

Question # 76
Answer: C

This solution applies the equivalence principle to each life. Applying the equivalence principle to the 100 life group just multiplies both sides of the first equation by 100, producing the same result for *P*.

$$EPV\left(\text{Prems}\right) = P = EPV\left(\text{Benefits}\right) = 10q_{70}v + 10\,p_{70}q_{71}v^2 + Pp_{70}\,p_{71}v^2$$
$$P = \frac{(10)(0.03318)}{1.08} + \frac{(10)(1-0.03318)(0.03626)}{1.08^2} + \frac{P(1-0.03318)(1-0.03626)}{1.08^2}$$
$$= 0.3072 + 0.3006 + 0.7988P$$
$$P = \frac{0.6078}{0.2012} = 3.02$$

(*EPV* above means Expected Present Value).

Question #77
Answer: E

Level benefit premiums can be split into two pieces: one piece to provide term insurance for *n* years; one to fund the reserve for those who survive.

Then,

$$P_x = P^{1}_{x:\overline{n|}} + P_{x:\overline{n|}}^{\ \ 1}\ {}_nV$$

And plug in to get

$$0.090 = P^{1}_{x:\overline{n|}} + (0.00864)(0.563)$$

$$P^{1}_{x:\overline{n|}} = 0.0851$$

Another approach is to think in terms of retrospective reserves. Here is one such solution:

$$\begin{aligned}{}_nV &= \left(P_x - P^{1}_{x:\overline{n|}}\right)\ddot{s}_{x:\overline{n|}} \\ &= \left(P_x - P^{1}_{x:\overline{n|}}\right)\frac{\ddot{a}_{x:\overline{n|}}}{{}_nE_x} \\ &= \left(P_x - P^{1}_{x:\overline{n|}}\right)\frac{\ddot{a}_{x:\overline{n|}}}{P_{x:\overline{n|}}^{\ \ 1}\ddot{a}_{x:\overline{n|}}} \\ &= \frac{\left(P_x - P^{1}_{x:\overline{n|}}\right)}{\left(P_{x:\overline{n|}}^{\ \ 1}\right)}\end{aligned}$$

$$0.563 = \left(0.090 - P^{1}_{x:\overline{n|}}\right) / \ 0.00864$$

$$\begin{aligned}P^{1}_{x:\overline{n|}} &= 0.090 - (0.00864)(0.563) \\ &= 0.0851\end{aligned}$$

Question #78
Answer: A

$\delta = \ln(1.05) = 0.04879$

$$\bar{A}_x = \int_0^{\omega-x} {}_t p_x \mu_{x+t} e^{-\delta t} dt$$
$$= \int_0^{\omega-x} \frac{1}{\omega - x} e^{-\delta t} dt \text{ for the given mortality function}$$
$$= \frac{1}{\omega - x} \bar{a}_{\overline{\omega-x}|}$$

From here, many formulas for the reserve could be used. One approach is:

Since

$$\bar{A}_{50} = \frac{\bar{a}_{\overline{50}|}}{50} = \frac{18.71}{50} = 0.3742 \text{ so } \bar{a}_{50} = \left(\frac{1-\bar{A}_{50}}{\delta}\right) = 12.83$$
$$\bar{A}_{40} = \frac{\bar{a}_{\overline{60}|}}{60} = \frac{19.40}{60} = 0.3233 \text{ so } \bar{a}_{40} = \left(\frac{1-\bar{A}_{40}}{\delta}\right) = 13.87$$
$$\bar{P}\left(\bar{A}_{40}\right) = \frac{0.3233}{13.87} = 0.02331$$
$$\text{reserve} = \left[\bar{A}_{50} - \bar{P}\left(\bar{A}_{40}\right)\bar{a}_{50}\right] = \left[0.3742 - (0.02331)(12.83)\right] = 0.0751.$$

Question #79
Answer: D

$$\bar{A}_x = E\left[v^{T_x}\right] = E\left[v^{T_x} \mid NS\right] \times \text{Prob}(\text{NS}) + E\left[v^{T_x} \mid S\right] \times \text{Prob}(\text{S})$$
$$= \left(\frac{0.03}{0.03+0.08}\right) \times 0.70 + \left(\frac{0.6}{0.06+0.08}\right) \times 0.30$$
$$= 0.3195$$

Similarly, ${}^2\bar{A}_x = \left(\frac{0.03}{0.03+0.16}\right) \times 0.70 + \left(\frac{0.06}{0.06+0.16}\right) \times 0.30 = 0.1923.$

$$\text{Var}\left(\bar{a}_{\overline{T(x)}|}\right) = \frac{{}^2\bar{A}_x - \bar{A}_x^2}{\delta^2} = \frac{0.1923 - 0.3195^2}{0.08^2} = 14.1.$$

Question #80
Answer: B

$_{2|}q_{\overline{80:84}} = {}_{2|}q_{80} + {}_{2|}q_{84} - {}_{2|}q_{80:84}$

$= 0.5\times 0.4\times(1-0.6)+0.2\times 0.15\times(1-0.1)$
= 0.10136

Using new p_{82} value of 0.3

$0.5\times 0.4\times(1-0.3)+0.2\times 0.15\times(1-0.1)$
= 0.16118

Change = 0.16118 – 0.10136 = 0.06

Alternatively,
$_2p_{80} = 0.5\times 0.4 = 0.20$
$_3p_{80} = {}_2p_{80}\times 0.6 = 0.12$
$_2p_{84} = 0.20\times 0.15 = 0.03$
$_3p_{84} = {}_2p_{84}\times 0.10 = 0.003$
$_2p_{\overline{80:84}} = {}_2p_{80} + {}_2p_{84} - {}_2p_{80}\ {}_2p_{84}$ since independent
$= 0.20+0.03-(0.20)(0.03) = 0.224$
$_3p_{\overline{80:84}} = {}_3p_{80} + {}_3p_{84} - {}_3p_{80}\ {}_3p_{84}$
$= 0.12+0.003-(0.12)(0.003) = 0.12264$
$_{2|}q_{\overline{80:84}} = {}_2p_{\overline{80:84}} - {}_3p_{\overline{80:84}}$
$= 0.224-0.12264 = 0.10136$

Revised
$_3p_{80} = 0.20\times 0.30 = 0.06$
$_3p_{\overline{80:84}} = 0.06+0.003-(0.06)(0.003)$
$= 0.06282$
$_{2|}q_{\overline{80:84}} = 0.224-0.06282 = 0.16118$
change $= 0.16118-0.10136 = 0.06$

Question #81 - Removed

Question #82
Answer: A

$$\begin{aligned} {}_5p_{50}^{(\tau)} &= {}_5p_{50}'^{(1)}\,{}_5p_{50}'^{(2)} \\ &= \left(\frac{100-55}{100-50}\right)e^{-(0.05)(5)} \\ &= (0.9)(0.7788) = 0.7009 \end{aligned}$$

Similarly

$$\begin{aligned} {}_{10}p_{50}^{(\tau)} &= \left(\frac{100-60}{100-50}\right)e^{-(0.05)(10)} \\ &= (0.8)(0.6065) = 0.4852 \end{aligned}$$

$$\begin{aligned} {}_{5|5}q_{50}^{(\tau)} = {}_5p_{50}^{(\tau)} - {}_{10}p_{50}^{(\tau)} &= 0.7009 - 0.4852 \\ &= 0.2157 \end{aligned}$$

Question #83
Answer: C

Only decrement 1 operates before $t = 0.7$

${}_{0.7}q_{40}'^{(1)} = (0.7)\,q_{40}'^{(1)} = (0.7)(0.10) = 0.07$ since UDD

Probability of reaching $t = 0.7$ is 1-0.07 = 0.93

Decrement 2 operates only at $t = 0.7$, eliminating 0.125 of those who reached 0.7

$q_{40}^{(2)} = (0.93)(0.125) = 0.11625$

Question #84
Answer: C

$$\pi\left(1+{}_2p_{80}v^2\right)=1000A_{80}+\frac{\pi v q_{80}}{2}+\frac{\pi v^3\,{}_2p_{80}q_{82}}{2}$$

$$\pi\left(1+\frac{0.83910}{1.06^2}\right)=665.75+\pi\left(\frac{0.08030}{2(1.06)}+\frac{0.83910\times 0.09561}{2(1.06)^3}\right)$$

$$\pi(1.74680)=665.75+\pi(0.07156)$$
$$\pi(1.67524)=665.75$$
$$\pi=397.41$$

Where ${}_2p_{80}=\dfrac{3{,}284{,}542}{3{,}914{,}365}=0.83910$

Or ${}_2p_{80}=(1-0.08030)(1-0.08764)=0.83910$

Question #85
Answer: E

At issue, expected present value (EPV) of benefits

$$=\int_0^\infty b_t v^t\,{}_tp_{65}\,\mu_{65+t}\,dt$$
$$=\int_0^\infty 1000\left(e^{0.04t}\right)\left(e^{-0.04t}\right){}_tp_{65}\,\mu_{65}(t)\,dt$$
$$=1000\int_0^\infty {}_tp_{65}\,\mu_{65}(t)\,dt=1000\,{}_\infty q_{65}=1000$$

EPV of premiums $=\pi\,\bar{a}_{65}=\pi\left(\dfrac{1}{0.04+0.02}\right)=16.667\pi$

Benefit premium $\pi=1000/16.667=60$

$${}_2\bar{V}=\int_0^\infty b_{2+u}\,v^u\,{}_up_{67}\,\mu_{65}(2+u)du-\pi\,\bar{a}_{67}$$
$$=\int_0^\infty 1000\,e^{0.04(2+u)}e^{-0.04u}\,{}_up_{67}\,\mu_{65}(2+u)du-(60)(16.667)$$
$$=1000e^{0.08}\int_0^\infty {}_up_{67}\mu_{65}(2+u)\,du-1000$$
$$=1083.29\,{}_\infty q_{67}-1000=1083.29-1000=83.29$$

Question #86
Answer: B

(1) $a_{x:\overline{20}|} = \ddot{a}_{x:\overline{20}|} - 1 + {}_{20}E_x$

(2) $\ddot{a}_{x:\overline{20}|} = \dfrac{1 - A_{x:\overline{20}|}}{d}$

(3) $A_{x:\overline{20}|} = A^1_{x:\overline{20}|} + A_{x:\overset{1}{\overline{20}|}}$

(4) $A_x = A^1_{x:\overline{20}|} + {}_{20}E_x\ A_{x+20}$

$$0.28 = A^1_{x:\overline{20}|} + (0.25)(0.40)$$
$$A^1_{x:\overline{20}|} = 0.18$$

Now plug into (3): $A_{x:\overline{20}|} = 0.18 + 0.25 = 0.43$

Now plug into (2): $\ddot{a}_{x:\overline{20}|} = \dfrac{1-0.43}{(0.05/1.05)} = 11.97$

Now plug into (1): $a_{x:\overline{20}|} = 11.97 - 1 + 0.25 = 11.22$

Question #87 - Removed

Question #88
Answer: B

$$e_x = p_x + p_x e_{x+1} \Rightarrow p_x = \frac{e_x}{1+e_{x+1}} = \frac{8.83}{9.29} = 0.95048$$
$$\ddot{a}_x = 1 + vp_x + v^2\ {}_2p_x + ...$$
$$\ddot{a}_{\overline{x:2}|} = 1 + v + v^2\ {}_2p_x + ...$$
$$\ddot{a}_{\overline{x:2}|} - \ddot{a}_x = vq_x = 5.6459 - 5.60 = 0.0459$$

$$v(1-0.95048) = 0.0459$$
$$v = 0.9269$$
$$i = \frac{1}{v} - 1 = 0.0789$$

Question #89 - Removed

Question #90 – Removed

Question #91
Answer: E

$$\mu_{60}^{M} = \frac{1}{75-60} = \frac{1}{15}$$

$$\mu_{60}^{F} = \frac{1}{\omega-60} = \frac{1}{15}\times\frac{3}{5} = \frac{1}{25} \Rightarrow \omega = 85$$

$${}_t p_{65}^{M} = 1-\frac{t}{10}$$

$${}_t p_{60}^{F} = 1-\frac{t}{25}$$

Let *x* denote the male and *y* denote the female.

$$\mathring{e}_x = 5 \ \left(\text{mean for uniform distribution over } (0,10)\right)$$

$$\mathring{e}_y = 12.5 \ \left(\text{mean for uniform distribution over } (0,25)\right)$$

$$\mathring{e}_{xy} = \int_0^{10}\left(1-\frac{t}{10}\right)\left(1-\frac{t}{25}\right)dt$$

$$= \int_0^{10}\left(1-\frac{7}{50}t+\frac{t^2}{250}\right)dt$$

$$= \left(t-\frac{7}{100}t^2+\frac{t^3}{750}\right)\Bigg|_0^{10} = 10-\frac{7}{100}\times100+\frac{1000}{750}$$

$$= 10-7+\frac{4}{3} = \frac{13}{3}$$

$$\mathring{e}_{\overline{xy}} = \mathring{e}_x+\mathring{e}_y-\mathring{e}_{xy} = 5+\frac{25}{2}-\frac{13}{3} = \frac{30+75-26}{6} = 13.17$$

Question #92
Answer: B

$$\bar{A}_x = \frac{\mu}{\mu+\delta} = \frac{1}{3}$$
$$^2\bar{A}_x = \frac{\mu}{\mu+2\delta} = \frac{1}{5}$$

$$\bar{P}\left(\bar{A}_x\right) = \mu = 0.04$$

$$\begin{aligned}\text{Var}(L) &= \left(1+\frac{\bar{P}\left(\bar{A}_x\right)}{\delta}\right)^2\left({}^2\bar{A}_x - \bar{A}_x^2\right)\\ &= \left(1+\frac{0.04}{0.08}\right)^2\left(\frac{1}{5}-\left(\frac{1}{3}\right)^2\right)\\ &= \left(\frac{3}{2}\right)^2\left(\frac{4}{45}\right)\\ &= \frac{1}{5}\end{aligned}$$

Question #93
Answer: A

Let π be the benefit premium
Let ${}_kV$ denote the benefit reserve at the end of year *k*.
For any $n, \left({}_nV+\pi\right)(1+i) = \left(q_{25+n} \times {}_{n+1}V + p_{25+n} \times {}_{n+1}V\right)$
$$= {}_{n+1}V$$
Thus ${}_1V = \left({}_0V+\pi\right)(1+i)$

$${}_2V = \left({}_1V+\pi\right)(1+i) = \left(\pi(1+i)+\pi\right)(1+i) = \pi\ddot{s}_{\overline{2|}}$$
$${}_3V = \left({}_2V+\pi\right)(1+i) = \left(\pi\ddot{s}_{\overline{2|}}+\pi\right)(1+i) = \pi\ddot{s}_{\overline{3|}}$$

By induction (proof omitted)
$${}_nV = \pi\ddot{s}_{\overline{n|}}$$
For $n = 35$, ${}_nV = \ddot{a}_{60}$ (expected present value of future benefits; there are no future premiums)
$$\ddot{a}_{60} = \pi\,\ddot{s}_{\overline{35|}}$$
$$\pi = \frac{\ddot{a}_{60}}{\ddot{s}_{\overline{35|}}} \quad \text{For} \quad n = 20, \quad {}_{20}V = \pi\,\ddot{s}_{\overline{20|}} = \left(\frac{\ddot{a}_{60}}{\ddot{s}_{\overline{35|}}}\right)\ddot{s}_{\overline{20|}}$$

Alternatively, as above

$\left({}_{n}V+\pi\right)\left(1+i\right)={}_{n+1}V$

Write those equations, for $n=0$ to $n=34$

$0:\left({}_{0}V+\pi\right)\left(1+i\right)={}_{1}V$

$1:\left({}_{1}V+\pi\right)\left(1+i\right)={}_{2}V$

$2:\left({}_{2}V+\pi\right)\left(1+i\right)={}_{3}V$

$\vdots$

$34:\left({}_{34}V+\pi\right)\left(1+i\right)={}_{35}V$

Multiply equation *k* by $\left(1+i\right)^{34-k}$ and sum the results:

$$\left({}_{0}V+\pi\right)\left(1+i\right)^{35}+\left({}_{1}V+\pi\right)\left(1+i\right)^{34}+\left({}_{2}V+\pi\right)\left(1+i\right)^{33}+\cdots+\left({}_{34}V+\pi\right)\left(1+i\right)=$$
$$\quad {}_{1}V\left(1+i\right)^{34}+{}_{2}V\left(1+i\right)^{33}+{}_{3}V\left(1+i\right)^{32}+\cdots+{}_{34}V\left(1+i\right)+{}_{35}V$$

For $k=1,2,\cdots,34,$ the ${}_{k}V\left(1+i\right)^{35-k}$ terms in both sides cancel, leaving

$${}_{0}V\left(1+i\right)^{35}+\pi\left[\left(1+i\right)^{35}+\left(1+i\right)^{34}+\cdots+\left(1+i\right)\right]={}_{35}V$$

Since ${}_{0}V=0$

$$\pi\,\ddot{s}_{\overline{35|}}={}_{35}V$$
$$=\ddot{a}_{60}$$

(see above for remainder of solution)

Question #94
Answer: B

$$\mu_{\overline{x+t:y+t}} = \frac{{}_tq_y \; {}_tp_x \mu_{x+t} + {}_tq_x \; {}_tp_y \; \mu_{y+t}}{{}_tq_x \times {}_tp_y + {}_tp_x \times {}_tq_y + {}_tp_x \times {}_tp_y}$$

For (x) = (y) = (50)

$$\mu_{\overline{50:50}}(10.5) = \frac{\left({}_{10.5}q_{50}\right)\left({}_{10}p_{50}\right)q_{60}\cdot 2}{\left({}_{10.5}q_{50}\right)\left({}_{10.5}p_{50}\right)\cdot 2 + \left({}_{10.5}p_{50}\right)^2} = \frac{(0.09152)(0.91478)(0.01376)(2)}{(0.09152)(0.90848)(2)+(0.90848)^2} = 0.0023$$

where

$${}_{10.5}p_{50} = \frac{\tfrac{1}{2}\left(l_{60}+l_{61}\right)}{l_{50}} = \frac{\tfrac{1}{2}(8{,}188{,}074 + 8{,}075{,}403)}{8{,}950{,}901} = 0.90848$$

$${}_{10.5}q_{50} = 1 - {}_{10.5}p_{50} = 0.09152$$

$${}_{10}p_{50} = \frac{8{,}188{,}074}{8{,}950{,}901} = 0.91478$$

${}_{10.5}p_{50} \;\mu(50+10.5) = \left({}_{10}p_{50}\right)q_{60}$ since UDD

Alternatively, ${}_{(10+t)}p_{50} = {}_{10}p_{50} \; {}_tp_{60}$

$${}_{(10+t)}p_{50:50} = \left({}_{10}p_{50}\right)^2 \left({}_tp_{60}\right)^2$$

$${}_{(10+t)}p_{\overline{50:50}} = 2\; {}_{10}p_{50} \; {}_tp_{60} - \left({}_{10}p_{50}\right)^2 \left({}_tp_{60}\right)^2$$

$$= 2\; {}_{10}p_{50}\left(1 - tq_{60}\right) - \left({}_{10}p_{50}\right)^2 \left(1 - tq_{60}\right)^2 \text{ since UDD}$$

$$\text{Derivative } = -2\; {}_{10}p_{50}\; q_{60} + 2\left({}_{10}p_{50}\right)^2 \left(1 - tq_{60}\right) q_{60}$$

