# FETE Model Solutions
# Spring 2013

**1.** **Learning Objectives:**

2. Corporate Financial Applications

3. Derivatives, Pricing and Modeling

**Learning Outcomes:**

(2c) Describe the process, methods and uses of insurance securitizations and recommend a structure that is appropriate for a given set of circumstances.

(3l) Recommend an equity or interest rate model for a given situation.

**Sources:**

FET-165-08: Integrated Risk Management, Doherty, Chapter 16: A Case Study: The Securitization of Catastrophic Risk

Options Futures & Other Derivatives, Hull J.C., 8th Edition, 2012

- Chapter 13, Wiener Processes and Ito's Lemma (Appendix, exclude multivariate material)
- Chapter 30, Interest Rate Derivatives: Models of the Short Rate (exclude 30.2 properties of Vasicek and CIR, exclude 30.3 BDT and Black-Karasinski)

**Commentary on Question:**

*This question tests how to apply the knowledge in Wiener processes and short-rate interest rate models to solve a real-world insurance related problem – model and hedge mortality risk.*

**Solution:**

(a) Design a continuous stochastic process to model the force of mortality $\mu_x(t)$ and explain all parameters in the process.

**Commentary on Question**:

*Candidates did well on this section. The most common mistake made was to use a mean reverting model for the force of mortality.*

$$d\mu_x(t) = a(t)\mu_x(t)dt + \sigma(t)\sqrt{\mu_x(t)}dz$$

*a(t)* is the drift of the process,

$\sigma(t)$ is the volatility of the process, proportional to $\mu_x(t)$,

**1.** **Continued**

*dz* is a Weiner process

Note: This answer is not unique. Full credit was also given to candidates as long as the stochastic process was:

1) not mean-reverting;
2) $\boldsymbol{\mu_x}$ is always non-negative (through square root of $\boldsymbol{\mu_x}$, or through the lognormal of $\boldsymbol{\mu_x}$);
3) the process is one-factor.

(b) Describe how each of the following instruments can be used to hedge catastrophe mortality:

(i) Catastrophe Bonds

(ii) Catastrophe Options

**Commentary on Question**:
*Candidates did well on this section. The most common mistake was to reverse the descriptions of each instrument.*

(i) reduce the debt payment by an embedded call option that will pay off if actual mortality is greater than some mortality index

index can be a composite of insurers' losses

value of cat bond = bond – embedded call option (forgiveness provision)

(ii) bought by insurance company to hedge adverse mortality

payoff is based on actual mortality being greater than some mortality index in the market

(c) Identify any credit risk considerations associated with the above instruments.

**Commentary on Question**:
*Candidates did extremely well on this section. The most common mistake was to reverse the considerations with each instrument. A smaller number of candidates identified the credit risk of the issuer of the bond was high.*

Credit risk for the issuer of catastrophe bonds is nil because the bond is issued by the company.

Credit risk for catastrophe options is high. Credit risk mitigation includes use of daily mark-to-market and/or a security fund.

**1.** **Continued**

(d) Determine if these alternatives are effective in your situation. Justify your answer.

**Commentary on Question**:
*Candidates did relatively well on this section. Most candidates were able to identify the policyholder considerations and provide a justification, however few were able to demonstrate an understanding of the catastrophe equity put.*

**Policyholder Considerations:** Effective because it will share the catastrophic risk with policyholders. Moral hazard decreases because policyholders will reduce the actions that cause or increase catastrophic mortality losses.

**Catastrophe Equity Put:** Effective because it reduces the cost of post-loss financing. Issued on preferred shares such that upon catastrophe, the preferreds are converted to equity (effectively funds catastrophic mortality losses).

## 2. Learning Objectives:

4. Efficient and Inefficient Markets, Complete and Incomplete Markets, Information Theory & Market Misbehavior

**Learning Outcomes:**

(4e) Define principal-agency theory and explain how it affects capital structure, portfolio management and risk management.

**Sources:**

Financial Theory and Corporate Policy, Copeland, Weston, Shastri, 4th Edition

- Chapter 12, Information Asymmetry and Agency Theory

**Commentary on Question:**

*This question tests information asymmetry and agency theory.*

**Solution:**

(a) Identify and explain the four major sources of conflict between stockholders and bondholders.

**Commentary on Question**:

*Generally, this question was extremely well answered by candidates. To receive maximum points, the candidate needed to identify the source and provide an explanation. Alternative source names were acceptable provided the candidate's explanation was consistent with the solution.*

1. Dividend payout - becomes a source of conflict if a firm unexpectedly changes its dividend payout and finances this increase by reducing he asset base or by reducing planned investments.
2. Claim dilution - If a firm chooses to issue new debt with equal or higher priority than existing debt, the claim of the existing debt holders is diluted.
3. Asset substitution - reduces the value of debt by making it more risky. Specifically, if a firm decided to substitute a high-risk investment for a low-risk investment, the risk faced by debt holders increases, and this results in a reduction in debt value.
4. Underinvestment - In a situation that a firm can potentially invest in a positive net present value project with the benefits of the investment accruing to bondholders, the firm may choose to pass up on this project, thus causing bonds to suffer an opportunity loss.

(b) Determine the amount of debt that you expect the manager to issue today. Show all your work.

**2.** **Continued**

**Commentary on Question**:
*Candidates overall did moderately well on this section. However candidates either got most of the points for this section or very few. Candidates were required to identify the proper initial perceived values of the firm, V(0), as a result of the two debt issue scenarios. Candidates that were able to correctly interpret the market signals were generally able to achieve maximum points for this question.*
*Common mistakes by candidates included:*

- *Using V(0)=50 under both scenarios. This resulted in partial credit.*
- *Calculating more than two scenarios with various combinations of V(0), V(1) and D. This would not result in any credit as the candidate was not demonstrating any knowledge. However, if the candidate properly identified the two required scenarios, maximum credit was obtained.*

Per the information provided:

- The manager believes the firm value is 50
- In one year's time the firm's value will be revealed and will not change from today.
- Thus, the value in one year's time is 50, regardless of the debt issue. V(1)=50

Scenario 1:
If a debt of 20 is issued (D=20), the firm is signaling to the market that the value of the firm is 50.
V(0)=50
Since V(1)=50 > D=20, the first equation is used
M=0.1*V(0) + 0.2*V(1)
M=0.1*50+0.2*50=15

Scenario 2:
If a debt of 60 is issued (D=60), the firm is signaling to the market that the value of the firm is 100.
V(0)=100
Since V(1)=50 < D=60, the second equation is used
M=0.1*V(0) + 0.2*(V(1)-10)
M=0.1*100+0.2*(50-10)=18;

Conclusion:
The manager maximizes compensation by issuing debt of 60
The manager will issue a debt of 60 even with risk of bankruptcy

(c) Discuss three ways for the board to adjust its agreement with the manager so that he has the incentive to issue the more appropriate amount of debt while the manager's compensation remains linked to the value of the firm.

## 2. Continued

**Commentary on Question**:
*This part of the question tested the candidate's understanding in how structuring a manager's compensation can affect a firm's short and long term performance. Further commentary is provided within the solution in italics.*

*Candidates overall did well on this section with many getting 2 out of 3 of the answers below.*

1. To make sure the manager issues a debt of 20, the manager's compensation should be higher when he issues debt equal to 20 than if he issues debt equal to 60.
   Solve for
   [0.1*V(0) + 0.2*(V(1) – C)] < [0.1*V(0) + 0.2*V(1)]
   [0.1*100 + 0.2*(50 – C)] < [0.1*50 + 0.2*50]
   Increase penalty to C>25
   *Commentary: partial credit was given for candidates that did not solve for "C" or just stated "penalty should be greater than 10".*

2. The board could remove, or lower, the market value at time 0 portion of the compensation formula. Compensation would only be based on the known value at time 1.
   *Commentary: credit was also given if the candidates discuss agreements where the manager created economic value or earnings benchmarks for the year. The intent of this answer was to get away from basing the manager's compensation on time zero, and weight it towards the performance of the company during the one year tenure.*

3. The board could offer deferred compensation beyond time 1, so that he will benefit from the firm remaining solvent.
   *Commentary: various alternatives to deferred compensation were acceptable (e.g., stock options) provided that reference was made to changing the manager's long-term view of the company.*

## 3. Learning Objectives:
3. Derivatives, Pricing and Modeling

**Learning Outcomes:**
(3o) Use numerical methods to effectively model complex assets or liabilities.

**Sources:**
Options Futures & Other Derivatives, Hull, J.C., $8^{th}$ Edition, 2012
- Chapter 19, Volatility Smiles

**Commentary on Question:**
*The question was to test whether candidates understood calculating option values using a binomial tree and then applying that result to calculate other metrics and make a recommendation whether or not to purchase protection.*

**Solution:**
(a) Calculate the growth factor implied by this model.

**Commentary on Question**:
*Candidates did relatively poorly on this section. Very few candidates correctly used first principles from the binomial tree to calculate the expected return over the first or both periods. If both periods were used, it was necessary to take the square root of the result to get the single period growth factor. Partial credit was given if the candidate recognized that the growth should have been derived from the general formula p = a – d / u – d. The common mistake was to put* $e^r = 1.1$ *instead of a in the formula.*

*Additionally. some candidates stated the result as 7% and a few solved for a continuous rate that would yield 7% which was an acceptable answer.*

P = (a-d) / (u-d) where
P=0.6 (given)
u=1.25 (given)
d = 1/u = 0.8 due to the Cox, Ross, and Rubenstien conditions.
0.6 = (a-0.8) / (1.25 – 0.8)
(a – 0.8) = 0.6 * 0.45
a = 0.27 + 0.8 = 1.07

(b) Calculate value of the put under this model.