Derivative at $10 + t = 10.5$ is

$$-2(0.91478)(0.01376) + (0.91478)^2 \left(1 - (0.5)(0.01376)\right)(0.01376) = -0.0023$$

$${}_{10.5}p_{\overline{50:50}} = 2\; {}_{10.5}p_{50} - \left({}_{10.5}p_{50}\right)^2$$

$$= 2(0.90848) - (0.90848)^2$$

$$= 0.99162$$

μ (for any sort of lifetime) $= \dfrac{-\dfrac{dp}{dt}}{p} = \dfrac{-(-0.0023)}{0.99162} = 0.0023$

Question #95
Answer: D

$$\mu_{x+t}^{(\tau)} = \mu_{x+t}^{(1)} + \mu_{x+t}^{(2)} = 0.01 + 2.29 = 2.30$$

$$P = P\int_0^2 v^t\ {}_tp_x^{(\tau)}\mu_{x+t}^{(2)}\ dt + 50,000\int_0^2 v^t\ {}_tp_x^{(\tau)}\mu_{x+t}^{(1)}\ dt + 50,000\int_2^\infty v^t\ {}_tp_x^{(\tau)}\mu_{x+t}^{(\tau)}\ dt$$

$$P = P\int_0^2 e^{-0.1t}e^{-2.3t}\times 2.29dt + 50,000\int_0^2 e^{-0.1t}e^{-2.3t}\times 0.01dt + 50,000\int_2^\infty e^{-0.1t}e^{-2.3t}\times 2.3dt$$

$$P\left[1 - 2.29\times\frac{1-e^{-2(2.4)}}{2.4}\right] = 50000\left[0.01\times\frac{1-e^{-2(2.4)}}{2.4} + 2.3\times\frac{e^{-2(2.4)}}{2.4}\right]$$

P = 11,194

Question #96
Answer: B

$$e_x = p_x + {}_2p_x + {}_3p_x + \ldots = 11.05$$

$$\begin{aligned}\text{Annuity} &= v^3\ {}_3p_x 1000 + v^4\ {}_4p_x \times 1000\times(1.04) + \ldots \\ &= \sum_{k=3}^{\infty} 1000(1.04)^{k-3} v^k\ {}_kp_x \\ &= 1000v^3\sum_{k=3}^{\infty} {}_kp_x \\ &= 1000v^3(e_x - 0.99 - 0.98) = 1000\left(\frac{1}{1.04}\right)^3 \times 9.08 = 8072\end{aligned}$$

Let π = benefit premium.

$$\begin{aligned}\pi\left(1 + 0.99v + 0.98v^2\right) &= 8072 \\ 2.8580\pi &= 8072 \\ \pi &= 2824\end{aligned}$$

Question #97
Answer: B

$$\pi\, \ddot{a}_{30:\overline{10}|} = 1000 A_{30} + P(IA)^{1}_{30:\overline{10}|} + (10\pi)\left({}_{10|}A_{30}\right)$$

$$\pi = \frac{1000 A_{30}}{\ddot{a}_{30:\overline{10}|} - (IA)^{1}_{30:\overline{10}|} - 10\, {}_{10|}A_{30}}$$

$$= \frac{1000(0.102)}{7.747 - 0.078 - 10(0.088)}$$

$$= \frac{102}{6.789}$$

$$= 15.024$$

Question #98
Answer: E

For the general survival function $S_0(t) = 1 - \frac{t}{\omega},\ 0 \le t \le \omega$,

$$\mathring{e}_{30} = \int_0^{\omega-30} \left(1 - \frac{t}{\omega - 30}\right) dt$$

$$= \left[t - \frac{t^2}{2(\omega - 30)} \right]_0^{\omega-30}$$

$$= \frac{\omega - 30}{2}$$

Prior to medical breakthrough $\quad \omega = 100 \Rightarrow \mathring{e}_{30} = \frac{100 - 30}{2} = 35$

After medical breakthrough $\quad \mathring{e}'_{30} = \mathring{e}_{30} + 4 = 39$

so $\quad \mathring{e}'_{30} = 39 = \frac{\omega' - 30}{2} \Rightarrow \omega' = 108$

Question #99
Answer: A

$L = 100{,}000 v^{2.5} - 4000 \ddot{a}_{\overline{3}|}$ @5%

= 77,079

Question #100
Answer: D

$\mu^{(accid)} = 0.001$

$\mu^{(total)} = 0.01$

$\mu^{(other)} = 0.01 - 0.001 = 0.009$

Expected present value $= \int_0^\infty 500,000\, e^{-0.05t}\ e^{-0.01t}\,(0.009)\,dt$

$$+10\int_0^\infty 50,000\, e^{0.04t}\ e^{-0.05t}\ e^{-0.01t}\,(0.001)\,dt$$

$$= 500,000\left[\frac{0.009}{0.06} + \frac{0.001}{0.02}\right] = 100,000$$

Question #101 Removed

Question #102
Answer: D

$$1000\ {}_{20}V = 1000 A_{x+20} = \frac{1000\left({}_{19}V + {}_{20}P_x\right)(1.06) - q_{x+19}(1000)}{p_{x+19}}$$

$$= \frac{(342.03 + 13.72)(1.06) - 0.01254(1000)}{0.98746} = 369.18$$

$$\ddot{a}_{x+20} = \frac{1 - 0.36918}{(0.06/1.06)} = 11.1445$$

$$so \quad 1000 P_{x+20} = 1000\frac{A_{x+20}}{\ddot{a}_{x+20}} = \frac{369.18}{11.1445} = 33.1$$

Question #103
Answer: B

$$ {}_k p_x^{(\tau)} = e^{-\int_0^k \mu_{x+t}^{(\tau)} dt} = e^{-\int_0^k 2\mu_{x+t}^{(1)} dt} $$

$$ = \left(e^{-\int_0^k \mu_{x+t}^{(1)} dt} \right)^2 $$

$$ = \left({}_k p_x \right)^2 \text{ where } {}_k p_x \text{ is from Illustrative Life Table, since } \mu^{(1)} \text{ follows I.L.T.} $$

$$ {}_{10} p_{60} = \frac{6{,}616{,}155}{8{,}188{,}074} = 0.80802 $$

$$ {}_{11} p_{60} = \frac{6{,}396{,}609}{8{,}188{,}074} = 0.78121 $$

$$ {}_{10|} q_{60}^{(\tau)} = {}_{10} p_{60}^{(\tau)} - {}_{11} p_{60}^{(\tau)} $$

$$ = \left({}_{10} p_{60} \right)^2 - \left({}_{11} p_{60} \right)^2 \text{ from I.L.T.} $$

$$ = 0.80802^2 - 0.78121^2 = 0.0426 $$

Question #104
Answer: C

$P_s = \dfrac{1}{\ddot{a}_s} - d$, where *s* can stand for any of the statuses under consideration.

$$ \ddot{a}_s = \frac{1}{P_s + d} $$

$$ \ddot{a}_x = \ddot{a}_y = \frac{1}{0.1 + 0.06} = 6.25 $$

$$ \ddot{a}_{\overline{xy}} = \frac{1}{0.06 + 0.06} = 8.333 $$

$$ \ddot{a}_{\overline{xy}} + \ddot{a}_{xy} = \ddot{a}_x + \ddot{a}_y $$

$$ \ddot{a}_{xy} = 6.25 + 6.25 - 8.333 = 4.167 $$

$$ P_{xy} = \frac{1}{4.167} - 0.06 = 0.18 $$

Question #105
Answer: A

$$d_0^{(\tau)} = 1000\int_0^1 e^{-(\mu+0.04)t}(\mu+0.04)dt$$
$$= 1000\left(1-e^{-(\mu+0.04)}\right) = 48$$

$$e^{-(\mu+0.04)} = 0.952$$
$$\mu + 0.04 = -\ln(0.952)$$
$$= 0.049$$
$$\mu = 0.009$$

$$d_3^{(1)} = 1000\int_3^4 e^{-0.049t}(0.009)dt$$
$$= 1000\frac{0.009}{0.049}\left(e^{-(0.049)(3)} - e^{-(0.049)(4)}\right) = 7.6$$

Question #106
Answer: B

This is a graph of $l_x\mu_x$.
μ_x would be increasing in the interval $(80,100)$.
The graphs of $l_x p_x$, l_x and l_x^2 would be decreasing everywhere.

Question #107
Answer: B

Variance $= v^{30}\,{}_{15}p_x\,{}_{15}q_x$ Expected value $= v^{15}\,{}_{15}p_x$

$$v^{30}\,{}_{15}p_x\,{}_{15}q_x = 0.065 \quad v^{15}\,{}_{15}p_x$$
$$v^{15}\,{}_{15}q_x = 0.065 \Rightarrow {}_{15}q_x = 0.3157$$

Since μ is constant

$${}_{15}q_x = \left(1-(p_x)^{15}\right)$$
$$(p_x)^{15} = 0.6843$$
$$p_x = 0.975$$
$$q_x = 0.025$$

Question #108
Answer: E

$$(1) \quad {}_{11}V^A = \left({}_{10}V^A + 0\right)\frac{(1+i)}{p_{x+10}} - \frac{q_{x+10}}{p_{x+10}} \times 1000$$

$$(2) \quad {}_{11}V^B = \left({}_{10}V^B + \pi^B\right)\frac{(1+i)}{p_{x+10}} - \frac{q_{x+10}}{p_{x+10}} \times 1000$$

$$(1)-(2) \quad {}_{11}V^A - {}_{11}V^B = \left({}_{10}V^A - {}_{10}V^B - \pi^B\right)\frac{(1+i)}{p_{x+10}}$$

$$= (101.35 - 8.36)\frac{(1.06)}{1-0.004}$$

$$= 98.97$$

Question #109
Answer: A

EPV (*x*'s benefits) $= \sum_{k=0}^{2} v^{k+1} b_{k+1} \, {}_k p_x \, q_{x+k}$

$$= 1000\left[300v(0.02) + 350v^2(0.98)(0.04) + 400v^3(0.98)(0.96)(0.06)\right]$$

$$= 36{,}829$$

Question #110
Answer: E

π denotes benefit premium

${}_{19}V = EPV$ future benefits - EPV future premiums

$$0.6 = \frac{1}{1.08} - \pi \Rightarrow \pi = 0.326$$

$${}_{11}V = \frac{\left({}_{10}V + \pi\right)(1.08) - (q_{65})(10)}{p_{65}}$$

$$= \frac{(5.0 + 0.326)(1.08) - (0.10)(10)}{1-0.10}$$

=5.28

Question #111
Answer: A

Expected present value Benefits $= \dfrac{(0.8)(0.1)(10,000)}{1.06^2} + \dfrac{(0.8)(0.9)(0.097)(9,000)}{1.06^3}$

$$= 1,239.75$$

$$1,239.75 = P\left(1 + \frac{(0.8)}{1.06} + \frac{(0.8)(0.9)}{1.06^2}\right)$$

$$= P(2.3955)$$

$$P = 517.53 \quad \Rightarrow 518$$

Question #112
Answer: A

$$1180 = 70\bar{a}_{30} + 50\bar{a}_{40} - 20\bar{a}_{30:40}$$

$$1180 = (70)(12) + (50)(10) - 20\bar{a}_{30:40}$$

$$\bar{a}_{30:40} = 8$$

$$\bar{a}_{\overline{30:40}} = \bar{a}_{30} + \bar{a}_{40} - \bar{a}_{30:40} = 12 + 10 - 8 = 14$$

$$100\bar{a}_{\overline{30:40}} = 1400$$

Question #113
Answer: B

$$\bar{a} = \int_0^\infty \bar{a}_{\overline{t}|}\, f(t)dt \;\; = \int_0^\infty \frac{1 - e^{-0.05t}}{0.05} \frac{1}{\Gamma(2)} te^{-t}dt$$

$$= \frac{1}{0.05}\int_0^\infty \left(te^{-t} - te^{-1.05t}\right)dt$$

$$= \frac{1}{0.05}\left[-(t+1)e^{-t} + \left(\frac{t}{1.05} + \frac{1}{1.05^2}\right)e^{-1.05t}\right]\Bigg|_0^\infty$$

$$= \frac{1}{0.05}\left[1 - \left(\frac{1}{1.05}\right)^2\right] = 1.85941$$

$$20,000 \times 1.85941 = 37,188$$

Question #114
Answer: C

Event	***Prob***	Present Value
$x=0$	(0.05)	15
$x=1$	$(0.95)(0.10)=0.095$	$15+20/1.06=33.87$
$x\geq 2$	$(0.95)(0.90)=0.855$	$15+20/1.06+25/1.06^2=56.12$

$$E[X]=(0.05)(15)+(0.095)(33.87)+(0.855)(56.12)=51.95$$

$$E\left[X^2\right]=(0.05)(15)^2+(0.095)(33.87)^2+(0.855)(56.12)^2=2813.01$$

$$Var[X]=E\left(X^2\right)-E(X)^2=2813.01-(51.95)^2=114.2$$

Question #115
Answer: B

Let *K* be the curtate future lifetime of (*x* + *k*)

$${}_kL=1000v^{K+1}-1000P_{x:\overline{3|}}\times\ddot{a}_{\overline{K+1|}}$$

When (as given in the problem), (*x*) dies in the second year from issue, the curtate future lifetime of $(x+1)$ is 0, so

$${}_1L=1000v-1000P_{x:\overline{3|}}\,\ddot{a}_{\overline{1|}}$$
$$=\frac{1000}{1.1}-279.21$$
$$=629.88\approx 630$$

The premium came from

$$P_{x:\overline{3|}}=\frac{A_{x:\overline{3|}}}{\ddot{a}_{x:\overline{3|}}}$$
$$A_{x:\overline{3|}}=1-d\,\ddot{a}_{x:\overline{3|}}$$
$$P_{x:\overline{3|}}=279.21=\frac{1-d\,\ddot{a}_{x:\overline{3|}}}{\ddot{a}_{x:\overline{3|}}}=\frac{1}{\ddot{a}_{x:\overline{3|}}}-d$$

Question #116
Answer: D

Let M = the force of mortality of an individual drawn at random; and $T =$ future lifetime of the individual.

$$\begin{aligned}
\Pr[T \le 1] &= E\{\Pr[T \le 1|M]\} \\
&= \int_0^\infty \Pr[T \le 1|M = \mu]\, f_M(\mu)d\mu \\
&= \int_0^2 \int_0^1 \mu e^{-\mu t} dt \frac{1}{2} d\mu \\
&= \int_0^2 \left(1 - e^{-\mu}\right)\frac{1}{2} du = \frac{1}{2}\left(2 + e^{-2} - 1\right) = \frac{1}{2}\left(1 + e^{-2}\right) \\
&= 0.56767
\end{aligned}$$

Question #117
Answer: E

For this model:

$$\mu_{40+t}^{(1)} = \frac{1/60}{1-t/60} = \frac{1}{60-t};\ \mu_{40+20}^{(1)} = 1/40 = 0.025$$

$$\mu_{40+t}^{(2)} = \frac{1/40}{1-t/40} = \frac{1}{40-t};\ \mu_{40+20}^{(1)} = 1/20 = 0.05$$

$$\mu_{40+20}^{(\tau)} = 0.025 + 0.05 = 0.075$$

Question #118
Answer: D

Let $\pi =$ benefit premium

Expected present value of benefits =

$$= (0.03)(200{,}000)v + (0.97)(0.06)(150{,}000)v^2 + (0.97)(0.94)(0.09)(100{,}000)v^3$$
$$= 5660.38 + 7769.67 + 6890.08$$
$$= 20{,}320.13$$

Expected present value of benefit premiums

$$= \ddot{a}_{x:\overline{3}|}\,\pi$$
$$= \left[1 + 0.97v + (0.97)(0.94)v^2\right]\pi$$
$$= 2.7266\,\pi$$
$$\pi = \frac{20{,}320.13}{2.7266} = 7452.55$$

$${}_1V = \frac{(7452.55)(1.06) - (200{,}000)(0.03)}{1 - 0.03}$$
$$= 1958.46$$

Initial reserve, year 2 = ${}_1V + \pi$

= 1958.56 + 7452.55

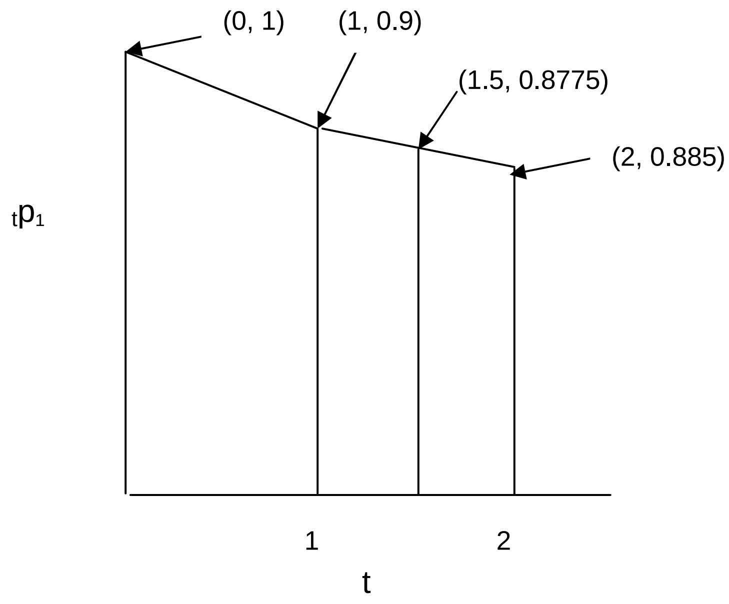
= 9411.01

Question #119
Answer: A

Let π denote the premium.

$$L = b_T v^T - \pi\,\bar{a}_{\overline{T}|} = (1+i)^T \times v^T - \pi\,\bar{a}_{\overline{T}|}$$
$$= 1 - \pi\,\bar{a}_{\overline{T}|}$$
$$E[L] = 1 - \pi\,\bar{a}_x = 0 \quad \Rightarrow \pi = 1/\bar{a}_x$$
$$\Rightarrow L = 1 - \pi\,\bar{a}_{\overline{T}|} = 1 - \frac{\bar{a}_{\overline{T}|}}{\bar{a}_x} = \frac{\delta\bar{a}_x - \left(1 - v^T\right)}{\delta\bar{a}_x}$$
$$= \frac{v^T - \left(1 - \delta\bar{a}_x\right)}{\delta\bar{a}_x} = \frac{v^T - \bar{A}_x}{1 - \bar{A}_x}$$

Question #120
Answer: D



$$_1p_1 = (1-0.1) = 0.9$$

$$_2p_1 = (0.9)(1-0.05) = 0.855$$

since uniform, $_{1.5}p_1 = (0.9+0.855)/2$
$= 0.8775$

$$\mathring{e}_{1:\overline{1.5}|} = \text{ Area between } t=0 \text{ and } t=1.5$$

$$= \left(\frac{1+0.9}{2}\right)(1) + \left(\frac{0.9+0.8775}{2}\right)(0.5)$$

$$= 0.95 + 0.444$$

$$= 1.394$$

Alternatively,

$$\mathring{e}_{1:\overline{1.5}|} = \int_0^{1.5} {}_t p_1 dt$$

$$= \int_0^1 {}_t p_1 dt + {}_1 p_1 \int_0^{0.5} {}_x p_2 dx$$

$$= \int_0^1 (1 - 0.1t) dt + 0.9 \int_0^{0.5} (1 - 0.05x) dx$$

$$= \left[t - \tfrac{0.1t^2}{2} \right]_0^1 + 0.9 \left[x - \tfrac{0.05x^2}{2} \right]_0^{0.5}$$

$$= 0.95 + 0.444 = 1.394$$

Question #121
Answer: A

$$10{,}000 A_{63} (1.12) = 5233$$

$$A_{63} = 0.4672$$

$$A_{x+1} = \frac{A_x (1+i) - q_x}{p_x}$$

$$A_{64} = \frac{(0.4672)(1.05) - 0.01788}{1 - 0.01788}$$

$$= 0.4813$$

$$A_{65} = \frac{(0.4813)(1.05) - 0.01952}{1 - 0.01952}$$

$$= 0.4955$$

Single gross premium at 65 = (1.12) (10,000) (0.4955)
= 5550

$$(1+i)^2 = \frac{5550}{5233} \qquad i = \sqrt{\frac{5550}{5233}} - 1 = 0.02984$$

Question #122A
Answer: C

Because your original survival function for (*x*) was correct, you must have

$$\mu_{x+t} = 0.06 = \mu^{02}_{x+t:y+t} + \mu^{03}_{x+t:y+t} = \mu^{02}_{x+t:y+t} + 0.02$$

$$\mu^{02}_{x+t:y+t} = 0.04$$

Similarly, for (*y*)

$$\mu_{y+t} = 0.06 = \mu^{01}_{x+t:y+t} + \mu^{03}_{x+t:y+t} = \mu^{01}_{x+t:y+t} + 0.02$$

$$\mu^{01}_{x+t:y+t} = 0.04$$

The first-to-die insurance pays as soon as State 0 is left, regardless of which state is next. The force of transition from State 0 is
$\mu^{01}_{x+t:y+t} + \mu^{02}_{x+t:y+t} + \mu^{03}_{x+t:y+t} = 0.04 + 0.04 + 0.02 = 0.10$.