**Commentary on Question**:
*Candidates did moderately well on this section. There ended up being a wide variety of answers depending on how candidates were interpreting how to apply the risk neutral condition. The intention was for the candidate to use the parameters given including p = 0.6 and just use the* $e^r = 1.1$ *for discounting only.*

## 3. Continued

*Many candidates went through the extra step of recalculating the p using the formula p = ($e^r$ – d) / (u –d). Most credit was given to this answer as well. Some other approaches recalculating the d using the above also were done.*

*Many candidates did not make it clear what assumptions they were using for the various calculations, especially the comparison at time 1 for early exercise or not. Most candidates used the correct exercise prices though a small minority forgot to use 105 at time 1 instead of 100 or 110. A handful of candidates did the entire calculation as a call.*

Populate the basic tree of the stock price

P=0.6
U = 125%
D = 80%
$E^r$ = 1.1
Put strike at t=2 is 110
Put strike at t=1 is 105

| | | |
|---|---|---|
| | | 156.25 |
| | 125 | |
| 100 | | 100 |
| | 80 | |
| | | 64 |

Find the option values at time 2:
f(2,j) = Max (0, 110 – S2,j) for j=0 to 2
f(2,2) = Max(0, 110 -156.25) = 0
f(2,1) = Max(0, 110 – 100) = 10
f(2,0) = Max(0, 110 - 64) = 46

Find the option values at time 1:
f(1,j) = Max ((0.6*f(2,j)+0.4*f(2,j+1))/1.1, 105 – S1,j) for j=0 to 1
f(1,1) = Max((0.6*0 + 0.4*10)/1.1, 105-125) = 4/1.1 = 3.64
F(1,0) = Max((0.6*10 + 0.4*46)/1.1, 105 – 80) = Max(22.18, 25) = 25

Find the option value at time 0:
F(0,0) = (0.6*3.64 + 0.4*25)/1.1
F(0,0) = (2.18 + 10)/1.1 = 11.07

(c) Estimate the value of Delta and Gamma of the American put option from this model.

**3.** **Continued**

**Commentary on Question**:
*Candidates did well on this section. As long as the approach to part (c) used the values calculated in part (b) then those solution were accepted, assuming the correct application of duration and gamma.*

*Most candidates wrote out the equations that they were using which made for much clearer understanding of the following work. Most mistakes in the calculations were with gamma and generally fell into two situations. 1) Using an approximation formula and not stating that an approximate solution was being calculated. 2) The application of the h in the formula below. Many candidates did not remember what this term was supposed to be. It was not on the formula sheet but its derivation just from the definition of gamma was attainable.*

Delta = (f1,1 – f1,0)/(Su-Sd) = (3.64-25)/(125 – 80) = (-21.36/45)=-0.47
Gamma = [(f2,2 – f2,1)/(Su2-S) – (f2,1 – f2,0)/(S-Sd2)] /h
Where h = 0.5*(Su2-Sd2) = .5 (156.25 -64) = 92.25/2 = 46.12
(0-10)/(156.25-100) – (10-46)/(100 – 64) / 46.12
(-10/56.25 +36/36) / 46.12 = 0.82/46.12 = 0.0178

Note that delta is negative which is consistent with a put.

(d) Recommend and justify whether the company should buy the option or not.

**Commentary on Question**:
*Candidates did well on this section. Given the varying solutions to part (b), most credit was awarded for recommendations clearly based on the result from part (b). Little credit was given for other types of answers. Full credit was reserved for candidates who justified the limitations of the selected model and methodology.*

Recommendation: Buy the option at $10.
Why? Based on the assumed model, it has a value of 11.07 which is less than the cost. This recommendation is based on a relatively simple model over a short time period. There are limitations to this model and other techniques may be explored to determine if the market option is a good value.

## 4. Learning Objectives:

3. Derivatives, Pricing and Modeling

**Learning Outcomes:**

(3m) Describe issues and best practices in the estimation or calibration of financial models.

**Sources:**

Investment Guarantees: Modeling and Risk Management for Equity-Linked Life Insurance, hardy, 2003

- Chapter 2, Modeling Long-Term Stock Returns

**Commentary on Question:**

*This question examines candidates' ability to estimate equity models with maximum likelihood. The pros and cons of maximum likelihood estimation are also examined.*

*Students generally did well on this question.*

*Most candidates demonstrated a good understanding of the concept of MLE and able to derive the model parameters via MLE.*

*To gain full credit, students are required to show all necessary steps for quantitative questions and explain well for qualitative questions.*

**Solution:**

(a) Show that the log-likelihood function $l(\mu,\sigma)$ is given by:

$$l(\mu,\sigma) = -\frac{n}{2}\ln(2\pi) - n\ln(\sigma) - \frac{1}{2}\sum_{t=1}^{n}\left(\frac{y_t-\mu}{\sigma}\right)^2.$$

**Commentary on Question**:

*Candidates did relatively well on this section. Candidates generally understood the MLE concept. Common mistakes included not identifying the underlying model was lognormal and not providing detail steps to derive the function.*

| Derivation of the log-likelihood function |
|---|
| - Because $\varepsilon_t$ follows N(0,1), $Y_t$ follows N($\mu,\sigma^2$) |
| - It follows that the contribution to the likelihood from each observation is (i.e. the lognormal model) $$\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{y_t-\mu}{\sigma}\right)^2\right)$$ |
| - Because $\varepsilon_t$ and $\varepsilon_s$ are uncorrelated for $t \neq s$, $Y_t$ and $Y_s$ are uncorrelated for $t \neq s$ |

# 4. Continued

| |
|---|
| - It follows that the aggregate likelihood is simply the product of the likelihood contributed from all observations. That is, $$\prod_{t=1}^{n}\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{y_t-\mu}{\sigma}\right)^2\right)$$ |
| - Taking log, we obtain the log-likelihood function: $$l(\mu,\sigma)=\sum_{t=1}^{n}\ln\left(\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{y_t-\mu}{\sigma}\right)^2\right)\right)$$ |
| - Simplifying, we obtain $$l(\mu,\sigma)=-\frac{n}{2}\ln(2\pi)-n\ln(\sigma)-\frac{1}{2}\sum_{t=1}^{n}\left(\frac{y_t-\mu}{\sigma}\right)^2$$ |

(b) Derive the maximum likelihood estimates of parameters $\mu$ and $\sigma$ in terms of $\bar{y}$, $n$, and $y_t, t=1,2,\ldots,n,$ using the result in (a).

**Commentary on Question**:
*Candidates did well on this section. Most candidates were able to demonstrate an understanding of the concept but a few did not successfully derive the estimate of the parameters due to an incorrect partial derivative calculation.*

| Getting expressions for the maximum likelihood estimates |
|---|
| ▪ Taking the first partial derivative with respect to $\mu$, we obtain $$\frac{\partial l(\mu,\sigma)}{\partial\mu}=\frac{1}{\sigma^2}\left(\sum_{t=1}^{n}y_i-n\mu\right)$$ Note: if candidate answers $$\frac{\partial l(\mu,\sigma)}{\partial\mu}=\frac{1}{\sigma}\left(\sum_{t=1}^{n}y_i-n\mu\right)$$ credits will also be given as this is printed in the Hardy textbook |
| ▪ Taking the first partial derivative with respect to $\sigma$, we obtain $$\frac{\partial l(\mu,\sigma)}{\partial\sigma}=-\frac{n}{\sigma}+\frac{1}{\sigma^3}\left(\sum_{t=1}^{n}(y_t-\mu)^2\right)$$ |
| ▪ The maximum likelihood estimates for $\mu$ and $\sigma$ are found by setting the partial derivatives equal to zero. |
| ▪ This gives $\hat{\mu}=\bar{y}$. |

# 4. Continued

| |
|---|
| ▪ Also, $\hat{\sigma} = \sqrt{\frac{\sum_{t=1}^{n}(y_t - \hat{\mu})^2}{n}} = \sqrt{\frac{\sum_{t=1}^{n}(y_t - \overline{y})^2}{n}}$ . |

(c) Show that the maximum likelihood estimate of $\mu$ is unbiased.

**Commentary on Question**:
*Candidates did relatively well on this section. The common mistake made was to not define an unbiased estimator which is a key element in the derivation.*

| |
|---|
| Generally speaking, the bias of an estimator $\hat{\theta}$ of a parameter $\theta$ is $b(\theta) = \mathrm{E}[\hat{\theta} - \theta]$ . |
| If an estimator is unbiased, then the expected value of $b(\theta)$ is zero. That is, it has expected value equal to the unknown parameter. |
| In this problem, $$\begin{aligned} \mathrm{E}(\hat{\mu}) &= \mathrm{E}(\overline{y}) \\ &= \mathrm{E}\left(\frac{1}{n}\sum_{t=1}^{n} y_t\right) \\ &= \frac{1}{n}\sum_{t=1}^{n}\mathrm{E}(y_t) \\ &= \frac{n\mu}{n} = \mu \end{aligned}$$ Hence, the maximum likelihood estimate of $\mu$ is unbiased. |

(d) Describe, other than the property of asymptotically unbiasedness, two other desirable properties of maximum likelihood estimators.

**Commentary on Question**:
*Candidates did relatively well on this section. Most candidates were able to list the two desirable properties, however many did not provide any descriptions of the properties.*

| Two desirable properties of maximum likelihood estimation |
|---|
| **Property 1: Asymptotic minimum variance** |
| Provided an estimator is unbiased or nearly unbiased, a low variance estimator is preferred. |
| The variance of an estimator measures how much the estimate will change from one sample to the next. |

## 4. Continued

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| A low variance indicates that different samples will give similar values for the parameter estimate |
| The asymptotic variance of a parameter is the inverse of the expected information. |
| The inverse information function is also the Cramer-Rao lower bound for the variance of an estimator |
| The asymptotic variance may be used as an approximate variance of a maximum likelihood estimator. |
| |
| **Property 2: Asymptotic Normal Distribution** |
| Maximum likelihood estimates are asymptotically normally distributed, |
| with mean equal to the parameter(s) being estimated, |
| and variance equal to the inverse of the information function. |
| For large samples, we can make use of this property to construct confidence intervals for the parameters. |

(e) Describe two limitations of maximum likelihood estimation.