With a constant force of transition, the expected present value is
$$\int_0^\infty e^{-\delta t}\, {}_t p^{00}_{xy} (\mu^{01}_{x+t:y+t} + \mu^{02}_{x+t:y+t} + \mu^{03}_{x+t:y+t}) dt = \int_0^\infty e^{-0.05t} e^{-0.10t} (0.10) dt = \frac{0.10}{0.15}$$

Question #122B
Answer: E

Because (x) is to have a constant force of 0.06 regardless of (y)'s status (and vice-versa) it must be that $\mu^{13}_{x+t:y+t} = \mu^{23}_{x+t:y+t} = 0.06$.

There are three mutually exclusive ways in which both will be dead by the end of year 3:

1: Transition from State 0 directly to State 3 within 3 years. The probability of this is
$$\int_0^3 {}_t p^{00}_{xy} \mu^{03}_{x+t:y+t} dt = \int_0^3 e^{-0.10t} 0.02 dt = -\frac{0.02}{0.10} e^{-0.10t} \Big|_0^3 = 0.2(1 - e^{-0.3}) = 0.0518$$
2: Transition from State 0 to State 1 and then to State 3 all within 3 years. The probability of this is
$$\int_0^3 {}_t p^{00}_{xy} \mu^{01}_{x+t:y+t}\, {}_{3-t} p^{13}_{x+t:y+t} dt = \int_0^3 e^{-0.10t} 0.04(1 - e^{-0.06(3-t)}) dt$$
$$= \int_0^3 0.04 \left[e^{-0.10t} - e^{-0.18} e^{-0.04t} \right] = -\frac{0.04}{0.10} e^{-0.10t} + \frac{0.04 e^{-0.18}}{0.04} e^{-0.04t} \Big|_0^3$$
$$= 0.4(1 - e^{-0.3}) - e^{-0.18}(1 - e^{-0.12}) = 0.00922$$

3: Transition from State 0 to State 2 and then to State 3 all within 3 years. By symmetry, this probability is 0.00922.

The answer is then 0.0518 + 2(0.00922) = 0.0702.

Question #122C
Answer: D

Because the original survival function continues to hold for the individual lives, with a constant force of mortality of 0.06 and a constant force of interest of 0.05, the expected present values of the individual insurances are

$$\bar{A}_x = \bar{A}_y = \frac{0.06}{0.06+0.05} = 0.54545,$$

Then,

$$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = 0.54545 + 0.54545 - 0.66667 = 0.42423$$

Alternatively, the answer can be obtained be using the three mutually exclusive outcomes used in the solution to Question 122B.

1: $\int_0^\infty e^{-0.05t}\,{}_t p_{xy}^{00}\mu_{x+t:y+t}^{03}dt = \int_0^\infty e^{-0.05t}e^{-0.10t}0.02dt = \dfrac{0.02}{0.15} = 0.13333$

2 and 3:
$$\int_0^\infty e^{-0.05t}\,{}_t p_{xy}^{00}\mu_{x+t:y+t}^{01}\int_0^\infty e^{-0.05r}\,{}_r p_{x+t:y+t}^{11}\mu_{x+t+r:y+t+r}^{13}drdt$$
$$= \int_0^\infty e^{-0.05t}e^{-0.10t}0.04\int_0^\infty e^{-0.05r}e^{-0.06r}0.06drdt = \frac{0.04}{0.15}\frac{0.06}{0.11} = 0.14545$$

The solution is 0.13333 + 2(0.14545) = 0.42423.

The fact that the double integral factors into two components is due to the memoryless property of the exponential transition distributions.

Question #123
Answer: B

$$
\begin{aligned}
{}_{5|}q_{\overline{35:45}} &= {}_{5|}q_{35} + {}_{5|}q_{45} - {}_{5|}q_{35:45} \\
&= {}_5p_{35}q_{40} + {}_5p_{45}q_{50} - {}_5p_{35:45}q_{40:50} \\
&= {}_5p_{35}q_{40} + {}_5p_{45}q_{50} - {}_5p_{35} \times {}_5p_{45}\left(1 - p_{40:50}\right) \\
&= {}_5p_{35}q_{40} + {}_5p_{45}q_{50} - {}_5p_{35} \times {}_5p_{45}\left(1 - p_{40}\,p_{50}\right) \\
&= (0.9)(.03) + (0.8)(0.05) - (0.9)(0.8)\left[1 - (0.97)(0.95)\right] \\
&= 0.01048
\end{aligned}
$$

Alternatively,

$$
\begin{aligned}
{}_6p_{35} &= {}_5p_{35} \times p_{40} = (0.90)(1 - 0.03) = 0.873 \\
{}_6p_{45} &= {}_5p_{45} \times p_{50} = (0.80)(1 - 0.05) = 0.76
\end{aligned}
$$

$$
\begin{aligned}
{}_{5|}q_{\overline{35:45}} &= {}_5p_{\overline{35:45}} - {}_6p_{\overline{35:45}} \\
&= \left({}_5p_{35} + {}_5p_{45} - {}_5p_{35:45}\right) - \left({}_6p_{35} + {}_6p_{45} - {}_6p_{35:45}\right) \\
&= \left({}_5p_{35} + {}_5p_{45} + {}_5p_{35} \times {}_5p_{45}\right) - \left({}_6p_{35} + {}_6p_{45} - {}_6p_{35} \times {}_6p_{45}\right) \\
&= \left(0.90 + 0.80 - 0.90 \times 0.80\right) - \left(0.873 + 0.76 - 0.873 \times 0.76\right) \\
&= 0.98 - 0.96952 \\
&= 0.01048
\end{aligned}
$$

Question #124 – Removed

Question #125 - Removed

Question #126
Answer: E

Let Y = present value random variable for payments on one life
$S = \sum Y =$ present value random variable for all payments

$$E[Y] = 10\ddot{a}_{40} = 148.166$$

$$\text{Var}[Y] = 10^2 \frac{\left({}^2A_{40} - A_{40}^2\right)}{d^2}$$

$$= 100\left(0.04863 - 0.16132^2\right)(1.06/0.06)^2$$

$$= 705.55$$

$$E[S] = 100E[Y] = 14{,}816.6$$

$$\text{Var}[S] = 100\,\text{Var}[Y] = 70{,}555$$

Standard deviation $[S] = \sqrt{70{,}555} = 265.62$

By normal approximation, need
E [*S*] + 1.645 Standard deviations = 14,816.6 + (1.645) (265.62)
= 15,254

Question #127
Answer: B

Initial Benefit Prem $= \dfrac{5A_{30} - 4\left(A^{1}_{30:\overline{20|}}\right)}{5\ddot{a}_{30:\overline{35|}} - 4\ddot{a}_{30:\overline{20|}}}$

$$= \frac{5(0.10248) - 4(0.02933)}{5(14.835) - 4(11.959)}$$

$$= \frac{0.5124 - 0.11732}{74.175 - 47.836} = \frac{0.39508}{26.339} = 0.015$$

Where

$$A^{1}_{30:\overline{20|}} = \left(A_{30:\overline{20|}} - A_{30:\overline{20|}}^{\ \ \ \ \ 1}\right) = 0.32307 - 0.29374 = 0.02933$$

and

$$\ddot{a}_{30:\overline{20|}} = \frac{1 - A_{30:\overline{20|}}}{d} = \frac{1 - 0.32307}{\left(\frac{0.06}{1.06}\right)} = 11.959$$

Comment: the numerator could equally well have been calculated as
$A_{30} + 4\ {}_{20}E_{30}\ A_{50}$ = 0.10248 + (4) (0.29374) (0.24905) = 0.39510

Question #128
Answer: B

$$
\begin{aligned}
{}_{0.75}p_x &= 1-(0.75)(0.05)\\
&= 0.9625\\
{}_{0.75}p_y &= 1-(0.75)(.10)\\
&= 0.925\\
{}_{0.75}q_{xy} &= 1-{}_{0.75}p_{xy}\\
&= 1-\left({}_{0.75}p_x\right)\left({}_{0.75}p_y\right) \text{ since independent}\\
&= 1-(0.9625)(0.925)\\
&= 0.1097
\end{aligned}
$$

Question #129
Answer: D

Let *G* be the expense-loaded premium.
Expected present value (EPV) of benefits = $100{,}000A_{35}$
EPV of premiums = $G\ddot{a}_{35}$
EPV of expenses = $\left[0.1G+25+(2.50)(100)\right]\ddot{a}_{35}$
Equivalence principle:

$$
\begin{aligned}
G\ddot{a}_{35} &= 100{,}000A_{35}+(0.1G+25+250)\ddot{a}_{35}\\
G &= 100{,}000\frac{A_{35}}{\ddot{a}_{35}}+0.1G+275\\
0.9G &= 100{,}000P_{35}+275\\
G &= \frac{(100)(8.36)+275}{0.9}\\
&= 1234
\end{aligned}
$$

Question #130
Answer: A

The person receives *K* per year guaranteed for 10 years $\Rightarrow K\ddot{a}_{\overline{10}|} = 8.4353K$

The person receives *K* per years alive starting 10 years from now $\Rightarrow {}_{10|}\ddot{a}_{40}K$

*Hence we have $10000 = \left(8.4353 + {}_{10}E_{40}\ddot{a}_{50}\right)K$

Derive ${}_{10}E_{40}$:

$$A_{40} = A^{1}_{40:\overline{10}|} + \left({}_{10}E_{40}\right)A_{50}$$

$${}_{10}E_{40} = \frac{A_{40} - A^{1}_{40:\overline{10}|}}{A_{50}} = \frac{0.30 - 0.09}{0.35} = 0.60$$

Derive $\ddot{a}_{50} = \dfrac{1 - A_{50}}{d} = \dfrac{1 - 0.35}{\frac{.04}{1.04}} = 16.90$

Plug in values:

$$10{,}000 = \left(8.4353 + (0.60)(16.90)\right)K$$

$$= 18.5753K$$

$$K = 538.35$$

Question #131
Answer: D

STANDARD: $\mathring{e}_{25:\overline{11}|} = \int_0^{11}\left(1-\frac{t}{75}\right)dt = t - \frac{t^2}{2\times 75}\Big|_0^{11} = 10.1933$

MODIFIED: $p_{25} = e^{-\int_0^1 0.1ds} = e^{-.1} = 0.90484$

$$\mathring{e}_{25:\overline{11}|} = \int_0^1 {}_tp_{25}dt + p_{25}\int_0^{10}\left(1-\frac{t}{74}\right)dt$$

$$= \int_0^1 e^{-0.1t}dt + e^{-0.1}\int_0^{10}\left(1-\frac{t}{74}\right)dt$$

$$= \frac{1-e^{-0.1}}{0.1} + e^{-0.1}\left(t-\frac{t^2}{2\times 74}\right)\Big|_0^{10}$$

$$= 0.95163 + 0.90484(9.32432) = 9.3886$$

Difference =0.8047

Question #132
Answer: B

Comparing B & D: Prospectively at time 2, they have the same future benefits. At issue, B has the lower benefit premium. Thus, by formula 7.2.2, B has the higher reserve.

Comparing A to B: use formula 7.3.5. At issue, B has the higher benefit premium. Until time 2, they have had the same benefits, so B has the higher reserve.

Comparing B to C: Visualize a graph C* that matches graph B on one side of t=2 and matches graph C on the other side. By using the logic of the two preceding paragraphs, C's reserve is lower than C*'s which is lower than B's.

Comparing B to E: Reserves on E are constant at 0.

Question #133
Answer: C

Since only decrements (1) and (2) occur during the year, probability of reaching the end of the year is

$$p'^{(1)}_{60} \times p'^{(2)}_{60} = (1-0.01)(1-0.05) = 0.9405$$

Probability of remaining through the year is

$$p'^{(1)}_{60} \times p'^{(2)}_{60} \times p'^{(3)}_{60} = (1-0.01)(1-0.05)(1-0.10) = 0.84645$$

Probability of exiting at the end of the year is

$$q^{(3)}_{60} = 0.9405 - 0.84645 = 0.09405$$

Question #134 - Removed

Question #135
Answer: D

EPV of regular death benefit $= \int_0^\infty (100000)\left(e^{-\delta t}\right)(0.008)\left(e^{-\mu t}\right)dt$

$= \int_0^\infty (100000)\left(e^{-0.06t}\right)(0.008)\left(e^{-0.008t}\right)dt$

$= 100000\left[0.008 / (0.06 + 0.008)\right] = 11{,}764.71$

EPV of accidental death benefit $= \int_0^{30} (100000)\left(e^{-\delta t}\right)(0.001)\left(e^{-\mu t}\right)dt$

$= \int_0^{30} (100000)\left(e^{-0.06t}\right)(0.001)\left(e^{-0.008t}\right)dt$

$= 100\left[1 - e^{-2.04}\right] / 0.068 = 1{,}279.37$

Total EPV $= 11765 + 1279 = 13044$

Question #136
Answer: B

$$l_{[60]+.6} = (.6)(79{,}954) + (.4)(80{,}625)$$
$$= 80{,}222.4$$
$$l_{[60]+1.5} = (.5)(79{,}954) + (.5)(78{,}839)$$
$$= 79{,}396.5$$

$$_{0.9}q_{[60]+.6} = \frac{80222.4 - 79{,}396.5}{80{,}222.4}$$
$$= 0.0103$$

Question #137 - Removed

Question #138
Answer: A

$$q_{40}^{(\tau)} = q_{40}^{(1)} + q_{40}^{(2)} = 0.34$$
$$= 1 - p_{40}'^{(1)} p_{40}'^{(2)}$$
$$0.34 = 1 - 0.75 p_{40}'^{(2)}$$

$$p_{40}'^{(2)} = 0.88$$
$$q_{40}'^{(2)} = 0.12 = y$$

$$q_{41}'^{(2)} = 2y = 0.24$$
$$q_{41}^{(\tau)} = 1 - (0.8)(1 - 0.24) = 0.392$$
$$l_{42}^{(\tau)} = 2000(1 - 0.34)(1 - 0.392) = 803$$

Question #139
Answer: C

$\Pr[L(\pi') > 0] < 0.5$

$\Pr\left[10{,}000v^{K+1} - \pi'\ddot{a}_{\overline{K+1}|} > 0\right] < 0.5$

From Illustrative Life Table, $_{47}p_{30} = 0.50816$ and $_{48}p_{30} = .47681$

Since *L* is a decreasing function of *K*, to have
$\Pr[L(\pi') > 0] < 0.5$ means we must have $L(\pi') \le 0$ for $K \ge 47$.
Highest value of $L(\pi')$ for $K \ge 47$ is at *K* = 47.

$L(\pi')[\text{at } K = 47] = 10{,}000\, v^{47+1} - \pi'\, \ddot{a}_{\overline{47+1}|}$

$= 609.98 - 16.589\pi'$

$L(\pi') \le 0 \Rightarrow (609.98 - 16.589\pi') \le 0$

$\Rightarrow \pi' > \dfrac{609.98}{16.589} = 36.77$

Question #140
Answer: B

$\Pr(K = 0) = 1 - p_x = 0.1$

$\Pr(K = 1) = {}_1p_x - {}_2p_x = 0.9 - 0.81 = 0.09$

$\Pr(K > 1) = {}_2p_x = 0.81$

$E(Y) = .1 \times 1 + .09 \times 1.87 + .81 \times 2.72 = 2.4715$

$E(Y^2) = .1 \times 1^2 + .09 \times 1.87^2 + .81 \times 2.72^2 = 6.407$

$VAR(Y) = 6.407 - 2.4715^2 = 0.299$

Question #141
Answer: E

$E[Z] = b\bar{A}_x$

since constant force $\bar{A}_x = \mu/(\mu+\delta)$

$E(Z) = \dfrac{b\mu}{\mu+\delta} = \dfrac{b(0.02)}{(0.06)} = b/3$

$\text{Var}[Z] = \text{Var}[bv^T] = b^2\text{Var}[v^T] = b^2\left({}^2\bar{A}_x - \bar{A}_x^2\right)$

$$= b^2\left(\frac{\mu}{\mu+2\delta} - \left(\frac{\mu}{\mu+\delta}\right)^2\right)$$

$$= b^2\left[\frac{2}{10} - \frac{1}{9}\right] = b^2\left(\frac{4}{45}\right)$$

$\text{Var}(Z) = E(Z)$

$b^2\left[\dfrac{4}{45}\right] = \dfrac{b}{3}$

$b\left[\dfrac{4}{45}\right] = \dfrac{1}{3} \Rightarrow b = 3.75$

Question #142
Answer: B

In general $\text{Var}(L) = \left(1+\frac{P}{\delta}\right)^2\left({}^2\bar{A}_x - \bar{A}_x^2\right)$

Here $\bar{P}(\bar{A}_x) = \dfrac{1}{\bar{a}_x} - \delta = \dfrac{1}{5} - .08 = .12$

So $\text{Var}(L) = \left(1+\dfrac{.12}{.08}\right)^2\left({}^2\bar{A}_x - \bar{A}_x^2\right) = .5625$

and $\text{Var}(L^*) = \left(1+\dfrac{\frac{5}{4}(.12)}{.08}\right)^2\left({}^2\bar{A}_x - \bar{A}_x^2\right)$

So $\text{Var}(L^*) = \dfrac{\left(1+\frac{15}{8}\right)^2}{\left(1+\frac{12}{8}\right)^2}(0.5625) = .744$

$\text{E}[L^*] = \bar{A}_x - .15\bar{a}_x = 1 - \bar{a}_x(\delta + .15) = 1 - 5(.23) = -.15$

$\text{E}[L^*] + \sqrt{\text{Var}(L^*)} = .7125$

Question #143 - Removed

Question #144
Answer: B

Let $l_0^{(\tau)} =$ number of students entering year 1
superscript (f) denote academic failure
superscript (w) denote withdrawal
subscript is "age" at start of year; equals year - 1

$$p_0^{(\tau)} = 1 - 0.40 - 0.20 = 0.40$$

$$l_2^{(\tau)} = 10\, l_2^{(\tau)}\, q_2^{(f)} \Rightarrow q_2^{(f)} = 0.1$$

$$q_2^{(w)} = q_2^{(\tau)} - q_2^{(f)} = (1.0 - 0.6) - 0.1 = 0.3$$

$$l_1^{(\tau)} q_1^{(f)} = 0.4\left[l_1^{(\tau)}\left(1 - q_1^{(f)} - q_1^{(w)}\right)\right]$$

$$q_1^{(f)} = 0.4\left(1 - q_1^{(f)} - 0.3\right)$$

$$q_1^{(f)} = \frac{0.28}{1.4} = 0.2$$

$$p_1^{(\tau)} = 1 - q_1^{(f)} - q_1^{(w)} = 1 - 0.2 - 0.3 = 0.5$$

$$_3q_0^{(w)} = q_0^{(w)} + p_0^{(\tau)} q_1^{(w)} + p_0^{(\tau)} p_1^{(\tau)} q_2^{(w)}$$

$$= 0.2 + (0.4)(0.3) + (0.4)(0.5)(0.3)$$

$$= 0.38$$

Question #145
Answer: D

$$e_{25} = p_{25}(1+e_{26})$$

$e_{26}^N = e_{26}^M$ due to having the same μ

$$p_{25}^N = \exp\left[-\int_0^1 \mu_{25+t}^M + 0.1(1-t)dt\right] = p_{25}^M e^{-0.05}$$

$$e_{25}^N = p_{25}^N(1+e_{26}) = e^{-0.05} p_{25}^M(1+e_{26}) = 0.951 e_{25}^M = 9.51$$

Question #146
Answer: D

$$\mathrm{E}\left[Y_{AGG}\right] = 100\mathrm{E}\left[Y\right] = 100(10{,}000)\bar{a}_x$$

$$= 100(10{,}000)\left(\frac{(1-\bar{A}_x)}{\delta}\right) = 10{,}000{,}000$$

$$\sigma_Y = \sqrt{\mathrm{Var}\left[Y\right]} = \sqrt{(10{,}000)^2 \frac{1}{\delta^2}\left({}^2\bar{A}_x - \bar{A}_x^2\right)}$$

$$= \frac{(10{,}000)}{\delta}\sqrt{(0.25)-(0.16)} = 50{,}000$$

$$\sigma_{AGG} = \sqrt{100}\sigma_Y = 10(50{,}000) = 500{,}000$$

$$0.90 = \Pr\left[\frac{F - \mathrm{E}\left[Y_{AGG}\right]}{\sigma_{AGG}} > 0\right]$$

$$\Rightarrow 1.282 = \frac{F - \mathrm{E}\left[Y_{AGG}\right]}{\sigma_{AGG}}$$

$$F = 1.282\sigma_{AGG} + \mathrm{E}\left[Y_{AGG}\right]$$

$$F = 1.282(500{,}000) + 10{,}000{,}000 = 10{,}641{,}000$$

Question #147
Answer: A

$$A^{1}_{30:\overline{3|}} = 1000vq_{30} + 500v^2\ {}_{1|}q_{30} + 250v^3\ {}_{2|}q_{30}$$

$$= 1000\left(\frac{1}{1.06}\right)\left(\frac{1.53}{1000}\right) + 500\left(\frac{1}{1.06}\right)^2(0.99847)\left(\frac{1.61}{1000}\right) + 250\left(\frac{1}{1.06}\right)^3(0.99847)(0.99839)\left(\frac{1.70}{1000}\right)$$

$$= 1.4434 + 0.71535 + 0.35572 = 2.51447$$

$$\ddot{a}^{(2)}_{30:\overline{1|}} = \tfrac{1}{2} + \tfrac{1}{2}\left(\frac{1}{1.06}\right)^{1/2}\left(1 - \tfrac{1}{2}q_{30}\right) = \frac{1}{2} + \frac{1}{2}(0.97129)\left(1 - \frac{0.00153}{2}\right)$$

$$= \frac{1}{2} + \frac{1}{2}(0.97129)(0.999235)$$

$$= 0.985273$$

Annualized premium $= \dfrac{2.51447}{0.985273}$

$= 2.552$

Each semiannual premium $= \dfrac{2.552}{2}$

$= 1.28$

Question: #148
Answer: E

$$(DA)^{1}_{80:\overline{20|}} = 20vq_{80} + vp_{80}\left((DA)^{1}_{81:\overline{19|}}\right)$$

$q_{80} = .2$ $\quad 13 = \dfrac{20(.2)}{1.06} + \dfrac{.8}{1.06}(DA)^{1}_{81:\overline{19|}}$

$$\therefore (DA)^{1}_{81:\overline{19|}} = \frac{13(1.06) - 4}{.8} = 12.225$$

$q_{80} = .1$ $\quad DA^{1}_{80:\overline{20|}} = 20v(.1) + v(.9)(12.225)$

$$= \frac{2 + .9(12.225)}{1.06} = 12.267$$

Question #149 - Removed

Question #150
Answer: A

$${}_t p_x = \exp\left[-\int_0^t \frac{ds}{100-x-s}\right] = \exp\left[\ln(100-x-s)\Big|_0^t\right] = \frac{100-x-t}{100-x}$$

$$\overset{\circ}{e}_{\overline{50:60}} = \overset{\circ}{e}_{50} + \overset{\circ}{e}_{60} - \overset{\circ}{e}_{50:60}$$

$$\overset{\circ}{e}_{50} = \int_0^{50} \frac{50-t}{50}\,dt = \frac{1}{50}\left[50t - \frac{t^2}{2}\right]_0^{50} = 25$$

$$\overset{\circ}{e}_{60} = \int_0^{40} \frac{40-t}{40}\,dt = \frac{1}{40}\left[40t - \frac{t^2}{2}\right]_0^{40} = 20$$

$$\overset{\circ}{e}_{50:60} = \int_0^{40} \left(\frac{50-t}{50}\right)\left(\frac{40-t}{40}\right)dt = \int_0^{40} \frac{1}{2000}\left(2000-90t+t^2\right)dt$$

$$= \frac{1}{2000}\left(2000t - 45t^2 + \frac{t^3}{3}\Big|_0^{40}\right) = 14.67$$

$$\overset{\circ}{e}_{\overline{50:60}} = 25 + 20 - 14.67 = 30.33$$

Question #151
Answer: C

Ways to go $0 \rightarrow 2$ in 2 years

$$0-0-2;\ p = (0.7)(0.1) = 0.07$$
$$0-1-2;\ p = (0.2)(0.25) = 0.05$$
$$0-2-2;\ p = (0.1)(1) = 0.1$$

Total = 0.22
Binomial m = 100 q = 0.22
Var = (100) (0.22) (0.78) = 17

Question #152
Answer: A

For death occurring in year 2

$EPV = \frac{0.3\times1000}{1.05} = 285.71$

For death occurring in year 3, two cases:

$$\begin{array}{ll} \text{(1) State 2} \rightarrow \text{State 1} \rightarrow \text{State 4:} & (0.2\times0.1) = 0.02 \\ \text{(2) State 2} \rightarrow \text{State 2} \rightarrow \text{State 4:} & (0.5\times0.3) = \underline{0.15} \\ \quad \text{Total} & 0.17 \end{array}$$

EPV = $\frac{0.17\times1000}{1.05^2} = 154.20$

Total. EPV = 285.71 + 154.20 = 439.91

Question #153 - Removed

Question #154
Answer: C

Let π denote the single benefit premium.

$\pi = {}_{30|}\ddot{a}_{35} + \pi A^{1}_{35:\overline{30|}}$

$$\pi = \frac{{}_{30|}\ddot{a}_{35}}{1 - A^{1}_{35:\overline{30|}}} = \frac{\left(A_{35:\overline{30|}} - A^{1}_{35:\overline{30|}}\right)\ddot{a}_{65}}{1 - A^{1}_{35:\overline{30|}}} =$$

$$= \frac{(.21 - .07)9.9}{(1 - .07)}$$

$$= \frac{1.386}{.93}$$

$$= 1.49$$

Question #155
Answer: E

$$
\begin{aligned}
{}_{0.4}p_0 = .5 &= e^{-\int_0^{0.4}\left(F+e^{2x}\right)dx} \\
&= e^{-.4F-\left[\frac{e^{2x}}{2}\right]_0^{.4}} \\
&= e^{-.4F-\left(\frac{e^{0.8}-1}{2}\right)} \\
.5 &= e^{-.4F-.6128}
\end{aligned}
$$

$$
\begin{aligned}
&\Rightarrow \ln(.5) = -.4F - .6128 \\
&\Rightarrow -.6931 = -.4F - .6128 \\
&\Rightarrow F = 0.20
\end{aligned}
$$

Question #156
Answer: C

$$
\begin{aligned}
\left({}_9V + P\right)(1.03) &= q_{x+9}b + \left(1 - q_{x+9}\right){}_{10}V \\
&= q_{x+9}\left(b - {}_{10}V\right) + {}_{10}V \\
(343)(1.03) &= 0.02904(872) + {}_{10}V \\
&\Rightarrow {}_{10}V = 327.97
\end{aligned}
$$

$$b = \left(b - {}_{10}V\right) + {}_{10}V = 872 + 327.97 = 1199.97$$

$$P = b\left(\frac{1}{\ddot{a}_x} - d\right) = 1200\left(\frac{1}{14.65976} - \frac{0.03}{1.03}\right)$$

= 46.92

${}_9V =$ benefit reserve at the start of year ten – P = 343 – 46.92 = 296.08

Question #157
Answer: B

d = 0.06 $\Rightarrow$ V = 0.94

Step 1 Determine p_x

$$
\begin{aligned}
668 + 258vp_x &= 1000vq_x + 1000v^2 p_x\left(p_{x+1} + q_{x+1}\right) \\
668 + 258(0.94)p_x &= 1000(0.94)\left(1 - p_x\right) + 1000(0.8836)p_x(1) \\
668 + 242.52p_x &= 940\left(1 - p_x\right) + 883.6p_x \\
p_x &= 272/298.92 = 0.91
\end{aligned}
$$

Step 2 Determine $1000P_{x:\overline{2|}}$

$$668 + 258(0.94)(0.91) = 1000P_{x:\overline{2|}}\left[1 + (0.94)(0.91)\right]$$

$$1000P_{x:\overline{2|}} = \frac{[220.69 + 668]}{1.8554} = 479$$

Question #158
Answer: D

$$100,000(IA)^1_{40:\overline{10|}} = 100,000\, v\, p_{40}\left[(IA)^1_{41:\overline{10|}} - 10v^{10}\, {}_9p_{41}\, q_{50}\right] + A^1_{40:\overline{10|}}(100,000) \quad [\text{see comment}]$$

$$= 100,000\frac{0.99722}{1.06}\left[0.16736 - \frac{10\left(\frac{8,950,901}{9,287,264}\right)}{1.06^{10}} \times (0.00592)\right]$$

$$+ (0.02766 \times 100,000)$$

$$=15,513$$

Where $A^1_{40:\overline{10|}} = A_{40} - {}_{10}E_{40}\, A_{50}$

$$= 0.16132 - (0.53667)(0.24905)$$

$$= 0.02766$$

Comment: the first line comes from comparing the benefits of the two insurances. At each of age 40, 41, 42,…,49 $(IA)^1_{40:\overline{10|}}$ provides a death benefit 1 greater than $(IA)^1_{41:\overline{10|}}$. Hence the $A^1_{40:\overline{10|}}$ term. But $(IA)^1_{41:\overline{10|}}$ provides a death benefit at 50 of 10, while $(IA)^1_{40:\overline{10|}}$ provides 0. Hence a term involving ${}_{9|}q_{41} = {}_9p_{41}\, q_{50}$. The various *v*'s and *p*'s just get all expected present values at age 40.

Question #159
Answer: A

$$1000\,{}_1V = \pi(1+i) - q_x(1000 - 1000\,{}_1V)$$
$$40 = 80(1.1) - q_x(1000 - 40)$$
$$q_x = \frac{88-40}{960} = 0.05$$

$$ {}_1AS = \frac{(\text{G} - \text{ expenses})(1+i) - 1000q_x}{p_x}$$
$$= \frac{(100-(0.4)(100))(1.1)-(1000)(0.05)}{1-0.05}$$
$$= \frac{60(1.1)-50}{0.95} = 16.8$$

Question #160
Answer: C

At any age, $p_x'^{(1)} = e^{-0.02} = 0.9802$

$q_x'^{(1)} = 1 - 0.9802 = 0.0198$, which is also $q_x^{(1)}$, since decrement 2 occurs only at the end of the year.

Expected present value (EPV) at the start of each year for that year's death benefits
= 10,000*0.0198 *v* = 188.1

$$p_x^{(\tau)} = 0.9802 * 0.96 = 0.9410$$
$$E_x = p_x^{(\tau)}v = 0.941\ v = 0.941 * 0.95 = 0.8940$$

EPV of death benefit for 3 years $188.1 + E_{40} * 188.1 + E_{40} * E_{41} * 188.1 = 506.60$

Question #161
Answer: B

$$\overset{\circ}{e}_{30:\overline{40|}} = \int_0^{40} {}_t p_{30} dt$$

$$= \int_0^{40} \frac{\omega - 30 - t}{\omega - 30} dt$$

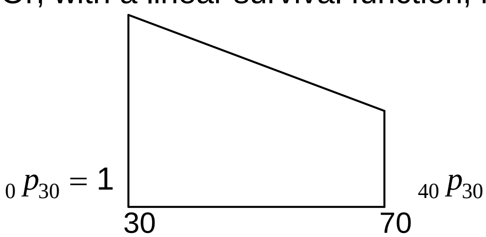
$$= t - \frac{t^2}{2(\omega - 30)} \Big|_0^{40}$$

$$= 40 - \frac{800}{\omega - 30}$$

$$= 27.692$$

$$\omega = 95$$

Or, with a linear survival function, it may be simpler to draw a picture:



$$\overset{\circ}{e}_{30:\overline{40|}} = \text{area} = 27.692 = 40 \frac{(1 + {}_{40} p_{30})}{2}$$

$$_{40} p_{30} = 0.3846$$

$$\frac{\omega - 70}{\omega - 30} = 0.3846$$

$$\omega = 95$$

$$_t p_{30} = \frac{65 - t}{65}$$

$$Var = E(T)^2 - (E(T))^2$$

Using a common formula for the second moment:

$$Var(T) = \int_0^\infty 2t\, {}_t p_x dt - \mathring{e}_x^2$$

$$= 2\int_0^{65} t \bullet \left(1 - \frac{t}{65}\right) dt - \left(\int_0^{65} \left(1 - \frac{t}{65}\right) dt^2\right.$$

$$= 2 * (2112.5 - 1408.333) - (65 - 65/2)^2$$

$$= 1408.333 - 1056.25 = 352.08$$

Another way, easy to calculate for a linear survival function is

$$Var(T) = \int_0^\infty t^2\, {}_t p_x \mu_x(t) dt - \left(\int_0^\infty t\, {}_t p_x \mu_x(t) dt\right)^2$$

$$= \int_0^{65} t^2 \times \frac{1}{65} dt - \left(\int_0^{65} t \times \frac{1}{65} dt\right)^2$$

$$= \frac{t^3}{3 \times 65}\Big|_0^{65} - \left(\frac{t^2}{2 \times 65}\Big|_0^{65}\right)^2$$

$$= 1408.33 - (32.5)^2 = 352.08$$

With a linear survival function and a maximum future lifetime of 65 years, you probably didn't need to integrate to get $E(T_{30}) = \mathring{e}_{30} = 32.5$

Likewise, if you realize (after getting $\omega = 95$) that T_{30} is uniformly distributed on (0, 65), its variance is just the variance of a continuous uniform random variable:

$$Var = \frac{(65 - 0)^2}{12} = 352.08$$

Question #162
Answer: E

$${}_1V = \frac{218.15(1.06) - 10{,}000(0.02)}{1 - 0.02} = 31.88$$

$${}_2V = \frac{(31.88 + 218.15)(1.06) - (9{,}000)(0.021)}{1 - 0.021} = 77.66$$

Question #163
Answer: D

$$e_x = e_y = \sum_{k=1}^{\infty} {}_t p_x = 0.95 + 0.95^2 + \ldots$$

$$= \frac{0.95}{1-0.95} = 19$$

$$e_{xy} = p_{xy} + {}_2 p_{xy} + \ldots$$

$$= 1.02(0.95)(0.95) + 1.02(0.95)^2(0.95)^2 + \ldots$$

$$= 1.02\left[0.95^2 + 0.95^4 + \ldots\right] = \frac{1.02(0.95)^2}{1-0.95^2} = 9.44152$$

$$e_{\overline{xy}} = e_x + e_y - e_{xy} = 28.56$$

Question #164 - Removed

Question #165 - Removed

Question #166
Answer: E

$$\bar{a}_x = \int_0^\infty e^{-0.08t} dt = 12.5$$

$$\bar{A}_x = \int_0^\infty e^{-0.08t}(0.03) dt = \frac{3}{8} = 0.375$$

$$^2\bar{A}_x = \int_0^\infty e^{-0.13t}(0.03) dt = \frac{3}{13} = 0.23077$$

$$\sigma\left(\bar{a}_{\overline{T|}}\right) = \sqrt{\text{Var}\left[\bar{a}_{\overline{T|}}\right]} = \sqrt{\frac{1}{\delta^2}\left[{}^2\bar{A}_x - \left(\bar{A}_x\right)^2\right]} = \sqrt{400\left[0.23077 - (0.375)^2\right]} = 6.0048$$

$$\Pr\left[\bar{a}_{\overline{T|}} > \bar{a}_x - \sigma\left(\bar{a}_{\overline{T|}}\right)\right] = \Pr\left[\bar{a}_{\overline{T|}} > 12.5 - 6.0048\right]$$

$$= \Pr\left[\frac{1-v^T}{0.05} > 6.4952\right] = \Pr\left[0.67524 > e^{-0.05T}\right]$$

$$= \Pr\left[T > \frac{-\ln\, 0.67524}{0.05}\right] = \Pr\left[T > 7.85374\right]$$

$$= e^{-0.03\times 7.85374} = 0.79$$

Question #167
Answer: A

$${}_5p_{50}^{(\tau)} = e^{-(0.05)(5)} = e^{-0.25} = 0.7788$$

$${}_5q_{55}^{(1)} = \int_0^5 \mu_{55+t}^{(1)} \times e^{-(0.03+0.02)t} dt = -\left(0.02/0.05\right) e^{-0.05t} \Big|_0^5$$

$$= 0.4\left(1 - e^{-0.25}\right)$$

= 0.0885

Probability of retiring before $60 = {}_5p_{50}^{(\tau)} \times {}_5q_{55}^{(1)}$
= 0.7788*0.0885
= 0.0689

Question #168
Answer: C

Complete the table:

$l_{81} = l_{[80]} - d_{[80]} = 910$

$l_{82} = l_{[81]} - d_{[81]} = 830$ (not really needed)

$\overset{\circ}{e}_x = e_x + \frac{1}{2} \qquad \left(\frac{1}{2} \text{ since UDD}\right)$

$\overset{\circ}{e}_{[x]} = e_{[x]} + 1/2$

$\overset{\circ}{e}_{[x]} = \left[\frac{l_{81} + l_{82} + l_{83} + \dots}{l_{[80]}}\right] + \frac{1}{2}$

$\left[\overset{\circ}{e}_{[80]} - \frac{1}{2}\right] l_{[80]} = l_{81} + l_{82} + \dots$ Call this equation (A)

$\left[\overset{\circ}{e}_{[81]} - \frac{1}{2}\right] l_{[81]} = l_{82} + \dots$ Formula like (A), one age later. Call this (B)

Subtract equation (B) from equation (A) to get

$$l_{81} = \left[\overset{\circ}{e}_{[80]} - \frac{1}{2}\right] l_{[80]} - \left[\overset{\circ}{e}_{[81]} - \frac{1}{2}\right] l_{[81]}$$

$$910 = [8.5 - 0.5]1000 - \left[\overset{\circ}{e}_{[81]} - 0.5\right]920$$

$$\overset{\circ}{e}_{[81]} = \frac{8000 + 460 - 910}{920} = 8.21$$

Alternatively, and more straightforward,

$$p_{[80]} = \frac{910}{1000} = 0.91$$
$$p_{[81]} = \frac{830}{920} = 0.902$$
$$p_{81} = \frac{830}{910} = 0.912$$

$$\mathring{e}_{[80]} = \frac{1}{2} q_{[80]} + p_{[80]} \left(1 + \mathring{e}_{81} \right)$$

where $q_{[80]}$ contributes $\frac{1}{2}$ since UDD

$$8.5 = \frac{1}{2}(1 - 0.91) + (0.91)\left(1 + \mathring{e}_{81} \right)$$

$$\mathring{e}_{81} = 8.291$$

$$\mathring{e}_{81} = \frac{1}{2} q_{81} + p_{81} \left(1 + \mathring{e}_{82} \right)$$

$$8.291 = \frac{1}{2}(1 - 0.912) + 0.912\left(1 + \mathring{e}_{82} \right)$$

$$\mathring{e}_{82} = 8.043$$

$$\mathring{e}_{[81]} = \frac{1}{2} q_{[81]} + p_{[81]} \left(1 + \mathring{e}_{82} \right)$$
$$= \frac{1}{2}(1 - 0.902) + (0.902)(1 + 8.043)$$
$$= 8.206$$

Or, do all the recursions in terms of *e*, not $\mathring{e}$, starting with $e_{[80]} = 8.5 - 0.5 = 8.0$, then final step $\mathring{e}_{[81]} = e_{[81]} + 0.5$

Question #169
Answer: A

T	p_{x+t}	${}_t p_x$	v^t	$v^t\ {}_t p_x$
0	0.7	1	1	1
1	0.7	0.7	0.95238	0.6667
2	–	0.49	0.90703	0.4444
3	–	–	–	–

From above $\ddot{a}_{x:\overline{3}|} = \sum_{t=0}^{2} v^t\ {}_t p_x = 2.1111$

$$1000\ {}_2V_{x:\overline{3}|} = 1000\left(1 - \frac{\ddot{a}_{x+2:\overline{1}|}}{\ddot{a}_{x:\overline{3}|}}\right) = 1000\left(1 - \frac{1}{2.1111}\right) = 526$$

Alternatively,

$$P_{x:\overline{3}|} = \frac{1}{\ddot{a}_{x:\overline{3}|}} - d = 0.4261$$

$$\begin{aligned} 1000\ {}_2V_{x:\overline{3}|} &= 1000\left(v - P_{x:\overline{3}|}\right) \\ &= 1000\left(0.95238 - 0.4261\right) \\ &= 526 \end{aligned}$$

You could also calculate $A_{x:\overline{3}|}$ and use it to calculate $P_{x:\overline{3}|}$.