**Commentary on Question**:
*Candidates did relatively well on this section. Most candidates were able to list the two limitation (with many identifying limitation 3 below), however many did not provide any descriptions of the limitations.*

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|---|
| Limitations of maximum likelihood estimation |
| |
| **Limitation 1: The asymptotic results do not apply for models that are not strictly stationary** |
| ▪ For non-stationary models, other methods may be more preferable |
| |
| **Limitation 2: The asymptotic results cannot be relied on if a parameter is estimated near the boundaries of the parameter space** |
| ▪ This problem could be significant for more complex models such as GARCH and RSLN with more than two regimes. |
| |
| **Limitation 3: The asymptotic properties are only useful if we have a reasonably large sample** |
| ▪ For small samples, other estimation methods may give a lower bias and variance. |
| ▪ This problem applies not only to the variances of the estimators, but also the relationships between them |
| |

## 4. **Continued**

| **Limitation 4: Maximum likelihood estimation does not tell the user how close the fit is** |
|---|
| ▪ It only finds parameters that fit the data |
| ▪ A small (asymptotic) variance / standard error does not mean that the fit is good |
| ▪ It just means that, given the assumed model, there is little uncertainty about the parameters |
| ▪ The assumed model may still provide a worse fit than another model with larger standard errors |

**5.** **Learning Objectives:**

3. Derivatives, Pricing and Modeling

**Learning Outcomes:**

(3g) Identify limitations of each option pricing technique.

**Sources:**

FET-167-09: How to Use the Holes in Black-Scholes by Black

Options Futures & Other Derivatives, Hull, J.C., 8th Edition, 2012

- Chapter 18, Greek Letters

**Commentary on Question:**

*This question tests the calculation of the hedge instrument value for a GMMB contract. It also tests the limitation of the Black-Scholes model and how to improve the volatility assumption.*

**Solution:**

(a) Calculate the GMMB value by using the Black-Scholes model and the above assumptions.

**Commentary on Question**:

*The candidates did extremely well on this section. Once the candidate identified that the GMMB is equivalent to a put option, this is a straightforward put option calculation.*

The embed option in Variable Annuity is a put option.

$p = Ke^{-rt}N(-d_2) - S_0N(-d_1)$

Where

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

or $d_2 = d_1 - \sigma\sqrt{T}$

K = 700 million, $S_0$= 500 million, r = 5%, T = 3, $\sigma$ = 0.2

$$d_1 = \frac{\ln\left(\frac{500}{700}\right) + \left(5\% + \frac{0.2^2}{2}\right) * 3}{0.2 * \sqrt{3}} = -0.365$$

**5. Continued**

$$d_2 = \frac{\ln\left(\frac{500}{700}\right) + \left(5\% - \frac{0.2^2}{2}\right) * 3}{0.2 * \sqrt{3}} = -0.712$$

or $d_2 = -0.365 - 0.2 * \sqrt{3} = -.712$

$$N(-d_1) = 0.642, \quad N(-d_2) = 0.762$$

$$p = 700 * e^{-0.05*3} * 0.762 - 500 * 0.642 = 137.6 \text{ million}$$

(b) Calculate the vega of the GMMB and interpret the result.

**Commentary on Question**:
*The candidates did relatively poorly on this section. Most candidates wrote the Vega formula correctly but did not know how to calculate N'(-d1). Many candidates interpreted the result incorrectly or did not interpret the result at all. A common mistake was to interpret the option value change by the full Vega amount ($323.03M) for a 1% change in the volatility. It should only be 1% of the Vega amount ($3.23M). In order to obtain full credit for this section, candidate needs to calculate the Vega and interpret the result correctly.*

The option's Vega is

$$V = S_0\sqrt{T}N'(d_1)$$

Where

$$N'(d_1) = \frac{1}{\sqrt{2\pi}}e^{-d_1^2/2}$$

$$N'(-0.365) = \frac{1}{\sqrt{2\pi}}e^{-(-0.365)^2/2} = 0.373$$

$$V = 500 * \sqrt{3} * 0.373 = 323.03$$

The Vega is used to measure the option value change with respect to the change of the volatility. Vega of 323.03 means for a 1% increase in the volatility, the guarantee value increases by approximately 1% * 323.03 = 3.23 million.

Alternative solution: Calculate GMMB at 21% and 19% volatility. Put option value is 141 at 21% volatility and 134.4 at 19% volatility. The volatility sensitivity of the put option (for 1% change in volatility) is approximately (141-134.4)/2 = 3.3.

(c) Describe potential problems with using the Black-Scholes model.

**Commentary on Question**:
*Candidates did relatively well on this section. Partial credit was given for stating any of the given answers below or other valid arguments listed elsewhere in FETE syllabus. Full points were received for getting most of the items below and/or providing other valid arguments.*

**5. Continued**

The B-S formula is derived based on the following unrealistic assumptions:

- The stock's volatility is known, and doesn't change over the life of the option.
- The stock price changes smoothly: it never jumps up or down a large amount in a short time.
- The short-term interest rate is constant.
- Anyone can borrow or lend as much as he wants at a single rate.
- An investor who sells the stock or the option short will have the use of all the proceeds of the sale and receive any returns from investing theses proceeds.
- There are no trading costs for either the stock or the option.
- An investor's trades do not affect the taxes he pays.
- The stock pays no dividends.
- An investor can exercise the option only at expiration.
- There are no takeovers or other events that can end the option's life early.

(d) Critique the volatility assumption that you were given.

**Commentary on Question**:
*Candidates did relatively poorly on this section. This section tests the limitation of the constant volatility assumption under the Black-Scholes formula. A common mistake was that most candidates commented on the appropriateness of using the S&P 500 volatility for the Emerging market index fund instead of addressing the constant volatility issue.*

The constant volatility assumption is not realistic. The option value will depend on the entire future path that we expect the volatility to take, and on the uncertainty about what the volatility will be at each point in the future.
The volatility assumption can be improved by quantifying the following measures:

1. Volatility of the volatility – a formula that takes account of changes in volatility will include both current and expected future levels of volatility.
2. Relationship between the future price and volatility – a decline in the stock price implies a substantial increase in volatility, while an increase in the stock price implies a substantial decrease in volatility.

## 6. Learning Objectives:

2. Corporate Financial Applications

**Learning Outcomes:**

(2d) Evaluate alternative options for utilizing capital and recommend the most appropriate use in a given situation.

**Sources:**

Financial Theory and Corporate Policy, Copeland, Weston, Shastri, 4th Edition, 2005

- Chapter 16, Dividend Policy: Theory and Evidence

**Commentary on Question:**

*This question tests the capital structure knowledge and how shareholder wealth is impacted by different dividend policies.*

**Solution:**

(a) List and describe the hypotheses associated with tender offers for share repurchase.

**Commentary on Question**:

*Candidates did well on this section. Most candidates were able to list the hypotheses, however the descriptions of the hypotheses were not as complete.*

**Information or Signaling Hypothesis:**

- Cash dividend may signal that the firm is expected to have increased future Cash flows
- May also imply that the firm has limited profitable investment opportunities
- May indicate the shares are undervalued

**Leverage Tax Shield Hypothesis:**

- If share repurchase is financed by debt issuance, then additional interest payments incur
- Interest payments may generate tax shield for the firm, and thus benefit shareholders

**Dividend Tax Avoidance Hypothesis:**

- Tender for share repurchase will be taxed as a capital gain rather than dividend
- This generates a tax incentive for repurchases as opposed to large dividend payments

**6.** **Continued**

**Bondholder Expropriation Hypothesis:**

- If the repurchase reduces asset base of company, bondholders are worse off as they have less collateral
- This hypothesis can be tested by looking at the firm's bond prices after a repurchase is announced.

**Wealth Transfers among Shareholders:**

- There may be wealth transfers between non-tendering and tendering shareholders

(b) Calculate the total wealth of an existing shareholder, who holds a single share, immediately after execution of the following:

(i) Retain all earnings and no dividend

(ii) Use $5 to pay out dividends

(iii) Issue $5 worth of new ex-dividend shares to finance dividend payment to existing shareholders

(iv) Use $5 to repurchase shares from shareholders in the market

**Commentary on Question**:
*Candidates did relatively well on this section. Most candidates were able to calculate part (i) and( ii) successfully. Some only calculated the total equity value but did not convert to a per share value as asked by the question. Most candidates did relatively poorly on part (iii) largely because of not calculating the ex-dividend number of shares or the ex-dividend price correctly. Most candidates did well on part iv)*

(i) *Retain all earnings and pay no dividend:*
Current share price = $15 million / 10 million shares = $1.5/share
Dividend per share = $0/share
Total wealth of shareholder = Dividend per share + Current share price = $1.5/share

(ii) *Use $5 million to pay out dividends:*
Dividend per share = $5 million / 10 million shares = $0.5/share

Ex-dividend price = (Existing cash – Cash used for dividends + Other assets)/No. shares
= ($10 - $5 + $5) million / 10 million shares = $1/share

**6. Continued**

Total wealth of shareholder = Dividend per share + Current share price = $1.5/share

(iii) *Issue $5 million worth of new shares to finance dividend payments to existing shareholders:*

Dividend per share = $5 million / 10 million shares = $0.5/share

Ex-dividend equity value = Existing equity value + Newly issued equity – Dividends paid = $15 + $5 - $5 million = $15 million

Ex-dividend price = Ex-dividend equity value / (# Newly issued shares + # Existing shares), where # Newly issued shares = Dividends paid / Ex-dividend price
Solving for ex-dividend price:
Ex-dividend price = $1/share

#Total shares = #Newly issued shares + #Existing shares = 5 + 10 million = 15 million

Ex-dividend price = Ex-dividend equity value/# Total shares
= $15 million / 15 million shares = $1/share

Total wealth of shareholder = Dividend per share + Current share price = $1.5/share

(iv) *Use $5 million to repurchase shares from shareholders in the market:*

Dividend per share = $0/share

#Shares repurchased = $5 million / $1.5/share = 3.33 million shares
#Shares remaining = 10 – 3.33 million = 6.67 million shares

Cash remaining = $10 - $5 million = $5 million

Total assets remaining = Cash remaining + Other assets = $10 million

Share price = $10 million / 6.67 million shares = $1.5/share

(c) Describe how dividend payments may or may not appeal to different types of investors.

## 6. **Continued**

**Commentary on Question**:
*Candidates did relatively well on this section. Most candidates were able to successfully identify how dividends can provide a tax advantage or help meet spending requirements; however most candidates missed the dividend preference of an endowment fund and trust.*

Dividend payments may appeal to:

- Investors who are in lower income tax bracket may prefer dividend payments as opposed to capital gains
- Endowment funds or trusts where only dividend income may be spent
- Investors who are spending from wealth and may find it cheaper and easier to receive dividends than to sell or borrow against their shares

**7.** **Learning Objectives:**

2. Corporate Financial Applications

**Learning Outcomes:**

(2f) Describe the process, methods and effects of a potential acquisition or reinsurance of a business including its effect on capital structure, return on equity, price/earnings multiples, and share price.

**Sources:**

Financial Theory and Corporate Policy, Copeland, Weston, Shastri, 4th Edition, 2005

- Chapter 18, Acquisitions, Divestitures, Restructuring, and Corporate Governance, pgs 781 – 806

**Commentary on Question:**

*This question was designed to test a candidate's understanding of the legal structures and financing approaches available for business expansion. Specifically, the candidate was expected to articulate differences between Joint-Venture and Acquisition as well as the impact of acquisition financing methodology on key metrics.*

**Solution:**

(a) Identify situations under which a joint venture would be a better approach.

**Commentary on Question**:

*Candidates did relatively well on this section. A small minority of candidates simply listed the benefits of joint venture rather than identifying situations where joint venture would be a better approach.*

**Joint Venture would be a better approach under the following situations:**

Objective is to exploit a distinctive or narrow opportunity
Objective is the joint production of a single product
Either player has limited risk appetite or would like to share the risk
Strategic flexibility is highly valued
Other hurdles such as limited financial resources, regulatory barriers, geographic constraints etc.
Corporate cultures are incompatible or difficult to integrate
Opportunity is relatively short term in nature

(b) Explain three post-acquisition benefits of ABank's acquisition.

**Commentary on Question**:

*Candidates did well on this section. Some candidates did not receive full credit because the question asked for an explanation whereas the candidates only provided a list.*

## 7. Continued

**Revenues synergies**
The combined firm should be able to increase revenues by increasing cross-selling and market share, using expanded distribution channels, and leveraging broader customer base

**Cost (Operational) synergies**
The combined firm should be able to reduce overhead by leveraging best practices, become more efficient by using scale and enhance operations by combining business activities such as IT, sales, and other operations
Financial synergies
ABank and ALife can better finance and allocate capital, reducing their cost of capital. In addition, the combined firm may be able to use the different taxation regime to its advantage
Diversification
ABank and ALife will benefit as they diversify their risk, profitability, cash flows, geographic presence and line of business exposures making the combined business model more profitable and resilient

(c) Determine the EPS under each funding method.

**Commentary on Question**:
*Candidates did relatively poorly on this section. Many candidates did well in identifying the components to the calculation, however most did not account for or calculate the components correctly. The components most often calculated incorrectly were: goodwill, interest expense, # of shares after-merger.*

**Acquisition financing based on debt**

| | | | |
|---|---|---|---|
| Net income of acquirer | = 400 | = | 400.0 |
| + net income of target | = 100 | = | 100.0 |
| + after-tax revenue and cost synergies | = 20 | = | 20.0 |
| - after-tax deal and integration costs | = - 10 | = | (10.0) |
| - additional after-tax interest expense | = - 1,600 * 10% | = | (160.0) |
| - after-tax goodwill amortization | = - 100 / 10 | = | (10.0) |

## 7. Continued

| | | | |
|---|---|---|---|
| After-Merger Income | = sum of above | = | 340.0 |
| Additional shares issued | = 0 | = | - |
| After-Merger outstanding shares | = 40 | = | 40.0 |
| After-Merger EPS | = 340 / 40 | = | 8.5 |

**Acquisition based on equity issue**

| | | | |
|---|---|---|---|
| Net income of acquirer | = 400 | = | 400.0 |
| + net income of target | = 100 | = | 100.0 |
| + after-tax revenue and cost synergies | = 20 | = | 20.0 |
| - after-tax deal and integration costs | = - 10 – 5 | = | (15.0) |
| - additional after-tax interest expense | = 0 | = | - |
| - after-tax goodwill amortization | = - 100 / 10 | = | (10.0) |
| After-Merger Income | = sum of above | = | 495.0 |
| Additional shares issued | = 1600 / 400 | = | 4.0 |
| After-Merger outstanding shares | = 40 + 4 | = | 44.0 |
| After-Merger EPS | = 495 / 44 | = | 11.25 |

**8.** **Learning Objectives:**

2. Corporate Financial Applications

**Learning Outcomes:**

(2g) Recommend an optimal capital structure and how to implement it for a given business or strategy and be able to justify the recommendation.

**Sources:**

FET-151-08: Real and Illusory Value Creation by Insurance Companies, D. Babbel and C. Merrill, Journal of Risk and Insurance, 2005, Vol. 72, No. 1, pgs 1 - 21

**Commentary on Question:**

*This question tests a candidate's understanding of Franchise Value and Net Tangible Value. Additionally, it tests the understanding of how embedded options and risks impact valuation of assets and liabilities.*

**Solution:**

(a) List approaches which trade away Franchise Value and Minimize Net Tangible Value.

**Commentary on Question**:

*Candidates did well on this section. This is straight list question. Candidate received credit for either listing the approaches or describing the approaches.*

- Inflate statutory surplus and reduce net tangible value: Sale asset with capital gain and hold the asset with capital loss.
- Surplus relief reinsurance: No actual risk transfer in the reinsurance
- Commission financing
- Purchase assets with book yields higher than their expected returns
- Engage in a duration mismatch strategy
- Actuarial Techniques: Using different actuary assumptions.

(b) Identify and explain the options embedded in the liability that account for the difference between B and C.

**Commentary on Question**:

*Candidates did relatively poorly on this section. Most of the candidates only identified the minimum interest rate guarantee, but missed the other options. Common mistakes: 1) Many candidates identified incorrectly that the credit worthiness of insurance company itself was a reason. 2) Many candidates confused the buying or selling of a put or call between the policyholder and the insurer due to the interest rate guarantee.*

## 8. Continued

- SPDA holders have the right to surrender their policies when interest rates change.
- SPDA holders may have:
  - the minimum credit rate guarantees or
  - the minimum death benefits associated with the contract.

(c) Identify and explain the risks that account for the difference in yield between D and F.

**Commentary on Question**:
*Candidates did relatively well on this section. Common mistakes: 1) Most of the candidate missed liquidity risk, 2) Some candidates confused asset risks with reinsurance or product risks.*

Since the D is invested in more risky assets, the difference in yield due to the

- Credit/Default risks
- Liquidity risk.

(d) Explain why when ABC Life writes liabilities and invests in assets to generate a stated spread of $D - B > 0$ the effective spread $E - A < 0$.

**Commentary on Question**:
*Candidates did poorly on this question. Many candidates got drawn to explain why D-B >0 or E-A <0 but did not explain what would cause the sign change. This question is a naturally derivation from part (b) and (c). If candidates did not get part (b) or (c), they usually missed this one too.*

On liability side, since there are the option features for the policy holder, the policy holder have to pay for these options. To reflect the cost in crediting rates B we have the effective yield A.

On the asset side, invested in the lower grade asset than Treasury bond, therefore it has higher yield than Treasury yield. The effective yield (E) will reduced to reflect the default cost.

The negative spread (E-A<0) is due to the duration mismatch risks and spread risks between asset and liability

(e) Describe the problems associated with funding life insurance and annuities with mortgage-backed securities in varying interest rate environments.

## 8. Continued

**Commentary on Question**:
*Candidates did well on this section. Candidates received full credit if they correctly stated the risks related to the asset and liability* <u>*and*</u> *also correctly described the behaviors of asset and liability at one direction of the interest rate changes (either up or down).*

There are two main risks: Prepayment risks for MBS and lapse risks of the life products. When the interest rate changes, the two risks make asset and liability behavior differently.

- Declining interest rate:
    - The MBS prepayment speeds up; maturity profile of the asset portfolio shortens.
    - The insurance products have lower lapse and lower surrender rates; the insurance policies lengthen due to the favorable portfolio yields that are being created on life policies,
- Rising interest rate:
    - The MBS prepayment slows down; maturity profile of the asset portfolio lengthens.
    - The insurance products have higher lapse and higher surrender rates; the insurance policies short since policy holders want to put the fund in higher yield investment.

**9.** **Learning Objectives:**

3. Derivatives, Pricing and Modeling

**Learning Outcomes:**

(3c) Identify embedded options in assets and liabilities.