Question #170
Answer: E

Let G be the gross premium.
Expected present value (EPV) of benefits = $1000A_{50}$.
EPV of expenses, except claim expense = $15+1\times\ddot{a}_{50}$
EPV of claim expense = $50A_{50}$ (50 is paid when the claim is paid)
EPV of premiums = $G\,\ddot{a}_{50}$
Equivalence principle: $G\ddot{a}_{50} = 1000A_{50} + 15 + 1\times\ddot{a}_{50} + 50A_{50}$

$$G = \frac{1050A_{50} + 15 + \ddot{a}_{50}}{\ddot{a}_{50}}$$

For the given survival function,

$$A_{50} = \frac{1}{l_{50}}\sum_{k=1}^{50} v^k (l_{50+k-1} - l_{50+k}) = \frac{1}{100}\sum_{k=1}^{50} v^k (2) = \frac{a_{\overline{50|}}}{50} = \frac{1-1.05^{-50}}{0.05(50)} = 0.36512$$

$$\ddot{a}_{50} = \frac{1-A_{50}}{d} = 13.33248$$

Solving for G, G = 30.88

Question #171
Answer: A

$${}_4p_{50} = e^{-(0.05)(4)} = 0.8187$$

$${}_{10}p_{50} = e^{-(0.05)(10)} = 0.6065$$

$${}_8p_{60} = e^{-(0.04)(8)} = 0.7261$$

$${}_{18}p_{50} = \left({}_{10}p_{50}\right)\left({}_8p_{60}\right) = 0.6065\times 0.7261$$

$$= 0.4404$$

$${}_{4|14}q_{50} = {}_4p_{50} - {}_{18}p_{50} = 0.8187 - 0.4404 = 0.3783$$

Question #172
Answer: D

$$\ddot{a}_{40:\overline{5}|} = \ddot{a}_{40} - {}_5E_{40}\,\ddot{a}_{45}$$
$$= 14.8166 - (0.73529)(14.1121)$$
$$= 4.4401$$

$$\pi\ddot{a}_{40:\overline{5}|} = 100{,}000 A_{45}\ v^5\ {}_5p_{40} + \pi\left(IA\right)^1_{40:\overline{5}|}$$
$$\pi = 100{,}000 A_{45} \times {}_5E_{40} / \left(\ddot{a}_{40:\overline{5}|} - \left(IA\right)^1_{40:\overline{5}|}\right)$$
$$= 100{,}000(0.20120)(0.73529)/(4.4401 - 0.04042)$$
$$= 3363$$

Question #173
Answer: B

Calculate the probability that both are alive or both are dead.
P(both alive) = ${}_kp_{xy} = {}_kp_x \cdot {}_kp_y$
P(both dead) = ${}_kq_{\overline{xy}} = {}_kq_x\,{}_kq_y$
P(exactly one alive) = $1 - {}_kp_{xy} - {}_kq_{\overline{xy}}$
Only have to do two year's worth so have table

Pr(both alive)	Pr(both dead)	Pr(only one alive)
1	0	0
(0.91)(0.91) = 0.8281	(0.09)(0.09) = 0.0081	0.1638
(0.82)(0.82) = 0.6724	(0.18)(0.18) = 0.0324	0.2952

$$EPV\ Annuity = 30{,}000\left(\frac{1}{1.05^0} + \frac{0.8281}{1.05^1} + \frac{0.6724}{1.05^2}\right) + 20{,}000\left(\frac{0}{1.05^0} + \frac{0.1638}{1.05^1} + \frac{0.2952}{1.05^2}\right) = 80{,}431$$

Alternatively,

$$\ddot{a}_{xy:\overline{3}|} = 1 + \frac{0.8281}{1.05} + \frac{0.6724}{1.05^2} = 2.3986$$
$$\ddot{a}_{x:\overline{3}|} = \ddot{a}_{y:\overline{3}|} = 1 + \frac{0.91}{1.05} + \frac{0.82}{1.05^2} = 2.6104$$
$$EPV = 20{,}000\,\ddot{a}_{x:\overline{3}|} + 20{,}000\,\ddot{a}_{y:\overline{3}|} - 10{,}000\,\ddot{a}_{xy:\overline{3}|}$$

(it pays 20,000 if *x* alive and 20,000 if y alive, but 10,000 less than that if both are alive)

$$= (20{,}000)(2.6104) + (20{,}000)(2.6104) - (10{,}000)2.3986\ \ = 80{,}430$$

Question #174
Answer: C

Let *P* denote the gross premium.

$$P = \overline{a}_x = \int_0^\infty e^{-\delta t} e^{-\mu t} dt = \int_0^\infty e^{-0.05t} dt = 20$$

$$E[L] = \overline{a}_x^{IMP} - P$$

$$\overline{a}_x^{IMP} = \int_0^{10} e^{-0.03t} e^{-0.02t} dt + e^{-0.03(10)} e^{-0.02(10)} \int_0^\infty e^{-0.03t} e^{-0.01t} dt$$

$$= \frac{l - e^{-0.5}}{0.05} + \frac{e^{-0.5}}{0.04} = 23$$

$$E[L] = 23 - 20 = 3$$

$$\frac{E[L]}{P} = \frac{3}{20} = 15\%$$

Question #175
Answer: C

$$A^{1}_{30:\overline{2}|} = 1000vq_{30} + 500v^2 {}_{1|}q_{30}$$

$$= 1000\left(\frac{1}{1.06}\right)(0.00153) + 500\left(\frac{1}{1.06}\right)^2 (0.99847)(0.00161)$$

$$= 2.15875$$

Initial fund $= 2.15875 \times 1000$ participants = 2158.75

Let F_n denote the size of Fund 1 at the end of year *n*.

$$F_1 = 2158.75(1.07) - 1000 = 1309.86$$

$$F_2 = 1309.86(1.065) - 500 = 895.00$$

Expected size of Fund 2 at end of year 2 = 0 (since the amount paid was the single benefit premium). Difference is 895.

Question #176
Answer: C

$$Var[Z] = E\left[Z^2\right] - E[Z]^2$$

$$E[Z] = \int_0^\infty \left(v^t b_t\right)_t p_x \mu_{x+t} dt = \int_0^\infty e^{-0.08t} e^{0.03t} e^{-0.02t} (0.02)\, dt$$

$$= \int_0^\infty (0.02) e^{-0.07t} dt = \frac{0.02}{0.07} = 2/7$$

$$E\left[Z^2\right] = \int_0^\infty \left(v_t\, b_t\right)^2{}_t p_x \mu_{x+t} dt = \int_0^\infty \left(e^{-0.05t}\right)^2 e^{-0.02t} (0.02)\, dt$$

$$= \int_0^\infty 0.02\, e^{-0.12t} \mu_{x+t} dt = 2/12 = 1/6$$

$$Var[Z] = \frac{1}{6} - \left(2/7\right)^2 = \frac{1}{6} - \frac{4}{49} = 0.08503$$

Question #177
Answer: C

From $A_x = 1 - d\ddot{a}x$ we have $A_x = 1 - \frac{0.1}{1.1}(8) = 3/11$

$$A_{x+10} = 1 - \frac{0.1}{1.1}(6) = 5/11$$

$$\bar{A}_x = A_x \times i/\delta$$

$$\bar{A}_x = \frac{3}{11} \times \frac{0.1}{\ln(1.1)} = 0.2861$$

$$\bar{A}_{x+10} = \frac{5}{11} \times \frac{0.1}{\ln(1.1)} = 0.4769$$

$${}_{10}V = \bar{A}_{x+10} - P\left(\bar{A}_x\right) \times \ddot{a}_{x+10}$$

$$= 0.4769 - \left(\frac{0.2861}{8}\right)6$$

= 0.2623

There are many other equivalent formulas that could be used.

Question #178
Answer: C

Regular death benefit $=\int_0^{\infty}100,000\times e^{-0.06t}\times e^{-0.001t}\,0.001dt$

$$=100,000\left(\frac{0.001}{0.06+0.001}\right)$$

$$=1639.34$$

Accidental death $=\int_0^{10}100,000\,e^{-0.06t}e^{-0.001t}\left(0.0002\right)dt$

$$=20\int_0^{10}e^{-0.061t}dt$$

$$=20\left[\frac{1-e^{-0.61}}{0.061}\right]=149.72$$

Expected Present Value $=1639.34+149.72=1789.06$

Question #179
Answer: C

$p_{61}^{00}=560/800=0.70$
$p_{61}^{01}=160/800=0.20$
$p_{61}^{10}=0,$ once dead, stays dead
$p_{61}^{11}=1,$ once dead by cause 1, stays dead by cause 1

$$p_{61}^{00}+p_{61}^{01}+p_{61}^{10}+p_{61}^{11}=0.70+0.20+0+1=1.90$$

Question #180 - Removed

Question #181
Answer: B

$$\Pr(\text{dies in year 1}) = p^{02} = 0.1$$
$$\Pr(\text{dies in year 2}) = p^{00}p^{02} + p^{01}p^{12} = 0.8(0.1) + 0.1(0.2) = 0.10$$
$$\Pr(\text{dies in year 3}) = p^{00}p^{00}p^{02} + p^{00}p^{01}p^{12} + p^{01}p^{11}p^{12} + p^{01}p^{10}p^{02} = 0.095$$
$$EPV(\text{benefits}) = 100,000[0.9(0.1) + 0.9^2(0.10) + 0.9^3(0.095)] = 24,025.5$$

$$\Pr(\text{in State 0 at time 0}) = 1$$
$$\Pr(\text{in State 0 at time 1}) = p^{00} = 0.8$$
$$\Pr(\text{in State 0 at time 2}) = p^{00}p^{00} + p^{01}p^{10} = 0.8(0.8) + 0.1(0.1) = 0.65$$
$$EPV(\$1 \text{ of premium}) = 1 + 0.9(0.8) + 0.9^2(0.65) = 2.2465$$

Benefit premium = 24,025.5/2.2465 = 10,695.

Question #182 - Removed
Answer: A

Question #183 - Removed

Question #184
Answer: B

$$1000P_{45}\ddot{a}_{45:\overline{15}|} + \pi\,\ddot{a}_{60:\overline{15}|} \times {}_{15}E_{45} = 1000A_{45}$$

$$1000\frac{A_{45}}{\ddot{a}_{45}}\left(\ddot{a}_{45} - {}_{15}E_{45}\,\ddot{a}_{60}\right) + \pi\left(\ddot{a}_{60} - {}_{15}E_{60}\,\ddot{a}_{75}\right)\left({}_{15}E_{45}\right) = 1000A_{45}$$

$$\frac{201.20}{14.1121}\left(14.1121 - (0.72988)(0.51081)(11.1454)\right)$$
$$+\pi\left(11.1454 - (0.68756)(0.39994)(7.2170)\right) \times (0.72988)(0.51081) = 201.20$$

where ${}_{15}E_x$ was evaluated as ${}_5E_x \times {}_{10}E_{x+5}$

$$14.2573(9.9568) + (\pi)(3.4154) = 201.20$$

$$\pi = 17.346$$

Question #185
Answer: A

$$ {}_1V = \left({}_0V + \pi\right)(1+i) - \left(1000 + {}_1V - {}_1V\right) q_x $$
$$ {}_2V = \left({}_1V + \pi\right)(1+i) - \left(2000 + {}_2V - {}_2V\right) q_{x+1} = 2000 $$
$$ \left(\left(\pi(1+i) - 1000 q_x\right) + \pi\right)(1+i) - 2000 q_{x+1} = 2000 $$
$$ \left(\left(\pi(1.08) - 1000 \times 0.1\right) + \pi\right)(1.08) - 2000 \times 0.1 = 2000 $$

$$ \pi = 1027.42 $$

Question #186
Answer: A

Let *Y* be the present value of payments to 1 person.
Let *S* be the present value of the aggregate payments.

$$ E[Y] = 500\,\ddot{a}_x = 500\frac{(1-A_x)}{d} = 5572.68 $$
$$ \sigma_Y = \sqrt{\text{Var}[Y]} = \sqrt{(500)^2 \frac{1}{d^2}\left({}^2A_x - A_x^2\right)} = 1791.96 $$
$$ S = Y_1 + Y_2 + \ldots + Y_{250} $$
$$ E(S) = 250E[Y] = 1{,}393{,}170 $$
$$ \sigma_S = \sqrt{250} \times \sigma_Y = 15.811388\,\sigma_Y = 28{,}333 $$
$$ 0.90 = \Pr(S \le F) = \Pr\left[\frac{S - 1{,}393{,}170}{28{,}333} \le \frac{F - 1{,}393{,}170}{28{,}333}\right] $$
$$ \approx \Pr\left[N(0,1) \le \frac{F - 1{,}393{,}170}{28{,}333}\right] $$
$$ 0.90 = \Pr\left(N(0,1) \le 1.28\right) $$
$$ F = 1{,}393{,}170 + 1.28(28{,}333) $$
=1.43 million

Question #187
Answer: A

$$q'^{(1)}_{41} = 1 - p'^{(1)}_{41} = 1 - \left(p_{41}^{(\tau)}\right)^{q_{41}^{(1)} / q_{41}^{(\tau)}}$$

$$l_{41}^{(\tau)} = l_{40}^{(\tau)} - d_{40}^{(1)} - d_{40}^{(2)} = 1000 - 60 - 55 = 885$$

$$d_{41}^{(1)} = l_{41}^{(\tau)} - d_{41}^{(2)} - l_{42}^{(\tau)} = 885 - 70 - 750 = 65$$

$$p_{41}^{(\tau)} = \frac{750}{885} \qquad \frac{q_{41}^{(1)}}{q_{41}^{(\tau)}} = \frac{65}{135}$$

$$q'^{(1)}_{41} = 1 - \left(\frac{750}{885}\right)^{65/135} = 0.0766$$

Question #188
Answer: D

$$S_0(t) = \left(1 - \frac{t}{\omega}\right)^{\alpha}$$

$$\mu_t = \frac{d}{dt} \log\left(S_0(t)\right) = \frac{\alpha}{\omega - t}$$

$$\mathring{e}_x = \int_0^{\omega - x}\left(1 - \frac{t}{\omega - x}\right)^{\alpha} dt = \frac{\omega - x}{\alpha + 1}$$

$$\mathring{e}_0^{\text{new}} = \frac{1}{2} \times \frac{\omega}{\alpha^{\text{old}} + 1} = \frac{\omega}{\alpha^{\text{new}} + 1} \Rightarrow \alpha^{\text{new}} = 2\alpha^{\text{old}} + 1$$

$$\mu_0^{(\text{new})} = \frac{2\alpha^{\text{old}} + 1}{\omega} = \frac{9}{4} \times \frac{\alpha^{\text{old}}}{\omega} \Rightarrow \alpha^{\text{old}} = 4$$

Question #189
Answer: C

Constant force implies exponential lifetime

$$Var[T] = E\left[T^2\right] - \left(E[T]\right)^2 = \frac{2}{\mu^2} - \left(\frac{1}{\mu}\right)^2 = \frac{1}{\mu^2} = 100$$

$$\mu = 0.1$$

$$E\left[\min(T,10)\right] = \int_0^{10} t(0.1)e^{-.1t}dt + \int_{10}^{\infty} 10(0.1)e^{-.1t}dt$$

$$= -te^{-.1t} - 10e^{-.1t}\Big|_0^{10} - 10e^{-.1t}\Big|_{10}^{\infty}$$

$$= -10e^{-1} - 10e^{-1} + 10 + 10e^{-1}$$

$$= 10\left(1 - e^{-1}\right) = 6.3$$

Question #190
Answer: A

$$G\ddot{a}_{x:\overline{15|}} = 100{,}000A_x + \overbrace{\left(0.08G + 0.02G\ddot{a}_{x:\overline{15|}}\right)}^{\text{\% premium amount for 15 years}} + \underbrace{\left(\left(x-5\right) + 5\ddot{a}_x\right)}_{\text{Per policy for life}}$$

$$4669.95(11.35) = 51{,}481.97 + (0.08)(4669.95) + (0.02)(11.35)(4669.95) + \left((x-5) + 5\ddot{a}_x\right)$$

$$\ddot{a}_x = \frac{1 - Ax}{d} = \frac{1 - 0.5148197}{0.02913} = 16.66$$

$$53{,}003.93 = 51{,}481.97 + 1433.67 + (x-5) + 83.30$$

$$4.99 = (x-5)$$

$$x = 9.99$$

The % of premium expenses could equally well have been expressed as $0.10G + 0.02G\,a_{x:\overline{14|}}$.

The per policy expenses could also be expressed in terms of an annuity-immediate.

Question #191
Answer: D

For the density where $T_x \neq T_y$,

$$\Pr\left(T_x < T_y\right) = \int_{y=0}^{40} \int_{x=0}^{y} 0.0005 dx dy + \int_{y=40}^{50} \int_{x=0}^{40} 0.0005 dx dy$$

$$= \int_{y=0}^{40} 0.0005\, x \Big|_0^y dy + \int_{y=40}^{50} 0.0005 x \Big|_0^{40} dy$$

$$= \int_0^{40} 0.0005 y dy + \int_{y=40}^{50} 0.02\, dy$$

$$= \frac{0.0005 y^2}{2} \Big|_0^{40} + 0.02 y \Big|_{40}^{50}$$

$$= 0.40 + 0.20 = 0.60$$

For the overall density,

$$\Pr\left(T_x < T_y\right) = 0.4 \times 0 + 0.6 \times 0.6 = 0.36$$

where the first 0.4 is the probability that $T_x = T_y$ and the first 0.6 is the probability that $T_x \neq T_y$.