**Sources:**

Options Futures & Other Derivatives, Hull, J.C., 8th Edition, 2012

- Chapter 28, Interest Rate Derivatives: The Standard Market Models

**Commentary on Question:**

*The purpose of this question was to test the candidate's understanding of embedded options in assets and liabilities as well as pricing techniques and theory for interest rate derivatives. In general, candidates did well on parts (b) and (c) of this question but struggled when applying the concepts.*

**Solution:**

(a) Identify any embedded options in BMD's offer and conditions that might cause the options to be exercised.

**Commentary on Question:**

*Candidates did relatively poorly on this section. Many candidates identified the 2 options but very few were able to identify that there was a* ***put*** *on a bond for the loan commitment option and a* ***call*** *on bond for repayment option.*

*The candidate had to understand that the student taking the loan was similar to a company issuing a bond. Hence, the prepayment option is equivalent to a call because it releases the student from future interest payment and the guaranteed interest rate is equivalent to putting a bond to an investor (the bank here).*

2 embedded options:

(1) **A loan commitment with interest guaranteed for 90 days.** This is equivalent to a put option on a bond in 90 days. Student will exercise the option if loan interest rate increase in 90 days period over 5%.

(2) **Early repayment option during the loan period.** Student may have repaid the loans with accrued interest at any time without penalty. This option is similar to call a bond before a maturity. If interest on loan is less than 5% the student may refinance with a new loan and reduce its payments.

(b) Calculate the price on January 1 of a derivative that caps the 12-month LIBOR rate, starting in 3 months, at 4.1% for 2 years, using the standard market model.

# 9. **Continued**

**Commentary on Question:**
*Candidates did relatively poorly on this section. Most candidates got the right formula, but failed to recognize that they had two caplets to value. Most common errors were related to $t_k$ and $\square_k$. A student having a good understanding of how a cap works should have had those time factors correct rather than simply rewriting the formula. Most candidates provided the basic formulas but were unable to correctly utilize them to arrive at the answer.*

An interest rate cap is a portfolio of put options on zero-coupon bonds
Price of the derivative cap = Price of caplet 1 + Price of caplet 2.

Price for each caplet = $L\delta kP(0, tk+1)$ $[FkN(d1) - RKN(d2)]$

For caplet 1
L = 50,000
$t_k = 0.25$
$\delta k = t_{k+1} - t_k = 1.25 - 0.25 = 1$
$P(0, t_{1.25}) = \exp(-0.040\text{x}1.25) = 0.951$
Fk = 4.0%, Rk = 4.1%
d1 = [ ln(Fk / Rk ) + σ2ktk / 2 ] / σk(tk)^0.5
= [ ln(0.04 / 0.041) + 0.3^2 x 0.25 / 2 ] / (0.3 x 0.25^0.5) = -0.090
d2 = d1 - $\sigma_k(t_k)$^0.5 = -0.09 – (0.3 x 0.25^0.5) = -0.240
N(d1) = 0.464
N(d2) = 0.405
Price caplet 1= 50,000 x 1 x 0.951 x ( 0.04 x 0.464 – 0.041 x 0.405 ) = 92.98

For caplet 2
$P(0, t_{2.25}) = \exp(-0.040\text{x}2.25) = 0.914$
d1 = [ ln(0.04 / 0.041) + 0.3^2 x 1.25 / 2 ] / (0.3 x 1.25^0.5) = 0.094
d2 = d1 - $\sigma_k(t_k)$^0.5 = 0.09 – (0.3 x 1.25^0.5) = -0.241
N(d1) = 0.537
N(d2) = 0.405
Price caplet 2= 50,000 x 1 x 0.914 x ( 0.04 x 0.537 – 0.041 x 0.405 ) = 222.77

Price of the Cap = 92.98 + 222.77 = 315.75

(c) Calculate the price on January 1 of a 3-month swaption giving BMD the right to enter a 2-year interest rate swap, paying 4.1% fixed interest in exchange for 12-month risk-free rate, using the standard market model.

# 9. Continued

**Commentary on Question**:
*Candidates did relatively poorly on this section. The most common error was related to $t_k$. A student having a good understanding of how a swap works should have understood how the time factor works rather than simply rewriting the formula. Most candidates provided the basic formulas but were unable to correctly utilize them to arrive at the answer.*

BMD will pay the fixed leg of the swap and receive the floating leg.
Price of swaption = LA[S0N(d1) − SKN(d2)] where A = 1 / m x sum i=1..mn P(0,Ti)

L = 50,000
m = 1
A = P(0,T1) + P(0,T2) = exp(-0.04x1.25) + exp(-0.04x2.25)
= 0.951 + 0.914 = 1.865
S0 = 0.04, Sk = 0.041
d1 = [ ln($S_0$ / Sk ) + $\sigma^2_k t_k$ / 2 ] / $\sigma_k(t_k)$^0.5
= [ ln(0.04 / 0.041) + 0.3^2 x 0.25 / 2 ] / (0.3 x 0.25^0.5) = -0.090
d2 = d1 - $\sigma_k(t_k)$^0.5 = -0.09 – (0.3 x 0.25^0.5) = -0.240
N(d1) = 0.464
N(d2) = 0.405

Price = 50,000 x 1.865 x (0.040 x 0.464 – 0.041 x 0.405) = 182.30

(d) Complete the template, from the perspective of BMD for the following scenarios:

**Commentary on Question**:
*Candidates did relatively poorly on this section overall. Candidate did much better on part (i) vs. parts (ii) and (iii). Candidates generally were able to get the correct decisions and the student loan cashflows. Common mistakes included:*
1) *Confusing sign of the cash flow for student and bank in both parts (i) and (ii)*
2) *Incorrect cashflows on the swaption/swap and Cap*
3) *Incorrect final loan cashflow which should have included the loan repayment.*

**9.** **Continued**

(i) Scenario 2: Interest rates increase 2.0% immediately after the loan offer.

| Time | 0 | 0,25 | 1,25 | 2,25 |
|---|---|---|---|---|
| Decision: Student Accept Loan Offer (Yes/No) | | Yes | | |
| A) Student Loan Cashflow | | -50 000,00 | 2 500,00 | 52 500,00 |
| B) Loan Funding Cashflow | | 50 000,00 | -3 000,00 | -53 000,00 |
| Net Loan Cashflow (A+B) | | 0,00 | -500,00 | -500,00 |
| Decision: Swaption Exercised (Yes/No) | | Yes | | |
| Swaption/Swap Cashflow | Buy Swaption @ $X | | 950,00 | 950,00 |
| Cap Cashflow | Buy Cap @ $Y | | 950,00 | 950,00 |
| Student Decision: Prepay the Loan (Yes/No) | | No | No | No |

- Student will accept the loan since 5% rate guarantee is less than 6%.
- Bank will exercise Swaption because rate (6%) is in the money at 0.25.
- Student will not prepay the loan because prevailing rates are higher than 5% loan rate.

(ii) Scenario 3: Interest rates are unchanged until loan is accepted and then increase 2.0% immediately after loan acceptance.

| Time | 0 | 0,25 | 1,25 | 2,25 |
|---|---|---|---|---|
| Decision: Student Accept Loan Offer (Yes/No) | | Yes | | |
| A) Student Loan Cashflow | | -50 000,00 | 2 500,00 | 52 500,00 |
| B) Loan Funding Cashflow | | 50 000,00 | -3 000,00 | -53 000,00 |
| Net Loan Cashflow (A+B) | | 0,00 | -500,00 | -500,00 |
| Decision: Swaption Exercised (Yes/No) | | No | | |
| Swaption/Swap Cashflow | Buy Swaption @ $X | | 1 000,00 | 1 000,00 |
| Cap Cashflow | Buy Cap @ $Y | | 950,00 | 950,00 |
| Student Decision: Prepay the Loan (Yes/No) | | No | No | No |

## 9. **Continued**

- Bank will not exercise Swaption because rate (4%) is out the money at 0.25.
- Student will not prepay the loan because prevailing rates are higher than 5% loan rate.

(iii) Scenario 4: Interest rates are unchanged until loan is accepted and then decrease 2.0% immediately after loan acceptance.

| Time | 0 | 0,25 | 1,25 | 2,25 |
|---|---|---|---|---|
| Decision: Student Accept Loan Offer (Yes/No) | | Yes | | |
| A) Student Loan Cashflow | | 0,00 | 0,00 | 0,00 |
| B) Loan Funding Cashflow | | 50 000,00 | -1 000,00 | -51 000,00 |
| Net Loan Cashflow (A+B) | | 50 000,00 | -1 000,00 | -51 000,00 |
| Decision: Swaption Exercised (Yes/No) | | No | | |
| Swaption/Swap Cashflow | Buy Swaption @ $X | | -1 000,00 | -1 000,00 |
| Cap Cashflow | Buy Cap @ $Y | | 0,00 | 0,00 |
| Student Decision: Prepay the Loan (Yes/No) | | Yes | n/a | n/a |

- Bank will not exercise Swaption because rate (4%) is out the money at 0.25.
- Student will immediately prepay the loan because prevailing rates are lower than 5% loan rate. So refinance at a lower rate.

(e) Describe the advantages and disadvantages to BMD of each interest rate derivative relative to the liability based on your observations above.