Question #192
Answer: B

The conditional expected value of the annuity, given μ, is $\frac{1}{0.01 + \mu}$.

The unconditional expected value is

$$\bar{a}_x = 100 \int_{0.01}^{0.02} \frac{1}{0.01 + \mu} d\mu = 100 \ln\left(\frac{0.01 + 0.02}{0.01 + 0.01}\right) = 40.5$$

100 is the constant density of μ on the interval $[0.01, 0.02]$. If the density were not constant, it would have to go inside the integral.

Question #193
Answer: E

Recall $\overset{\circ}{e}_x = \dfrac{\omega - x}{2}$

$$\overset{\circ}{e}_{\overline{x:x}} = \overset{\circ}{e}_x + \overset{\circ}{e}_x - \overset{\circ}{e}_{x:x}$$

$$\overset{\circ}{e}_{x:x} = \int_0^{\omega - x}\left(1 - \frac{t}{\omega - x}\right)\left(1 - \frac{t}{\omega - y}\right)dt$$

Performing the integration we obtain

$$\overset{\circ}{e}_{x:x} = \frac{\omega - x}{3}$$

$$\overset{\circ}{e}_{\overline{x:x}} = \frac{2(\omega - x)}{3}$$

(i) $\dfrac{2(\omega - 2a)}{3} = 3 \times \dfrac{2(\omega - 3a)}{3} \Rightarrow 2\omega = 7a$

(ii) $\dfrac{2}{3}(\omega - a) = k \times \dfrac{2(\omega - 3a)}{3}$

$$3.5a - a = k(3.5a - 3a)$$

$$k = 5$$

The solution assumes that all lifetimes are independent.

Question #194
Answer: B

Although simultaneous death is impossible, the times of death are dependent as the force of mortality increases after the first death. There are two ways for the benefit to be paid. The first is to have (*x*) die prior to (*y*). That is, the transitions are State 0 to State 2 to State 3. The EPV can be written with a double integral

$$\int_0^\infty e^{-0.04t}\,{}_t p_{xy}^{00}\mu_{x+t:y+t}^{02}\int_0^\infty e^{-0.04r}\,{}_r p_{x+t:y+t}^{22}\mu_{x+t+r:y+t+r}^{23}drdt$$

$$= \int_0^\infty e^{-0.04t}e^{-0.12t}0.06\int_0^\infty e^{-0.04r}e^{-0.10r}0.10drdt = \frac{0.06}{0.16}\frac{0.10}{0.14} = 0.26786$$

By symmetry, the second case (State 0 to State 1 to State 3) has the same EPV. Thus the total EPV is 10,000(0.26786+0.26786) = 5,357.

Question #195
Answer: E

Let ${}_k p_0 =$ Probability someone answers the first *k* problems correctly.

$${}_2 p_0 = (0.8)^2 = 0.64 \qquad {}_4 p_0 = (0.8)^4 = 0.41$$

$${}_2 p_{0:0} = ({}_2 p_0)^2 = 0.64^2 = 0.41 \qquad {}_4 p_{0:0} = (0.41)^2 = 0.168$$

$${}_2 p_{\overline{0:0}} = {}_2 p_0 + {}_2 p_0 - {}_2 p_{0:0} = 0.87 \qquad {}_4 p_{\overline{0:0}} = 0.41 + 0.41 - 0.168 = 0.652$$

Prob(second child loses in round 3 or 4) = ${}_2 p_{\overline{0:0}} - {}_4 p_{\overline{0:0}}$
= 0.87-0.652
= 0.218

Prob(second loses in round 3 or $4 |$ second loses after round 2) = $\dfrac{{}_2 p_{\overline{0:0}} - {}_4 p_{\overline{0:0}}}{{}_2 p_{\overline{0:0}}}$

= $\dfrac{0.218}{0.87} = 0.25$

Question #196
Answer: E

If (40) dies before 70, he receives one payment of 10, and *Y* = 10. The probability of this is (70 – 40)/(110 – 40) = 3/7

If (40) reaches 70 but dies before 100, he receives 2 payments.
$Y = 10 + 20v^{30} = 16.16637$
The probability of this is also 3/7.

If (40) survives to 100, he receives 3 payments.
$Y = 10 + 20v^{30} + 30v^{60} = 19.01819$
The probability of this is 1 – 3/7 – 3/7 = 1/7

$$E(Y) = (3/7)\times 10 + (3/7)\times 16.16637 + (1/7)\times 19.01819 = 13.93104$$

$$E(Y^2) = (3/7)\times 10^2 + (3/7)\times 16.16637^2 + (1/7)\times 19.01819^2 = 206.53515$$

$$Var(Y) = E(Y^2) - [E(Y)]^2 = 12.46$$

Since everyone receives the first payment of 10, you could have ignored it in the calculation.

Question #197
Answer: C

$$\mathrm{E}(Z) = \sum_{k=0}^{2} \left(v^{k+1} b_{k+1}\right) \quad {}_k p_x \quad q_{x+k}$$

$$= \left[v(300) \times 0.02 + v^2 (350)(0.98)(0.04) + v^3 400 (0.98)(0.96)(0.06) \right]$$

$$= 36.8$$

$$E\left(Z^2\right) = \sum_{k=0}^{2} \left(v^{k+1} b_{k+1}\right)^2 \quad {}_k p_x \quad q_{x+k}$$

$$= v^2 (300)^2 \times 0.02 + v^4 (350)^2 (0.98)(0.04) + v^6 400^2 (0.98)(0.96) 0.06$$

$$= 11,773$$

$$Var[Z] = E\left(Z^2\right) - E(Z)^2$$

$$= 11,773 - 36.8^2$$

$$= 10,419$$

Question #198
Answer: A

Benefits + Expenses – Premiums

$${}_0L_e = 1000v^3 + \left(0.20G + 8\right) + \left(0.06G + 2\right)v + \left(0.06G + 2\right)v^2 - G\ddot{a}_{\overline{3|}}$$

at $G = 41.20$ and $i = 0.05$,

${}_0L$ (for $K = 2$) $= 770.59$

Question #199
Answer: D

$P = 1000P_{40}$

$(235 + P)(1+i) - 0.015(1000-255) = 255$ [A]

$(255 + P)(1+i) - 0.020(1000-272) = 272$ [B]

Subtract [A] from [B]

$$20(1+i) - 3.385 = 17$$

$$1+i = \frac{20.385}{20} = 1.01925$$

Plug into [A] $(235 + P)(1.01925) - 0.015(1000-255) = 255$

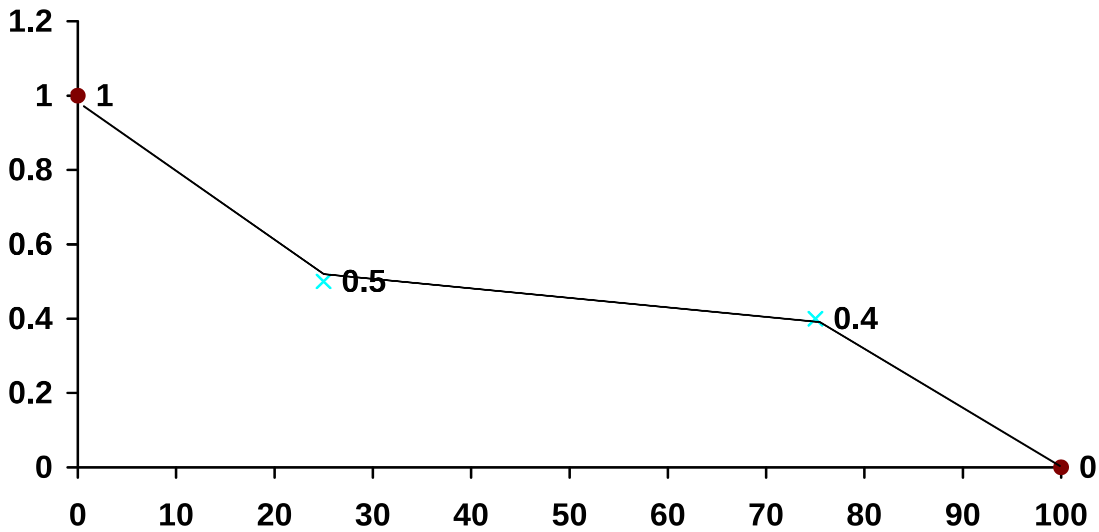
$$235 + P = \frac{255 + 11.175}{1.01925}$$

$$P = 261.15 - 235 = 26.15$$

$$1000\ {}_{25}V = \frac{(272 + 26.15)(1.01925) - (0.025)(1000)}{1 - 0.025}$$

$$= 286$$

Question #200
Answer: A



	Given		Given		Given		Given
x	0	15	25	35	75	90	100
$s(x)$	1	0.70	0.50	0.48	0.4	0.16	0
		Linear Interpolation		Linear Interpolation		Linear Interpolation	

$$_{55}q_{35} = 1 - \frac{s(90)}{s(35)} = 1 - \frac{0.16}{0.48} = \frac{32}{48} = 0.6667$$

$$_{20|55}q_{15} = \frac{s(35) - s(90)}{s(15)} = \frac{0.48 - 0.16}{0.70} = \frac{32}{70} = 0.4571$$

$$\frac{_{20|55}q_{15}}{_{55}q_{35}} = \frac{0.4571}{0.6667} = 0.6856$$

Alternatively,

$$\frac{{}_{20|55}q_{15}}{{}_{55}q_{35}} = \frac{{}_{20}p_{15} \times {}_{55}q_{35}}{{}_{55}q_{35}} = {}_{20}p_{15} = \frac{s(35)}{s(15)}$$
$$= \frac{0.48}{0.70}$$
$$= 0.6856$$

Question #201
Answer: A

$$S_0(80) = \tfrac{1}{2} * \left(e \wedge (-0.16*50) + e \wedge (-0.08*50)\right) = 0.00932555$$
$$S_0(81) = \tfrac{1}{2} * \left(e \wedge (-0.16*51) + e \wedge (-0.08*51)\right) = 0.008596664$$

$$p_{80} = s(81)/s(80) = 0.008596664/0.00932555 = 0.9218$$
$$q_{80} = 1 - 0.9218 = 0.078$$

Alternatively (and equivalent to the above)
For non-smokers, $p_x = e^{-0.08} = 0.923116$
$${}_{50}p_x = 0.018316$$
For smokers, $p_x = e^{-0.16} = 0.852144$
$${}_{50}p_x = 0.000335$$

So the probability of dying at 80, weighted by the probability of surviving to 80, is

$$\frac{0.018316 \times (1 - 0.923116) + 0.000335 \times (1 - 0.852144)}{0.018316 + 0.000335} = 0.078$$

Question #202
Answer: B

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
40	2000	20	60
41	1920	30	50
42	1840	40	

because $2000-20-60=1920$; $1920-30-50=1840$

Let premium = P

EPV premiums = $\left(\frac{2000}{2000}+\frac{1920}{2000}v+\frac{1840}{2000}v^2\right)P=2.749P$

EPV benefits = $1000\left(\frac{20}{2000}v+\frac{30}{2000}v^2+\frac{40}{2000}v^3\right)=40.41$

$P=\frac{40.41}{2.749}=14.7$

Question #203
Answer: A

$$\bar{a}_{30} = \int_0^{10} e^{-0.08t}\, e^{-0.05} dt + {}_{10}E_x \int_0^{\infty} e^{-0.08t} e^{-0.08t} dt$$

$$= \int_0^{10} e^{-0.13t} dt + e^{-1.3} \int_0^{\infty} e^{-0.16t} dt$$

$$\frac{-e^{-0.13t}}{0.13}\Big|_0^{10} + \left(e^{-1.3}\right)\frac{-e^{-0.16t}}{0.16}\Big|_0^{\infty}$$

$$= \frac{-e^{1.3}}{0.13} + \frac{1}{0.13} + \frac{e^{-1.3}}{0.16}$$

= 7.2992

$$\bar{A}_{30} = \int_0^{10} e^{-0.08t} e^{-0.05t}\left(0.05\right) dt + e^{-1.3}\int_0^{\infty} e^{-0.16t}\left(0.08\right) dt$$

$$= 0.05\left(\frac{1}{0.13} - \frac{e^{-1.3}}{0.13}\right) + \left(0.08\right)\frac{e^{-1.3}}{0.16}$$

$$= 0.41606$$

$$= \bar{P}\left(\bar{A}_{30}\right) = \frac{\bar{A}_{30}}{\bar{a}_{30}} = \frac{0.41606}{7.29923} = 0.057$$

$$\bar{a}_{40} = \frac{1}{0.08 + 0.08} = \frac{1}{0.16}$$

$$\bar{A}_{40} = 1 - \delta\, \bar{a}_{40}$$

$$= 1 - \left(0.08 / 0.16\right) = 0.5$$

$${}_{10}\bar{V}\left(\bar{A}_{30}\right) = \bar{A}_{40} - \bar{P}\left(\bar{A}_{30}\right)\bar{a}_{40}$$

$$= 0.5 - \frac{\left(0.057\right)}{0.16} = 0.14375$$

Question #204
Answer: C

Let T be the future lifetime of Pat, and [T] denote the greatest integer in T. ([T] is the same as K, the curtate future lifetime).

$L = 100,000\, v^T - 1600\, \ddot{a}_{\overline{[T]+1}\rceil}$	$0 < T \le 10$
$= 100,000 v^T - 1600\, \ddot{a}_{\overline{10}\rceil}$	$10 < t \le 20$
$-1600\, \ddot{a}_{\overline{10}\rceil}$	20<t

Minimum is $-1600\, \ddot{a}_{\overline{10}\rceil}$ when evaluated at *i* = 0.05

$= -12,973$

Question #205 - Removed

Question #206
Answer: A

$P\ddot{a}_{x:\overline{3}|} = EPV$ (stunt deaths)

$$P\left[\frac{2500+2486/1.08+2466/(1.08)^2}{2500}\right] = 500000\left(\frac{4/1.08+5/(1.08)^2+6/(1.08)^3}{2500}\right)$$

$$P(2.77) = 2550.68$$

$$\Rightarrow p = 921$$

Question #207
Answer: D

$$\mathring{e}_{30:\overline{50}|} = \frac{\int_{30}^{80} s(x)\,dx}{s(30)} = \frac{\int_{30}^{80}\left(1-\frac{x^2}{10,000}\right)dx}{1-\left(\frac{30}{100}\right)^2}$$

$$= \frac{\left(x-\frac{x^3}{30,000}\right)\Big|_{30}^{80}}{0.91}$$

$$= \frac{33.833}{0.91}$$

$$= 37.18$$

Question #208
Answer: B

$$A_{60} = v\times(p_{60}\times A_{61}+q_{60})$$

$$= (1/1.06)\times(0.98\times 0.440+0.02)$$

$$= 0.42566$$

$$\ddot{a}_{60} = (1-A_{60})/d$$

$$= (1-0.42566)/(0.06/1.06)$$

$$= 10.147$$

$$1000\,{}_{10}V = 1000A_{60} - 1000P_{50}\times\ddot{a}_{60}$$

$$= 425.66 - 10.147\times 25$$

$$= 172$$

Question #209
Answer: E

Let $Y_{65} =$ present value random variable for an annuity due of one on a single life age 65.
Thus $E(Y_{65}) = \ddot{a}_{65}$
Let $Y_{75} =$ present value random variable for an annuity due of one on a single life age 75.
Thus $E(Y_{75}) = \ddot{a}_{75}$

E (X) $= 50(2)\ddot{a}_{65} + 30(1)\ddot{a}_{75}$

$$= 100(9.8969) + 30(7.217) = 1206.20$$

Var (X) $= 50 \times 2^2 Var[Y_{65}] + 30(1)^2 Var[Y_{75}] = 200(13.2996) + 30(11.5339) = 3005.94$

where $Var[Y_{65}] = \frac{1}{d^2}\left({}^2A_{65} - A_{65}^2\right) = \frac{1}{(0.05660)^2}\left[0.23603 - (0.4398)^2\right] = 13.2996$

and $Var[Y_{75}] = \frac{1}{d^2}\left({}^2A_{75} - A_{75}^2\right) = \frac{1}{(0.05660)^2}\left[0.38681 - (0.59149)^2\right] = 11.5339$

$$95^{th} \text{ percentile} = E(\mathrm{X}) + 1.645\sqrt{Var[\mathrm{X}]}$$

$$= 1206.20 + 1.645(54.826)$$

$$= 1296.39$$

Question #210
Answer: C

$$\bar{a} = \int_0^\infty e^{-\delta t} \times e^{-\mu t} dt = \frac{1}{\delta + \mu}$$

$$EPV = 50,000 \times \frac{1}{0.5} \int_{0.5}^{1} \frac{1}{\delta + \mu} d\mu = 100,000 \times \left[\ln(\delta + 1) - \ln(\delta + 0.5)\right]$$

$$= 100,000 \times \ln\left(\frac{0.045 + 1}{0.045 + 0.5}\right)$$

$$= 65,099$$

Question #211 - Removed

Question #212 - Removed

Question #213 - Removed

Question #214
Answer: A

Let π be the benefit premium at issue.

$$\pi = 10,000 \frac{A_{45:\overline{20|}}}{\ddot{a}_{45:\overline{20|}}} = 10,000 \frac{\left[0.20120 - 0.25634\left(0.43980\right) + 0.25634\right]}{14.1121 - 0.25634\left(9.8969\right)}$$
$$= 297.88$$

The expected prospective loss at age 60 is

$$10,000\,{}_{15}V_{45:\overline{20|}} = 10,000 A_{60:\overline{5|}} - 297.88\,\ddot{a}_{60:\overline{5|}}$$
$$= 10,000 \times 0.7543 - 297.88 \times 4.3407$$
$$= 6250$$

where $A^{1}_{60:\overline{5|}} = 0.36913 - 0.68756\left(0.4398\right) = 0.06674$

$A_{60:\overline{5|}}^{\;\;\;\;1} = 0.68756$

$A_{60:\overline{5|}} = 0.06674 + 0.68756 = 0.7543$

$\ddot{a}_{60:\overline{5|}} = 11.1454 - 0.68756 \times 9.8969 = 4.3407$

After the change, expected prospective loss = $10,000 A^{1}_{60:\overline{5|}} + (\text{Reduced Amount})\, A_{60:\overline{5|}}^{\;\;\;\;1}$

Since the expected prospective loss is the same

$$6250 = \left(10,000\right)\left(0.06674\right) + \left(\text{Reduced Amount}\right)\left(0.68756\right)$$

Reduced Amount = 8119

Question #215
Answer: D

$$\bar{A}_x = \bar{A}^{1}_{x:\overline{5}|} + {}_5E_x\,\bar{A}^{\;1}_{x+5:\overline{7}|} + {}_{12}E_x\,\bar{A}_{x+12}$$

where

$${}_5E_x = e^{-5(0.04+0.02)} = 0.7408$$

$$\bar{A}^{1}_{x:\overline{5}|} = \frac{0.04}{0.04+0.02}\times(1-0.7408) = 0.1728$$

$${}_7E_{x+5} = e^{-7(0.05+0.02)} = 0.6126$$

$$\bar{A}^{\;1}_{x+5:\overline{7}|} = \left(\frac{0.05}{0.05+0.02}\right)(1-0.6126) = 0.2767$$

$${}_{12}E_x = {}_5E_x \times {}_7E_{x+5} = 0.7408\times 0.6126 = 0.4538$$

$$\bar{A}_{x+12} = \frac{0.05}{0.05+0.03} = 0.625$$

$$\bar{A}_x = 0.1728 + (0.7408)(0.2767) + (0.4538)(0.625)$$

= 0.6614

Question #216
Answer: A

EPV of Accidental death benefit and related settlement expense =

$$(2000\times 1.05)\times\frac{0.004}{0.004+0.04+0.05} = 89.36$$

EPV of other DB and related settlement expense = $(1000\times 1.05)\times\dfrac{0.04}{0.094} = 446.81$

EPV of Initial expense = 50

EPV of Maintenance expense = $\dfrac{3}{0.094} = 31.91$

EPV of future premiums $= \dfrac{100}{0.094} = 1063.83$

EPV of ${}_0L = 89.36 + 446.81 + 50 + 31.91 - 1063.83$

$= -445.75$

Question #217 - Removed

Question #218 - Removed

Question #219
Answer: E

$${}_{0.25|1.5}q_x = {}_{0.25}p_x - {}_{1.75}p_x$$

Let μ be the force of mortality in year 1, so 3μ is the force of mortality in year 2.
Probability of surviving 2 years is 10%

$$\begin{cases} 0.10 = p_x \; p_{x+1} = e^{-\mu}e^{-3\mu} = e^{-4\mu} \\ \mu = \dfrac{\ln(0.1)}{4} = 0.5756 \end{cases}$$

$${}_{0.25}p_x = e^{-1/4(0.5756)} = 0.8660$$

$${}_{1.75}p_x = p_x \times {}_{0.75}p_{x+1} = e^{-\mu}e^{-\frac{3}{4}(3\mu)} = e^{-\frac{13}{4}(0.5756)} = 0.1540$$

$${}_{1.5}q_{x+0.25} = \frac{{}_{0.25|1.5}q_x}{{}_{0.25}p_x} = \frac{{}_{0.25}p_x - {}_{1.75}p_x}{{}_{0.25}p_x} = \frac{0.866-0.154}{0.866} = 0.82$$

Question #220
Answer: C

$$\mu_x^{NS} = \frac{500}{500(110-x)} = \frac{1}{110-x}$$

$$= 1/2 \; \mu_x^S \Rightarrow \mu_x^S = \frac{2}{110-x}$$

$\Rightarrow l_x^S = l_0^S\left(110-x\right)^2$ [see note below]

Thus $${}_tp_{20}^S = \frac{l_{20+t}^S}{l_{20}^S} = \frac{\left(90-t\right)^2}{90^2}$$

$${}_tp_{25}^{NS} = \frac{l_{25+t}^{NS}}{l_{25}^{NS}} = \frac{\left(85-t\right)}{85}$$

$$\overset{\circ}{e}_{20:25} = \int_0^{85} {}_t p_{20:25} dt$$

$$= \int_0^{85} {}_t p_{20}^{S} \; {}_t p_{25}^{NS} dt = \int_0^{85} \frac{(90-t)^2}{(90)^2} \frac{(85-t)}{85} dt$$

$$= \frac{1}{688,500} \int_0^{85} (90-t)^2 (90-t-5) dt$$

$$= \frac{1}{688,500} \left[\int_0^{85} (90-t)^3 dt - 5 \int_0^{85} (90-t)^2 dt \right.$$

$$= \frac{1}{688,500} \left[\frac{-(90-t)^4}{4} + \frac{5(90-t)^3}{3} \right]_0^{85}$$

$$= \frac{1}{688,500} [-156.25 + 208.33 + 16,402,500 - 1,215,000]$$

= 22.1

[There are other ways to evaluate the integral, leading to the same result].

The $S_0(x)$ form is derived as $S_0(x) = e^{-\int_0^x \left(\frac{2}{110-t}\right) dt} = e^{2\ln(110-t)\big|_0^x} = \left(\frac{110-x}{110}\right)^2$

The l_x form is equivalent.

Question #221
Answer: B

$\ddot{a}_{30:\overline{20}|} = \ddot{a}_{30:\overline{10}|} + {}_{10}E_{30} \times \ddot{a}_{40:\overline{10}|}$

$15.0364 = 8.7201 + {}_{10}E_{30} \times 8.6602$

${}_{10}E_{30} = (15.0364 - 8.7201) / 8.6602 = 0.72935$

Expected present value (EPV) of benefits =

$= 1000 \times A^{1}_{40:\overline{10}|} + 2000 \times {}_{10}E_{30} \times A^{1}_{50:\overline{10}|}$

$= 16.66 + 2 \times 0.72935 \times 32.61 = 64.23$

EPV of premiums = $\pi \times \ddot{a}_{30:\overline{10}|} + 2\pi \times 0.72935 \times \ddot{a}_{40:\overline{10}|}$

$= \pi \times 8.7201 + 2 \times \pi \times 0.72935 \times 8.6602$

$= 21.3527\pi$

$\pi = 64.23 / 21.3527 = 3.01$

Question #222
Answer: A

$${}_{15}V = P_{25}\,\ddot{s}_{25:\overline{15}|} - \frac{A^{1}_{25:\overline{15}|}}{{}_{15}E_{25}} \qquad \text{(this is the retrospective reserve calculation)}$$

$$P^{1}_{25:\overline{15}|} = P_{25:\overline{15}|} - P_{25:\overset{1}{\overline{15}|}} = 0.05332 - 0.05107$$
$$= 0.00225$$
$$= \frac{A^{1}_{25:\overline{15}|}}{\ddot{a}_{25:\overline{15}|}}$$

$$0.05107 = P_{25:\overset{1}{\overline{15}|}} = \frac{{}_{15}E_{25}}{\ddot{a}_{25:\overline{15}|}} = \frac{1}{\ddot{s}_{25:\overline{15}|}}$$

$$\frac{A^{1}_{25:\overline{15}|}}{{}_{15}E_{25}} = \frac{A^{1}_{25:\overline{15}|}/\ddot{a}_{25:\overline{15}|}}{{}_{15}E_{25}/\ddot{a}_{25:\overline{15}|}} = \frac{0.00225}{0.05107} = 0.04406$$

$$P_{25}\,\ddot{s}_{25:\overline{15}|} = \frac{0.01128}{0.05107} = 0.22087$$

$$25{,}000\,{}_{15}V = 25{,}000\left(0.22087 - 0.04406\right) = 25{,}000\left(0.17681\right) = 4420$$

There are other ways of getting to the answer, for example writing

A: the retrospective reserve formula for ${}_{15}V$.

B: the retrospective reserve formula forthe 15th benefit reserve for a 15-year term insurance issued to (25), which = 0

Subtract B from A to get

$$\left(P_{25} - P^{1}_{25:\overline{15}|}\right)\ddot{s}_{25:\overline{15}|} = {}_{15}V$$

Question #223
Answer: C

ILT:

We have $p_{70} = 6{,}396{,}609/6{,}616{,}155 = 0.96682$

${}_{2}p_{70} = 6{,}164{,}663/6{,}616{,}155 = 0.93176$

$e_{70:\overline{2}|} = 0.96682 + 0.93176 = 1.89858$

CF: 0.93176 $= {}_{2}p_{70} = e^{-2\mu} \;\Rightarrow\; \mu = 0.03534$

Hence $e_{70:\overline{2}|} = p_{70} + {}_{2}p_{71} = e^{-\mu} + e^{-2\mu} = 1.89704$

DM: Since l_{70} and ${}_{2}p_{70}$ for the DM model equal the ILT, therefore l_{72} for the DM model also equals the ILT. For DM we have $l_{70} - l_{71} = l_{71} - l_{72} \Rightarrow l_{71}^{(\text{DM})} = 6{,}390{,}409$

Hence $e_{70:\overline{2|}} = 6{,}390{,}409/6{,}616{,}155 + 6{,}164{,}663/6{,}616{,}155 = 1.89763$

So the correct order is CF < DM < ILT

You could also work with *p*'s instead of *l*'s. For example, with the ILT,

$$p_{70} = (1 - 0.03318) = 0.96682$$
$${}_2p_{70} = (0.96682)(1 - 0.03626) = 0.93176$$

Note also, since $e_{70:\overline{2|}} = p_{70} + {}_2p_{70}$, and ${}_2p_{70}$ is the same for all three, you could just order p_{70}.

Question #224
Answer: D

$$l_{60}^{(\tau)} = 1000$$
$$l_{61}^{(\tau)} = 1000(0.99)(0.97)(0.90) = 864.27$$
$$d_{60}^{(\tau)} = 1000 - 864.27 = 135.73$$
$$d_{60}^{(3)} = 135.73 \times \frac{-\ln(0.9)}{-\ln\left[(0.99)(0.97)(0.9)\right]} = 135.73 \times \frac{0.1054}{0.1459} = 98.05$$
$$l_{62}^{(\tau)} = 864.27(0.987)(0.95)(0.80) = 648.31$$
$$d_{61}^{(\tau)} = 864.27 - 648.31 = 215.96$$
$$d_{61}^{(3)} = 215.96 \times \frac{-\ln(0.80)}{-\ln\left[(0.987)(0.95)(0.80)\right]} = 215.96 \times \frac{0.2231}{0.2875} = 167.58$$

So $d_{60}^{(3)} + d_{61}^{(3)} = 98.05 + 167.58 = 265.63$

Question #225
Answer: B

$${}_t p_{40} = e^{-0.05t}$$
$${}_t p_{50} = (60-t)/60$$
$$\mu_{50+t} = 1/(60-t)$$
$$\int_0^{10} {}_t p_{40:50}\, \mu_{50+t}\, dt = \int_0^{10} \frac{e^{-0.05t}}{60} dt = -\frac{1}{60} \frac{e^{-0.05t}}{(0.05)}\Bigg|_0^{10}$$

$$= \frac{20}{60}\left(1-e^{-0.5}\right) = 0.13115$$

Question #226
Answer: A

Actual payment (in millions) $= \dfrac{3}{1.1} + \dfrac{5}{1.1^2} = 6.860$

$$q_3 = 1 - \frac{0.30}{0.60} = 0.5$$

$${}_{1|}q_3 = \frac{0.30-0.10}{0.60} = 0.333$$

Expected payment $= 10\left(\dfrac{0.5}{1.1} + \dfrac{0.333}{1.1^2}\right) = 7.298$

Ratio $\dfrac{6.860}{7.298} = 94\%$

Question #227
Answer: E

At duration 1

K_x	${}_1L$	Prob
1	$v - P^{1}_{x:\overline{2}\rceil}$	q_{x+1}
>1	$0 - P^{1}_{x:\overline{2}\rceil}$	$1-q_{x+1}$

So $Var\left({}_1L\right) = v^2 q_{x+1}\left(1-q_{x+1}\right) = 0.1296$

That really short formula takes advantage of
$Var(aX + b) = a^2 Var(X)$, if *a* and *b* are constants.

Here *a* = *v*; $b = P^{1}_{x:\overline{2}|}$; *X* is binomial with $p(X = 1) = q_{x+1}$.

Alternatively, evaluate $P^{1}_{x:\overline{2}|} = 0.1303$

${}_1L = 0.9 - 0.1303 = 0.7697$ if $K_x = 1$

${}_1L = 0 - 0.1303 = -0.1303$ if $K_x > 1$

$E({}_1L) = (0.2)(0.7697) + (0.8)(-0.1303) = 0.0497$

$E({}_1L^2) = (0.2)(0.7697)^2 + (0.8)(-0.1303)^2 = 0.1320$

$Var({}_1L) = 0.1320 - (0.0497)^2 = 0.1295$

Question #228
Answer: C

$$\bar{P}(\bar{A}_x) = \frac{\bar{A}_x}{\bar{a}_x} = \frac{\bar{A}_x}{\left(\dfrac{1-\bar{A}_x}{\delta}\right)} = \frac{\delta \bar{A}_x}{1-\bar{A}_x} = \frac{(0.1)(1/3)}{(1-1/3)} = 0.05$$

$$Var(L) = \left(1 + \frac{\bar{P}(\bar{A}_x)}{\delta}\right)^2 \left({}^2\bar{A}_x - \bar{A}_x^2\right)$$

$$\frac{1}{5} = \left(1 + \frac{0.05}{0.10}\right)^2 \left({}^2\bar{A}_x - \bar{A}_x^2\right)$$

$$\left({}^2\bar{A}_x - \bar{A}_x^2\right) = 0.08888$$

$$Var[L'] = \left(1 + \frac{\pi}{\delta}\right)^2 \left({}^2\bar{A}_x - \bar{A}_x^2\right)$$

$$\frac{16}{45} = \left(1 + \frac{\pi}{0.1}\right)^2 (0.08888)$$

$$\left(1 + \frac{\pi}{0.1}\right)^2 = 4$$

$$\pi = 0.1$$

Question #229
Answer: E

Seek g such that $\Pr\left\{\bar{a}_{\overline{T(40)}|} > g\right\} = 0.25$

$\bar{a}_{\overline{T}|}$ is a strictly increasing function of T.

$$\Pr\{T(40) > 60\} = 0.25 \text{ since } {}_{60}p_{40} = \frac{100-40}{120-40} = 0.25$$

$$\therefore \Pr\left\{\bar{a}_{\overline{T(40)}|} > \bar{a}_{\overline{60}|}\right\} = 0.25$$

$$g = \bar{a}_{\overline{60}|} = 19.00$$

Question 230
Answer: B

$$A_{51:\overline{9}|} = 1 - d\ddot{a}_{51:\overline{9}|} = 1 - \left(\frac{0.05}{1.05}\right)(7.1) = 0.6619$$

$${}_{11}V = (2000)(0.6619) - (100)(7.1) = 613.80$$

$$\left({}_{10}V + P\right)(1.05) = {}_{11}V + q_{50}\left(2000 - {}_{11}V\right)$$

$$\left({}_{10}V + 100\right)(1.05) = 613.80 + (0.011)(2000 - 613.80)$$

$${}_{10}V = 499.09$$

where $q_{50} = (0.001)(10) + (0.001) = 0.011$

Alternatively, you could have used recursion to calculate $A_{50:\overline{10}|}$ from $A_{51:\overline{9}|}$, then $\ddot{a}_{50:\overline{10}|}$ from $A_{50:\overline{10}|}$, and used the prospective reserve formula for ${}_{10}V$.

Question #231
Answer: C

$$1000A_{81} = (1000A_{80})(1+i) - q_{80}(1000 - A_{81})$$

$$689.52 = (679.80)(1.06) - q_{80}(1000 - 689.52)$$

$$q_{80} = \frac{720.59 - 689.52}{310.48} = 0.10$$

$$q_{[80]} = 0.5q_{80} = 0.05$$

$$1000A_{[80]} = 1000vq_{[80]} + vp_{[80]}\,1000A_{81}$$
$$= 1000 \times \frac{0.05}{1.06} + 689.52 \times \frac{0.95}{1.06} = 665.14$$

Question #232
Answer: D

	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
42	776	8	16
43	752	8	16

$l_{42}^{(\tau)}$ and $l_{43}^{(\tau)}$ came from $l_{x+1}^{(\tau)} = l_x^{(\tau)} - d_x^{(1)} - d_x^{(2)}$

$$EPV \text{ Benefits} = \frac{2000(8v + 8v^2) + 1000(16v + 16v^2)}{776} = 76.40$$

$$EPV \text{ Premiums} = 34\left(\frac{776 + 752v}{776}\right) = (34)(1.92) = 65.28$$

$${}_2V = 76.40 - 65.28 = 11.12$$

Question #233
Answer: B

$$p_{xx} = 1 - q_{xx} = 0.96$$

$$p_x = \sqrt{0.96} = 0.9798$$

$$p_{x+1:x+1} = 1 - q_{x+1:x+1} = 0.99$$

$$p_{x+1} = \sqrt{0.99} = 0.995$$

$$\ddot{a}_x = 1 + vp_x + v^2 \times {}_2p_x = 1 + \frac{0.9798}{1.05} + \frac{(0.9798)(0.995)}{1.05^2}$$
$$= 2.8174$$

$$\ddot{a}_{xx} = 1 + vp_{xx} + v^2 \times {}_2p_{xx} = 1 + \frac{0.96}{1.05} + \frac{(0.96)(0.99)}{1.05^2} = 2.7763$$

EPV = $2000\,\ddot{a}_x + 2000\,\ddot{a}_x + 6000\,\ddot{a}_{xx}$
$$= (4000)(2.8174) + (6000)(2.7763)$$
$$= 27{,}927$$

Notes: The solution assumes that the future lifetimes are identically distributed. The precise description of the benefit would be a special 3-year temporary life annuity-due.

Question #234
Answer: B

$${}_t p_x'^{(1)} \mu_x^{(1)}(t) = q_x'^{(1)} = 0.20$$
$${}_t p_x'^{(2)} = 1 - t q_x'^{(2)} = 1 - 0.08t$$
$${}_t p_x'^{(3)} = 1 - t q_x'^{(3)} = 1 - 0.125t$$
$$q_x^{(1)} = \int_0^1 {}_t p_x^{(\tau)} \mu_x^{(1)}(t)\,dt = \int_0^1 {}_t p_x'^{(2)}\, {}_t p_x'^{(3)}\, {}_t p_x'^{(1)} \mu_{x+t}^{(1)}\,dt$$
$$= \int_0^1 (1 - 0.08t)(1 - 0.125t)(0.20)\,dt$$
$$= 0.2\int_0^1 \left(1 - 0.205t + 0.01t^2\right) dt$$
$$= 0.2\left[t - \frac{0.205t^2}{2} + \frac{0.01t^3}{3}\right]_0^1$$
$$= (0.2)\left[1 - 0.1025 + \frac{0.01}{3}\right] = 0.1802$$

Question #235
Answer: B

$$ {}_1AS = \frac{\left(G - 0.1G - (1.50)(1)\right)(1.06) - 1000q_{40}^{(d)} - 2.93 \times q_{40}^{(w)}}{1 - q_{40}^{(d)} - q_{40}^{(w)}} $$

$$ = \frac{(0.9G - 1.50)(1.06) - (1000)(0.00278) - (2.93)(0.2)}{1 - 0.00278 - 0.2} $$

$$ = \frac{0.954G - 1.59 - 2.78 - 0.59}{0.79722} $$

$$ = 1.197G - 6.22 $$

$$ {}_2AS = \frac{\left({}_1AS + G - 0.1G - (1.50)(1)\right)(1.06) - 1000q_{41}^{(d)} - {}_2CV \times q_{41}^{(w)}}{1 - q_{41}^{(d)} - q_{41}^{(w)}} $$

$$ = \frac{(1.197G - 6.22 + G - 0.1G - 1.50)(1.06) - (1000)(0.00298) - {}_2CV \times 0}{1 - 0.00298 - 0} $$

$$ = \frac{(2.097G - 7.72)(1.06) - 2.98}{0.99702} $$

$$ = 2.229G - 11.20 $$

$$ 2.229G - 11.20 = 24 $$
$$ G = 15.8 $$

Question #236
Answer: A

$$ {}_5AS = \frac{\left({}_4AS + G(1-c_4) - e_4\right)(1+i) - 1000q_{x+4}^{(1)} - {}_5CV \times q_{x+4}^{(2)}}{1 - q_{x+4}^{(1)} - q_{x+4}^{(2)}} $$

$$ = \frac{\left(396.63 + 281.77(1-0.05) - 7\right)(1+i) - 90 - 572.12 \times 0.26}{1 - 0.09 - 0.26} $$

$$ = \frac{(657.31)(1+i) - 90 - 148.75}{0.65} $$

$$ = 694.50 $$

$$ (657.31)(1+i) = 90 + 148.75 + (0.65)(694.50) $$

$$ 1+i = \frac{690.18}{657.31} = 1.05 $$

$$ i = 0.05 $$

Question #237 - Removed

Question #238 – Removed

Question #239
Answer: B

Let P denote the annual premium

The EPV of benefits is $25,000\bar{A}_{x:\overline{20}|} = 25,000(0.4058) = 10,145$.
The EPV of premiums is $P\ddot{a}_{x:\overline{20}|} = 12.522P$
The EPV of expenses is

$$\left(0.25-0.05\right)P + 0.05P\,\ddot{a}_{x:\overline{20}|} + \left[\left(2.00-0.50\right)+0.50\,\ddot{a}_{x:\overline{20}|}\right]\frac{25,000}{1,000} + (15-3) + 3\ddot{a}_{x:\overline{20}|}$$
$$= 0.20P + 0.6261P + 194.025 + 12 + 37.566 = 0.8261P + 243.591$$

Equivalence principle:

$$12.522\,P = 10,145 + 0.8261P + 243.591$$
$$P = \frac{10,389.591}{12.522-0.8261}$$
$$= 888.31$$

Question #240
Answer: D

Let G denote the premium.
Expected present value (EPV) of benefits = $1000\bar{A}_{40:\overline{20}|}$

EPV of premiums = $G\,\ddot{a}_{40:\overline{10}|}$

EPV of expenses $= \left(0.04+0.25\right)G + 10 + \left(0.04+0.05\right)G\,a_{40:\overline{9}|} + 5a_{40:\overline{19}|}$
$$= 0.29G + 10 + 0.09G\,a_{40:\overline{9}|} + 5a_{40:\overline{19}|}$$
$$= 0.2G + 10 + 0.09G\,\ddot{a}_{40:\overline{10}|} + 5a_{40:\overline{19}|}$$

(The above step is getting an $\ddot{a}_{40:\overline{10}|}$ term since all the answer choices have one. It could equally well have been done later on).