**Commentary on Question**:
*Candidates did poorly on this section. Very few candidates provided adequate understanding of the advantages and disadvantages from BMD's point of view. Many candidates listed general advantages and disadvantages without relating it to BMD.*

CAP
*Advantage*
- Risk of floating rate rising and eroding interest spread is reduced
- Can profit from declines in floating rate after the loan offer is accepted

## 9. Continued

*Disadvantages*

- Expensive relative to swaption
- Pre-commitment problem – if rates drop in next 90 days, loan offer likely not accepted and will not need the derivative (and its value will have declined)
- Risk of loan pre-payment – if rates drop after 90 days, loan could be prepaid and cap will no longer be needed (and its value will have declined)

SWAPTION

*Advantages*

- Can lock-in a particular funding cost now – if rates rise and loan is accepted, can exercise swaption and pay locked-in fixed rate
- Do not need to pre-commit to borrowing – if rates drop in next 90 days and loan is not accepted, do not need to exercise swaption. If loan is exercised despite decline in interest rates, can still purchase a swap at market rates.

*Disadvantages*

- Risk of loan pre-payment – if loan is accepted and a swap entered, this derivative does not protect from early loan pre-payment. Then swap not a good hedge for prepayment risk (could have negative value)

**10.** **Learning Objectives:**

1. Modern Corporate Financial Theory

2. Corporate Financial Applications

**Learning Outcomes:**

(1b) Calculate the cost of capital for a venture or a firm using the most appropriate method for given circumstances and justify the choice of method.

(2g) Recommend an optimal capital structure and how to implement it for a given business or strategy and be able to justify the recommendation.

**Sources:**

Financial Theory and Corporate Policy, Copeland, Weston, Shastri, $4^{th}$ Edition, 2005

- Chapter 15, Capital Structure and the Cost of Capital: Theory and Evidence

FET-109-07: Integrated Risk Management, Doherty, Chapter 13: Contingent Leverage Strategies and Hybrid Debt

**Commentary on Question:**

*This question tested the candidate's ability to calculate hurdle rates and WACC under multiple capital structures. It also tested the candidate's understanding of what rate to use when evaluating a project.*

**Solution:**

(a) Calculate the hurdle rate the Company should use in evaluating Project X before and after the debt increase.

**Commentary on Question**:

*Candidates did relatively well on this section. Candidates who were able to correctly apply the WACC formulas generally got full credit. A common mistake was that candidates used the company's beta instead of the project's beta. Also, several candidates only calculated the expected return but did not solve the WACC.*

*There were several ways to arrive at the hurdle rate. The two below were most commonly used with solution 2 being more prevalent.*

**Solution 1:**

Before debt increase:

- Debt Ratio = 20%
- Expected Return = Rf + (Rm – Rf) × βu
- Expected Return = 5% + (12% – 5%) × 0.5
- Expected Return = 8.5%
- WACC20% = ρ × (1 – tax × debt ratio)

## 10. **Continued**

- $\rho$ = Expected Return = 8.5%
- WACC20% = 8.5% × (1 – 50% × 20%)
- WACC20% = 7.65%

After debt increase:
- Debt Ratio = 50%
- Expected Return = Rf + (Rm – Rf) × $\beta$u
- Expected Return = 5% + (12% – 5%) × 0.5
- Expected Return = 8.5%
- WACC50% = $\rho$ × (1 – tax × debt ratio)
- $\rho$ = Expected Return = 8.5%
- WACC50% = 8.5% × (1 – 50% × 50%)
- WACC50% = 6.38%

**Solution 2:**

Before debt increase:
- $\beta_L$ = [1 + (1 – tax) x (debt %) / (1 – debt %)] x $\beta$u
- - $\beta_L$ = [1 + (1 – 50%) x (20%) / (1 – 20%)] x 0.5
- - $\beta_L$ = 0.5625
- - $k_s = r_f + ( E(R_m) – r_f) \times \beta_L$
- $k_s$ = 5% + ( 12% - 5%) x 0.5625
- $k_s$ = 0.0894
- $WACC_{20\%}$ = $k_b$ x (1-tax) x (debt ratio) + $k_s$ x (equity ratio)
- $WACC_{20\%}$ = 5% x (1 – 50%) x (20%) + 0.0894 x (80%)
- $WACC_{20\%}$ = 7.65%

After debt increase:
- $\beta_L$ = [1 + (1 – tax) x (debt %) / (1 – debt %)] x $\beta$u
- - $\beta_L$ = [1 + (1 – 50%) x (50%) / (1 – 50%)] x 0.5
- - $\beta_L$ = 0.75
- - $k_s = r_f + ( E(R_m) – r_f) \times \beta_L$
- $k_s$ = 5% + ( 12% - 5%) x 0.75
- $k_s$ = 0.1025
- $WACC_{50\%}$ = $k_b$ x (1-tax) x (debt ratio) + $k_s$ x (equity ratio)
- $WACC_{50\%}$ = 5% x (1 – 50%) x (50%) + 0.1025 x (50%)
- $WACC_{50\%}$ = 6.38%

(b) Calculate the WACC of the Company using the new targeted capital structure.

## 10. **Continued**

**Commentary on Question**:
*Most candidates did well on this question. A common mistake was that candidates used the projects' beta instead of the company's beta.*
*There were several ways to arrive at the WACC. The two below were most commonly used with solution 2 being more prevalent.*

**Solution 1:**

WACC of Company

- $k_s = R_f + (R_m - R_f) \times \beta_L$
- $k_s = 5\% + (12\% - 5\%) \times 0.8$
- $k_s = 10.6\%$
- $WACC_{20\%} = k_b$ (1-tax) (debt ratio) + $k_s$ (equity ratio)
- $WACC_{20\%} = 5\%\ (1\text{-}50\%)\ (20\%) + 10.6\%\ (80\%)$
- $WACC_{20\%} = 8.98\%$
- $\rho = WACC20\% \div (1 - tax \times \text{debt ratio})$
- $\rho = 8.98\% \div (1 - 50\% \times 20\%)$
- $\rho = 9.98\%$
- $WACC_{50\%} = \rho \times (1 - tax \times \text{debt ratio})$
- $WACC_{50\%} = 9.98\% \times (1\text{-}50\% \times 50\%)$
- $WACC_{50\%} = 7.48\%$

**Solution 2:**

- $\beta_L = [\ 1 + (1 - tax)\ (debt / equity)\ ]\ Bu$
- $0.8 = [\ 1 + (1 - 50\%)\ (20\% / 80\%)\ ]\ Bu$
- $Bu = 0.711$
- $\beta_{L50\%} = [1 + (1 - tax)\ (debt / equity)\ ]\ Bu$
- $\beta_{L50\%} = [1 + (1 - 50\%)\ (50\% / 50\%)\ ]\ 0.711$
- $\beta_{L50\%} = 1.067$
- $k_s = R_f + (R_m - R_f) \times \beta_L$
- $k_s = 5\% + (12\% - 5\%) \times 1.067$
- $k_s = 12.47\%$
- $WACC_{50\%} = k_b$ (1-tax) (debt ratio) + $k_s$ (equity ratio)
- $WACC_{50\%} = 5\%\ x\ (1 - 50\%)\ (50\%) + (12.47\%)\ x\ (50\%)$
- $WACC_{50\%} = 7.48\%$

(c) Explain if the Company should accept Project X.

## 10. Continued

**Commentary on Question**:
*Candidates did well on this section overall. Partial credit was given if a candidate evaluated the project against one of the hurdle rates above. However to get full credit the candidate needed to use the company's WACC from part (b). About half the candidates incorrectly used the hurdle rate of the project instead of the company's WACC to evaluate this decision.*

The company's WACC is 7.48%. Shareholders would not accept a project with return lower than 7.48%. Since management's interest is aligned with shareholders, the company should not accept Project X, since it returns only 7%.

**11.** **Learning Objectives:**

1. Modern Corporate Financial Theory

**Learning Outcomes:**

(1d) Define and compare risk metrics used to quantify economic capital and describe their limitations.

(1e) Apply the concept of economic capital and describe methodologies for allocating capital within a financial organization.

**Sources:**

FET-114-07: Capital Allocation in Financial Firms

FET-170-09: Theory of Risk Capital in Financial Firms, by Merton & Perold

FET-178-12: Economic Capital Modeling: practical Considerations

**Commentary on Question:**

*This question tests candidates' knowledge of allocating capital within a financial organization and demonstration of their understanding by solving a numerical problem. Overall candidates performed well on the numerical problem; however, candidates often earned less credit on parts of the question asking to "justify," "compare" and "explain".*

**Solution:**

(a) Calculate the expected returns on a stand-alone basis, fully allocated risk capital, and marginal risk capital for business unit B.

**Commentary on Question:**

*Candidates did well on this section. Most candidates correctly calculated the aggregate risk capital. Those candidates who did not allocate the risk capital appropriately could still have received partial credit.*

Stand-alone basis:
Expected return = 10 / 100 = 10%

Fully allocated risk capital:
Since the two business projections are independent, the standard deviation of the aggregate firm is the square root of the sum of stand-alone standard deviations.
Aggregate risk capital = $(200^2 + 100^2)^{1/2}$ = 223.6
Fully allocate risk capital = 100 / (100+200) * 223.6 = 74.5
Expected return = 10 / 74.5 = 13.4%

Marginal risk capital:
Marginal risk capital = 223.6 – 200 = 23.6
Expected return = 10 / 23.6 = 42.4%

## 11. Continued

(b) Recommend and justify whether or not adding business unit B based on a comparison of the expected profit after deducting the deadweight cost of risk capital, assuming that the deadweight cost of risk capital is 20%.

**Commentary on Question:**
*Candidates did relatively well on this section. Common mistakes included:*
*a) The candidate recommended a decision but did not show or attempt any numerical calculations to justify the decision.*
*b) The candidate did not perform the appropriate calculations, i.e., determining the profit after accounting for the deadweight cost of risk capital.*
*c) The candidate's calculations did not support the recommendation.*

Stand-alone B:
Expected profit after deadweight cost of risk capital = 10 – 20% * 100 = -10

Recommend and justify: Looking at a stand-alone basis, I would not recommend adding business unit B due to a negative expected profit after deadweight cost of risk capital.