Equivalence principle:

$$G\,\ddot{a}_{40:\overline{10}|} = 1000\bar{A}_{40:\overline{20}|} + 0.2G + 10 + 0.09G\,\ddot{a}_{40:\overline{10}|} + 5\,a_{40:\overline{19}|}$$

$$G\left(\ddot{a}_{40:\overline{10}|} - 0.2 - 0.09\,\ddot{a}_{40:\overline{10}|}\right) = 1000\bar{A}_{40:\overline{20}|} + 10 + 5\,a_{40:\overline{19}|}$$

$$G = \frac{1000\bar{A}_{40:\overline{20}|} + 10 + 5\,a_{40:\overline{19}|}}{0.91\ddot{a}_{40:\overline{10}|} - 0.2}$$

Question #241 - Removed

Question #242
Answer: C

$${}_{11}AS = \frac{\left({}_{10}AS + G - c_{10}\,G - e_{10}\right)(1+i) - 10,000q_{x+10}^{(d)} - {}_{11}CV\,q_{x+10}^{(w)}}{1 - q_{x+10}^{(d)} - q_{x+10}^{(w)}}$$

$$= \frac{\left(1600 + 200 - (0.04)(200) - 70\right)(1.05) - (10,000)(0.02) - (1700)(0.18)}{1 - 0.02 - 0.18}$$

$$= \frac{1302.1}{0.8}$$

$$= 1627.63$$

Question #243
Answer: E

The benefit reserve at the end of year 9 is the certain payment of the benefit one year from now, less the premium paid at time 9. Thus, it is 10,000*v* – 76.87.

The gross premium reserve adds expenses paid at the beginning of the tenth year and subtracts the gross premium rather than the benefit premium. Thus it is 10,000*v* + 5 + 0.1*G* – *G* where *G* is the gross premium.

Then,

$$10,000v - 76.87 - (10,000v + 5 - 0.9G) = 1.67$$
$$0.9G - 81.87 = 1.67$$
$$0.9G = 83.54$$
$$G = 92.82$$

Question #244
Answer: C

$${}_4AS = \frac{\left({}_3AS + G - c_4G - e_4\right)\left(1+i\right) - 1000q_{x+3}^{(d)} - {}_4CVq_{x+3}^{(w)}}{1 - q_{x+3}^{(d)} - q_{x+3}^{(w)}}$$

Plugging in the given values:

$${}_4AS = \frac{\left(25.22 + 30 - \left(0.02\right)\left(30\right) - 5\right)\left(1.05\right) - 1000\left(0.013\right) - 75\left(0.05\right)}{1 - 0.013 - 0.05}$$

$$= \frac{35.351}{0.937}$$

= 37.73

With higher expenses and withdrawals:

$$ {}_4AS^{\text{revised}} = \frac{25.22+30-(1.2)((0.02)(30)+5)(1.05)-1000(0.013)-75(1.2)(0.05)}{1-0.013-(1.2)(0.05)} $$

$$ = \frac{(48.5)(1.05)-13-4.5}{0.927} $$

$$ = \frac{33.425}{0.927} $$

= 36.06

$$ {}_4AS - {}_4AS^{\text{revised}} = 37.73-36.06 $$
= 1.67

Question #245
Answer: E

Let G denote the gross premium.
EPV (expected present value) of benefits $=1000\,{}_{10|20}A_{30}$.

EPV of premiums $=G\ddot{a}_{30:\overline{5}|}$.

EPV of expenses $=(0.05+0.25)G+20$ first year

$+\left[(0.05+0.10)G+10\right]a_{30:\overline{4}|}$ years 2-5

$+10\,{}_{5|}\ddot{a}_{35:\overline{4}|}$ years 6-10 (there is no premium)

$$ =0.30G+0.15G\,a_{30:\overline{4}|}+20+10\,a_{30:\overline{4}|}+10\,{}_{5|}\ddot{a}_{30:\overline{5}|} $$

$$ =0.15G+0.15G\ddot{a}_{30:\overline{5}|}+20+10\,a_{30:\overline{9}|} $$

(The step above is motivated by the form of the answer. You could equally well put it that form later).

Equivalence principle:

$$ G\ddot{a}_{30:\overline{5}|}=1000\,{}_{10|20}A_{30}+0.15G+0.15G\,\ddot{a}_{30:\overline{5}|}+20+10\,a_{30:\overline{9}|} $$

$$ G=\frac{\left(1000\,{}_{10|20}A_{30}+20+10\,a_{30:\overline{9}|}\right)}{(1-0.15)\ddot{a}_{30:\overline{5}|}-0.15} $$

$$ =\frac{\left(1000\,{}_{10|20}A_{30}+20+10\,a_{30:\overline{9}|}\right)}{0.85\ddot{a}_{30:\overline{5}|}-0.15} $$

Question #246
Answer: E

Let G denote the gross premium
EPV (expected present value) of benefits

$$=(0.1)(3000)v+(0.9)(0.2)(2000)v^2+(0.9)(0.8)1000v^2$$

$$=\frac{300}{1.04}+\frac{360}{1.04^2}+\frac{720}{1.04^2}=1286.98$$

EPV of premium = G
EPV of expenses $=0.02G+0.03G+15+(0.9)(2)v$

$$=0.05G+16.73$$

Equivalence principle: $G=1286.98+0.05G+16.73$

$$G=\frac{1303.71}{1-0.05}=1372.33$$

Question #247
Answer: C

EPV (expected present value) of benefits = 3499 (given)

EPV of premiums $=G+(0.9)(G)v$

$$=G+\frac{0.9G}{1.05}=1.8571G$$

EPV of expenses, except settlement expenses,

$$=\left[25+(4.5)(10)+0.2G\right]+(0.9)\left[10+(1.5)(10)+0.1G\right]v+(0.9)(0.85)\left[10+(1.5)(10)\right]v^2$$

$$=70+0.2G+\frac{0.9(25+0.1G)}{1.05}+\frac{0.765(25)}{1.05^2}$$

=108.78+0.2857G

Settlement expenses are $20+(1)(10)=30$, payable at the same time the death benefit is paid.

So EPV of settlement expenses $=\left(\dfrac{30}{10,000}\right)$ EPV of benefits

$$=(0.003)(3499)$$
$$=10.50$$

Equivalence principle:

$$1.8571G = 3499+108.78+0.2857G+10.50$$
$$G=\frac{3618.28}{1.8571-0.2857}=2302.59$$

Question #248
Answer: D

$$\ddot{a}_{50:\overline{20}|}=\ddot{a}_{50}-{}_{20}E_{50}\,\ddot{a}_{70}$$
$$=13.2668-(0.23047)(8.5693)$$
$$=11.2918$$
$$A_{50:\overline{20}|}=1-d\,\ddot{a}_{50:\overline{20}|}=1-\left(\frac{0.06}{1.06}\right)(11.2918)$$
$$=0.36084$$

Expected present value (EPV) of benefits $=10,000A_{50:\overline{20}|}$

$$=3608.40$$

EPV of premiums $=495\ddot{a}_{50:\overline{20}|}$

$$=5589.44$$

EPV of expenses $=(0.35)(495)+20+(15)(10)+\left[(0.05)(495)+5+(1.50)(10)\right]a_{50:\overline{19}|}$

$$=343.25+(44.75)(11.2918-1)$$
$$=803.81$$

EPV of amounts available for profit and contingencies
= EPV premium – EPV benefits – APV expenses
= 5589.44 – 3608.40 – 803.81
= 1177.23

Question #249
Answer: B

$$q_{xy}^{\;1} = \int_0^1 {}_t p_{xy}\mu_{x+t}dt = \int_0^1 {}_t p_x \, {}_t p_y \mu_{x+t}dt$$

$$= \int_0^1 q_x e^{-0.25t}dt \text{ (under UDD, } {}_t p_x \mu_{x+t} = q_x)$$

$$0.125 = q_x \,(-4e^{-0.25t})\Big|_0^1 = q_x(4)(1-e^{-0.25}) = 0.8848 q_x$$

$$q_x = 0.1413$$

Question #250
Answer: C

$${}_2 p_{[x]+1}^{11} = p_{[x]+1}^{11} p_{[x]+2}^{11} + p_{[x]+1}^{12} p_{[x]+2}^{21}$$

$$= \left(0.7+\frac{0.1}{2}\right)\left(0.7+\frac{0.1}{3}\right)+\left(0.3-\frac{0.1}{2}\right)\left(0.4-\frac{0.2}{3}\right)$$

$$= 0.75(0.7333)+0.25(0.3333) = 0.6333$$

Note that Anne might have changed states many times during each year, but the annual transition probabilities incorporate those possibilities.

Questions #251-260 – Removed

Question #261
Answer: A

The insurance is payable on the death of (*y*) provided (*x*) is already dead.

$$E(Z) = \bar{A}_{x\overset{2}{y}} = \int_0^\infty e^{-\delta t} \, {}_t q_x \, {}_t p_y \mu_{y+t} dt$$

$$= \int_0^\infty e^{-0.06t}(1-e^{-0.07t})e^{-0.09t}\, 0.09 dt$$

$$= 0.09\int_0^\infty e^{-0.15t} - e^{-0.22t} dt$$

$$= 0.09\left(\frac{1}{0.15}-\frac{1}{0.22}\right) = 0.191$$

Question #262
Answer: C

$${}_t p_x = \frac{95-x-t}{95-x}, \quad \mu_{x+t} = \frac{1}{95-x-t}, \quad {}_t p_y = e^{-\mu t}$$

$$\text{Pr(x dies within n years and before y)} = \int_0^n {}_t p_x \, {}_t p_y \mu_{x+t} dt$$

$$= \int_0^n \frac{95-x-t}{95-x} e^{-\mu t} \frac{1}{95-x-t} dt = \frac{1}{95-x}\int_0^n e^{-\mu t} dt = \frac{1-e^{-\mu n}}{\mu(95-x)}$$

Question #263
Answer: A

$${}_{0.25}q^{\ 2}_{30.5:40.5} = \int_0^{0.25} {}_t p_{30.5} \mu_{30.5+t} \, {}_t q_{40.5} dt$$

$$= \int_0^{0.25} \frac{0.4}{1-0.5(0.4)} \frac{0.6t}{1-0.5(0.6)} dt$$

$$= \frac{0.4(0.6)}{0.8(0.7)} \frac{t^2}{2}\Bigg|_0^{0.25} = 0.0134$$

Question #264 – Removed

Question #265
Answer: D

$${}_t p_x = \exp\left[-\int_0^t 5r dr\right] = e^{-2.5t^2}$$

$${}_t p_y = \exp\left[-\int_0^t r dr\right] = e^{-0.5t^2}$$

$$q^1_{x:y} = \int_0^1 {}_t p_y \, {}_t p_x \mu_{x+t} dt = \int_0^1 e^{-0.5t^2} e^{-2.5t^2} 5t dt = 5\int_0^1 e^{-3t^2} t dt$$

$$= \frac{5}{6} e^{-3t^2}\Bigg|_0^1 = \frac{5}{6}(1-e^{-3}) = 0.7918$$

Question #266
Answer: B

$$G=\int_0^5 \frac{1}{30}\frac{t}{25}dt=\frac{t^2}{30(25)(2)}\Bigg|_0^5=\frac{1}{60}$$

$$H={}_5p_{80:85}-{}_{10}p_{80:85}=\frac{25}{30}\frac{20}{25}-\frac{20}{30}\frac{15}{25}=\frac{200}{750}=\frac{4}{15}$$

$$G+H=\frac{1}{60}+\frac{16}{60}=\frac{17}{60}=0.2833$$

Question #267
Answer: D

$$S_0(t)=\exp\left[-\int_0^t(80-x)^{-0.5}dx\right]=\exp\left[2(80-x)^{0.5}\Big|_0^t\right]=\exp\left[2((80-t)^{0.5}-80^{0.5})\right]$$

$$F=S_0(10.5)=\exp\left[2(69.5^{0.5}-80^{0.5})\right]=0.29665$$

$$S_0(10)=0.31495$$

$$S_0(11)=0.27935$$

$$G=S_0(10.5)^{\exp}=[0.31495(0.27935)]^{0.5}=0.29662$$

$$F-G=0.00003$$

Question #268
Answer: A

$$E(Z)=500\int_0^4 0.2(1-0.25t)dt+1000\int_0^4 0.25(0.2t)dt$$

$$=500(0.2)\left(t-0.125t^2\right)\Big|_0^4+1000(0.25)\,(0.1t^2)\Big|_0^4$$

$$=100(4-2)+250(1.6)=600$$

Question #269
Answer: A

$${}_{10}q_{\overline{30:50}}={}_{10}q_{30}\,{}_{10}q_{50}=(1-e^{-0.5})(1-e^{-0.5})=0.1548$$

Question #270
Answer: C

$$\mathring{e}_{\overline{30:50}} = \mathring{e}_{30} + \mathring{e}_{50} - \mathring{e}_{30:50}$$

$$\mathring{e}_{30} = \mathring{e}_{50} = \int_0^\infty e^{-0.05t}dt = 20$$

$$\mathring{e}_{30:50} = \int_0^\infty e^{-0.10t}dt = 10$$

$$\mathring{e}_{\overline{30:50}} = 20 + 20 - 10 = 30$$

Question #271
Answer: B

$$\bar{A}^{1}_{30:50} = \int_0^\infty e^{-\delta t}\, {}_tp_{30}\, {}_tp_{50}\mu_{30+t}dt = \int_0^\infty e^{-0.03t}e^{-0.05t}e^{-0.05t}0.05dt = \frac{0.05}{0.13} = 0.3846$$

Question #272
Answer: B

$T_{30:50}$ has the exponential distribution with parameter 0.05 + 0.05 = 0.10 and so its mean is 10 and its variance is 100.

Question #273
Answer: D

$$Cov[T_{30:50}, T_{\overline{30:50}}] = \left(\mathring{e}_{30} - \mathring{e}_{30:50}\right)\left(\mathring{e}_{50} - \mathring{e}_{30:50}\right) = (20-10)(20-10) = 100$$

See solution to Question #270 for the individual values.

Question #274
Answer: E

$$_3V = ({}_2V + \pi_3)(1+i_3) - q_{x+2}(b_3 - {}_3V)$$
$$96 = (84+18)(1.07) - q_{x+2}(240-96)$$
$$q_{x+2} = 13.14/144 = 0.09125$$

Question #275
Answer: A

$$ {}_4V = \frac{({}_3V + \pi_4)(1+i_4) - q_{x+3}b_4}{p_{x+3}} = \frac{(96+24)(1.06) - 0.101(360)}{0.899} = 101.05 $$

Question #276
Answer: D

Under UDD:

$$ {}_{0.5}q_{x+3.5} = \frac{0.5q_{x+3}}{1-0.5q_{x+3}} = \frac{0.5(0.101)}{1-0.5(0.101)} = 0.0532 $$

Question #277
Answer: E

$$ \begin{aligned} {}_{3.5}V &= v^{0.5}\,{}_{0.5}p_{3.5}\,{}_4V + v^{0.5}\,{}_{0.5}q_{3.5}b_4 \\ &= 1.06^{-0.5}(0.9468)(101.05) + 1.06^{-0.5}(0.0532)(360) \\ &= 111.53 \end{aligned} $$

Question #278
Answer: D

$$ 1 - {}_{10}p_{30:40} = 1 - {}_{10}p_{30}\,{}_{10}p_{40} = 1 - \frac{60}{70}\frac{50}{60} = \frac{2}{7} $$

Question #279
Answer: A

$$ {}_{10}q^{2}_{30:40} = \int_0^{10} {}_tp_{30}\mu_{30+t}(1 - {}_tp_{40})dt = \int_0^{10} \frac{1}{70}\frac{t}{60}dt = \frac{50}{70(60)} = 0.0119 $$

Question #280
Answer: A

$$ \begin{aligned} &\int_{10}^{20} {}_tp_{30}\mu_{30+t}\,{}_tq_{40}dt + \int_{10}^{20} {}_tp_{40}\mu_{40+t}\,{}_tq_{30}dt \\ &= \int_{10}^{20} \frac{1}{70}\frac{t}{60}dt + \int_{10}^{20} \frac{1}{60}\frac{t}{70}dt = \frac{1}{70}\frac{400-100}{2(60)} + \frac{1}{60}\frac{400-100}{2(70)} = 0.0714 \end{aligned} $$

Question #281
Answer: C

$$140{,}000\int_0^{30} {}_tp_{30}\mu_{30+t}\,{}_tp_{40}dt+180{,}000\int_0^{30} {}_tp_{40}\mu_{40+t}\,{}_tp_{30}dt$$
$$=140{,}000\int_0^{30}\frac{1}{70}\frac{60-t}{60}dt+180{,}000\int_0^{30}\frac{1}{60}\frac{70-t}{70}dt$$
$$=140{,}000\frac{1}{70}\frac{60^2-30^2}{2(60)}+180{,}000\frac{1}{60}\frac{70^2-40^2}{2(70)}=115{,}714$$

Question #282
Answer: B

$$P\int_0^{20} {}_tp_{30}\,{}_tp_{40}dt=P\int_0^{20}\frac{70-t}{70}\frac{60-t}{60}dt=\frac{P}{4200}\int_0^{20}4200-130t+t^2dt$$
$$=\frac{P}{4200}[4200(20)-130(200)+8000/3]=14.444P$$

Question #283
Answer: A

Note that this is the same as Question 33, but using multi-state notation rather than multiple-decrement notation.

The only way to be in State 2 one year from now is to stay in State 0 and then make a single transition to State 2 during the year.

$$p_x^{02}=\int_0^1 {}_tp_x^{00}\mu_{x+t}^{02}dt=\int_0^1 e^{-(0.3+0.5+0.7)t}0.5dt=0.5\left.\frac{e^{-1.5t}}{-1.5}\right|_0^1=\frac{1}{3}(1-e^{-1.5})=0.259$$