Stand-alone A:
Expected profit after deadweight cost of risk capital = 50 – 20% * 200 = 10

Business units A + B:
Aggregate risk capital = 223.6 (from part a)
Expected profit after deadweight cost of risk capital = 50 + 10 – 20% * 223.6 = 15.3

Recommend and justify: Since $15.3 is greater than $10, I would recommend adding business unit B due to a greater expected profit after deadweight cost of risk capital.

(c) Determine the marginal risk capital for business unit B using the above-mentioned Merton-Perold's approximate model with $T = 1$.

**Commentary on Question**
*Candidates did relatively well on this section. Many candidates understood the method needed to calculate the aggregate capital excluding business B. There were two common mistakes: (1) failing to calculate the aggregate volatility for units A + C; (2) failing to calculate the correct aggregate volatility. If either mistake was made partial credit was given.*

Aggregate volatility = $\sqrt{\sum\sum w_i w_j \sigma_i \sigma_j \rho_{ij}}$

# 11. Continued

Aggregate volatility for all 3 units

$$= \sqrt{\begin{array}{l}1/3*1/3*37.5\%*37.5\%+1/3*1/3*50\%*50\% \\ +1/3*1/3*62.5\%*62.5\%+2*1/3*1/3*37.5\%*50.0\%*0.5\end{array}}$$

= 32.8%

Total risk capital for all 3 units = 0.4 * 6000 * 32.8% * 1 = 787

In order to calculate the marginal risk capital for business unit B, one must determine the aggregate capital without business unit B.

Aggregate volatility for units A + C

$$= \sqrt{\begin{array}{l}1/2*1/2*37.5\%*37.5\%+2*1/2*1/2*37.5\%*62.5\%*0 \\ +1/2*1/2*62.5\%*62.5\%\end{array}}$$

= 36.4%

Total risk capital for units A + C = 0.4 * 4000 * 36.4% * 1 = 583

Marginal risk capital for business unit B = 787 – 583 = 204

(d) Compare VaR to Tail VaR based on the above-mentioned paper.

**Commentary on Question**:
*Candidates did relatively poorly on this section. The common mistake was a failure to define VaR and Tail VaR which is instrumental in a comparison. Candidates who merely listed the table of properties for VaR and Tail VaR received partial credit because they did not provide a full comparison.*

VaR (Value at Risk) assesses the probability of ruin at a given quartile of the probability distribution.

Tail VaR (Tail Value at Risk) considers both the probability and severity of losses that exceed a given quartile.

VaR is commonly used in banking; Tail VaR is more common in insurance due to skewed risk distributions. However, Tail VaR presents more challenges in finding data to accurately model the tail of the probability distribution.

(e) Explain why a real-world technique would be preferred over a risk-neutral technique in an economic capital calculation.

# 11. Continued

**Commentary on Question**

*Candidates did relatively poorly on this section.  Many candidates correctly stated that economic capital was more appropriately calculated using real-world methods.  However, most candidates also performed a "brain dump" of information which was not relevant to the question and therefore no points could be awarded for this.*

Real-world is a method to calculate present value of cash flows by discounting projected cash flows with risk discount rates based on multiple scenarios.

Under real-world assumptions, projected cash flows are not adjusted for uncertainty risk.  To reflect "price" of uncertainty risk, it is common to set the discount rate higher than a risk free rate.

Reasons real-world would be preferred over risk-neutral:

- Under risk-neutral assumptions, an adjustment is made to be consistent with observable market prices of securities;
- Under risk-neutral assumptions, economic capital needs to be defined as expected value of cash flows;
- Under risk-neutral assumptions, it is often difficult to interpret the meaning of results since loss amount is derived as expected value under an adequate risk-neutral probability distribution.

**12.** **Learning Objectives:**

4. Efficient and Inefficient Markets, Complete and Incomplete Markets, Information Theory & Market Misbehavior

**Learning Outcomes:**

(4a) Define capital market efficiency and the value of information.

(4c) Describe empirical evidence and results regarding market efficiency.

**Sources:**

Financial Theory and Corporate Policy, Copeland, Weston, Shastri, 4th Edition

- Chapter 10, Efficient Capital Markets: Theory

**Commentary on Question:**

*This question asks the candidates to apply an understanding of efficient markets to a specific situation and adapt the concept of binomial trees in a situation other than option pricing.*

**Solution:**

(a) Describe and differentiate these two positions specifically with respect to tax-exempt bonds.

**Commentary on Question**:

*Candidates did well in this section. They were able to describe intrinsic value hypothesis and rational expectations hypothesis. Some candidates listed all four hypotheses as described in the source material. However credit was only given for the relevant items. Even if exact definitions given by candidates were incomplete, partial credit could be earned for recognizing the differentiating feature between the two hypotheses, namely the consideration of resale value to other individuals under the rational expectation hypothesis.*

*Some candidates were less successful in relating the definitions specifically to tax exempt bonds, which was required to earn full credit. Both "cashflows" and "coupon and/or principal payments" earned equal credit. Additional credit was earned for mentioning taxes saved under intrinsic value hypothesis, as well as for mentioning resale value under rational expectations hypothesis.*

Under intrinsic value hypothesis, prices will be determined by each individual's estimate of the payoffs of an asset without consideration of its resale value to other individuals. With respect to tax-exempt bonds, the payoffs are coupon and principal payments, as well as taxes saved by the investor.

Under rational expectations hypothesis, prices will be determined based on expected future payoffs of an asset, including the resale value to third parties. With respect to tax-exempt bonds, the resale value must consider whether investors gain some, all, or no tax benefits from the tax-exempt bonds.

## 12. Continued

(b) Differentiate between markets that fully aggregate information and those that only average it.

**Commentary on Question**:
*Candidates did relatively poorly on this section. They generally equated aggregating information to efficient markets and equated averaging information to inefficient markets. They then proceeded to differentiate efficient and inefficient markets. While it is true that a rational expectations market is an efficient market because prices reflect all information, this approach was unsuccessful towards answering the question.*

Markets that average information have asset prices that average the expectations of all market participants. Markets that aggregate information have asset prices that reflect all available information.

In a fully aggregating market even insiders who possess private information would not be able to profit by it.

(c) Question statement.

(i) Identify the hypothesis that you should seek in the bond traders to be hired.

(ii) Outline a bond trading strategy that will be advantageous given the competing trading firms.

**Commentary on Question**:
*(i) Candidates did relatively well on this section. Most candidates identified rational expectations hypothesis as the desired hypothesis.*
*(ii) Candidates did relatively poorly on this section. Candidates generally were not successful at clearly outlining a strategy along with a reasoning as to its advantage. Many candidates suggested trading with traders who held different hypothesis view and exploiting that difference. This misses the point to some degree. For example, while intrinsic hypothesis may ignore some information, namely, the benefit of tax-exempt status, a bond that provides no tax benefits might be valued equally under intrinsic hypothesis and rational expectations hypothesis, if the resale value is neutral. Also, a trader can't readily identify another trader's hypothesis to their advantage. All they can do is recognize that the bond provides different tax benefits to differently taxed investors.*

## 12. Continued

(i) Traders should be those that utilize rational expectations hypothesis. The new bonds do not have the same value to all investors (tax treatment). Traders utilizing rational expectations hypothesis are best able to evaluate the value that a bond has to different investors.

(ii) On January 1, foreign companies holding the new bonds will not receive tax benefits. Good trading strategy is to trade with foreign firms who value the tax-exempt bonds the least. This will obtain the best price for the bonds.

(d) Recommend and justify mathematically the expected number of each type of trader.

**Commentary on Question**:
*Candidates did relatively well on this section. While many candidates did not provide an answer, those that did at least identified the possible value of "d" at January 1. A common mistake was to use a binomial tree with one node instead of two.*

*Some candidates recommended an expected number of each type of trader with little justification, mathematical or otherwise. No marks could be awarded in this case, as the candidate was not able to demonstrate to the grader an understanding of the underlying theory.*

*If a candidate made an arithmetic error at some point along the way, then partial credit could still be obtained as long as the incorrect values were correctly carried through. An incorrect number based on correct logic was accepted, for partial credit.*

Current competitive advantage "d" = 4.00 at Nov 1. Calculate possible values of d at January
(the following solution relies on a binomial tree structure with two nodes):

At Jan 1 with 50% * 50% = 25% probability (up twice), d = 4.00 + 0.75 + 0.75 = 5.5
At Jan 1 with 50% * 50% = 25% probability (up/down), d = 4.00 + 0.75 - 0.75 = 4.0
At Jan 1 with 50% * 50% = 25% probability (down/up), d = 4.00 - 0.75 + 0.75 = 4.0
(alternatively, equal credit was given for d = 4.0 once with 50% probability)
At Jan 1 with 50% * 50% = 25% probability (down twice), d = 4.00 - 0.75 - 0.75 = 2.5

## 12. **Continued**

At each node, calculate the allocation to the analyst strategy "p" = $(rd-c_2)/(rd-r)$

Node d=5.5 → p = (.04*5.5 - .11)/(.04*5.5 - .04) = .11/.18 = 61.1%
Node d=4.0 → p = (.04*4.0 - .08)/(.04*4.0 - .04) = .08/.12 = 66.7%
Node d=2.5 → p = (.04*2.5 - .05)/(.04*2.5 - .04) = .05/.06 = 83.3%

Expected allocation = 25%*61.1 + 50%*66.7 + 25%*83.3 = 69.45%

This is approximately 70% which means you should hire 7 Analysts and 3 Random Selectors

**13. Learning Objectives:**
1. Modern Corporate Financial Theory

2. Corporate Financial Applications

**Learning Outcomes:**
(1b) Calculate the cost of capital for a venture or a firm using the most appropriate method for given circumstances and justify the choice of method.

(2a) Recommend a specific approach to raising capital for a given project or firm, including the steps needed to obtain funds.

**Sources:**
Financial Theory and Corporate Policy, Copeland, Weston, Shastri, 4th Edition, 2005
- Chapter 2, Investment Decisions: The Certainty Case

FET-108-07: Integrated Risk Management, Doherty, Chapter 13: Contingent Leverage strategies and Hybrid Debt

**Commentary on Question:**
This question tests the capital structure and risk management, integrated with capital budgeting techniques.

**Solution:**
(a) Prepare a briefing to the management identifying and describing the following:

(i) Benefits to TGI of a convertible bond issue relative to equity and conventional debt issues.

(ii) Drawbacks of raising capital using convertible bonds.

**Commentary on Question**:
*Candidates did relatively well on this section. Common omissions were: 1) Many candidates didn't provide enough explanation to convey the meaning of the point. For example, "costs less" is not sufficient. 2) Candidates should have answered the question from the firm's perspective as requested in the question.*

(i) Benefits are:

- Positive signal to market compared to equity issue since includes features of straight debt
- Convertible bonds are often issued with a call provision that allows a firm to force conversion.
- better tailored to the cash flow patterns of a rapidly growing firm
- lower coupon rate than comparable straight debt due to positive value of right to convert

## 13. **Continued**

- bondholders are less concerned about possibility of undertaking risky projects as conversion privilege allows bondholders to participate in the value created
- Reduced convexity of payoff compared to straight debt means convertible bond is relative insensitivity to the risk of issuing firm making valuation of bond easier
- Conversion allows debt investor to share the potential increase in firm.

(ii) Drawbacks are:

- Not as cheap as straight debt because convertible bonds are riskier and that their true cost of capital is greater
- Dilution to existing share value on equity side once conversion is exercised because the original shareholders have to give up some upside potential of firm value and this makes the equity holders a bit less risk tolerant.
- From original equity holder's point of view, a lower default point Dc on convertible debt means the shareholders have a bit more downside risk. (If the firms value falls below Dc, the firm is insolvent)

(b) Calculate the convertible bond's before-tax cost of capital $k_{CV}$.

**Commentary on Question**:
*Candidates did well on this section. While no candidates converted the interest rate to a continuous rate as required to accurately apply Black-Scholes, most candidates still received full marks for doing everything else correctly.*

Convertible bond value = straight debt + warrant (CV=B+W)
Using given bond value, the value of warrant piece
W = CV-B=1000-580 = $420
Using the relationship between $d_1$ and $d_2$,
$d_2 = d_1 - \sigma\sqrt{T} = 1.288 = d_1 - 0.3\sqrt{20}$
$d_1 = 2.63$
$N(d_1) = 0.9957 \; and \; N(d_2) = 0.9015$
$\lambda = ln1.14 = 13.1\%$
Call = $N(d_1)S - N(d_2)Ke^{-rT}$ =20.2
$\beta_c = \frac{S}{C} \times N(d_1) \times \beta_s = \frac{22}{20.2} \times 0.9957 \times 1.8 = 1.95$
$k_c = r_f + \beta_c \times (E[r_M] - r_f) = 14\% + 1.95 \times (20\% - 14\%) = 25.7\%$
Cost of capital of warrant equals cost of capital of the call option on firm's value, i.e. $k_w = k_c$

## 13. **Continued**

The before tax cost of capital for the convertible bond is

$$k_{cv} = k_d \frac{B}{B+W} + k_w \frac{W}{B+W} = 15\% \frac{580}{1000} + 25.7\% \frac{420}{1000} = 19.49\%$$

Later, after the convertible bond has been issued, but before any conversions are exercised, the following is known:

- The firm had 80,000 shares outstanding.
- Total debt before conversion was $1,500,000 of which $500,000 is senior debt and the remainder is convertible bond with face amount $1,000,000.
- Per $1000 par value, the conversion ratio is 20:1.
- Marginal tax rate is 30%.
- Critical firm value is $3,000,000

Also assume a +1% change in stock price due to conversion causes the systematic risk to *decrease* by 0.25%.

(c) Calculate the maximum value of the convertible bond below which investors should exercise the conversion at the given critical firm value.

**Commentary on Question**:
*Candidates did relatively well on this section. Some candidates incorrectly solved for the value of the firm instead of the value of the convertible bond. Other candidates miscalculated the number of shares converted.*

Number of convertible bond issued =(1500K-500K)/1000=1K
Number of converted share n=1K*20=20K

$$\frac{n}{n+m}(V(F) - senior\ debt) > D_c$$

$$V(F) = 3\text{million}$$

$$\frac{20}{20+80}(3\ million - 0.5million) > D_c$$

To make the investors to exercise the conversion now, the maximum value of convertible bond is $D_c = 500K$

(d) Demonstrate mathematically that the beta after the conversion is 1.89.

**Commentary on Question**:
*No candidates received marks for this part because the stock price at conversion was not supplied until part (e).*

## 13. **Continued**

Beta after conversion is calculated by using the given sensitivity factor:
First, need to calculate the stock price after conversion:
Price of stock after conversion = \$30 *80K /(80K+20K)=\$24

$$\beta_{s\,new} = 1.8 * \left\{1 + \left(\frac{stock\ price\ after\ conversion}{stock\ price\ at\ conversion} - 1\right) * 1\%\right\}$$
$$= 1.8 * \left\{1 + \left(\frac{\$24}{\$30} - 1\right) * (-0.25\%)\right\} = 1.89$$

(e) Calculate the WACC after conversion, assuming the stock price increases to \$30 per share at conversion.

**Commentary on Question**:
*Candidates did relatively poorly on this section. While no candidates calculated the weighting in the cost of capital properly, it was clear that candidates generally understood the structure and application of the WACC formula and received credit for this. Some candidates incorrectly used the risk free rate instead of the bond rate in the WACC calculation.*

WACC after conversion:
$k_{s\,new} = r_f + \beta_{s\,new} \times \left(E[r_M] - r_f\right) = 14\% + 1.89 \times (20\% - 14\%) = 25.34\%$

*(Comments: Simply CAPM formula.)*
WACC after conversion
=
$k_{bond} \frac{B}{B+S}(1 - \tau) + k_{s\,new} \frac{S}{B+S} =$
$\left\{\frac{500}{500+80*24+20*24} 15\%\right\}(1 - .3) + 25.34\% \frac{80*24+20*24}{500+80*24+20*24} = 22.78\%$

*(Comments: Simply WACC formula.)*

**14.** **Learning Objectives:**

1. Modern Corporate Financial Theory

3. Derivatives, Pricing and Modeling

**Learning Outcomes:**

(1d) Define and compare risk metrics used to quantify economic capital and describe their limitations.

(3a) Define the cash flow characteristics of complex derivatives including exotic options, interest rate derivatives, swaps, and other non-traditional derivatives.

(3c) Identify embedded options in assets and liabilities.

(3f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.

**Sources:**

Chapter 9 CSFB Handbook, Risk Measures: How Long is a Risky Piece of String?
https://www.credit-suisse.com/investment_banking/platforms_applications/doc/credit_portfolio_modeling.pdf

Options Futures & Other Derivatives, Hull, J.C., 8th Edition, 2012

- Chapter 13, Wiener Processes and Ito's Lemma (Appendix, exclude multivariate material)
- Chapter 14, Black –Scholes-Merton Model
- Chapter 25, Exotic Options (25.1, 25.2, 25.4, 25.6 – 25.11, 25.13 – 25.16 only)

**Commentary on Question:**

*Part (a) of the question asks the candidate to identify the embedded option in liabilities, apply delta hedging technique to determine the cost of hedging.*

*Part (b) of the question asks the candidate to use the Value at risk concept to quantify economic capital.*

*Part (c) of the question asks the candidate to define the cash flow characteristics of traditional option and exotic option.*

*Part (d) of the question asks the candidate to compare the costs and risks of various hedging strategies.*

**Solution:**

(a) Calculate the number of futures contracts required to hedge your company's position at issue for Product A using Strategy 2.

## 14. Continued

**Commentary on Question**:
*Candidates generally did poorly on this part.*
*Common mistakes: 1) A put option delta was calculated, 2) 1.1 was not multiplied in calculating the option delta, 3) strike price of 1.1 million was used instead of 1 million.*

Payoff = 1 million + 1.1 max (S (T) – 1 million, 0) = 1 million + 1.1 call option with strike price (K) = 1 million

So, the embedded hedged option = 1.1 call option with strike price (K) = 1 million
$d_1 = (\ln (S (0) / K + (r_f + \sigma^2/2)*T) / (\sigma (T^{0.5})$
$r_f = 4\%$, $\sigma = 18\%$, T = 5, S (0) = 1 million
$d_1 = 0.6982$
$N (d_1) = 0.7575$
$1.1 * N (d_1) = 0.8332$
Number of future contracts = 1 million * 1.1 $N (d_1)$ * exp (-4%) / 100,000 = 8

(b) Calculate the fund amount in Strategy 1 so that there will be a 99% probability of being able to pay the guarantee out of the accumulated value of this fund for Product A.

**Commentary on Question**:
*Candidates generally did poorly on this part.*
*Common mistakes: 1) 15% was used instead of $r_f$ in determining S (5),2) Incorrect Z was calculated, 3) 1.1 was not multiplied in determining the increase*

Let A be the Fund amount
Pr (S (0) + A exp (4%*5) > (S (5) – S (0)) * 1.1 + S (0)) = 99%
$S (5) = S (0) (\exp ((r_f - 1/2\sigma^2) * 5 + \sigma Z (5)^{0.5}))$
Z ~ Normal (0, 1)
N (2.326) = 99%
Pr (A * 1.2214 / 1.1 > (S (0) * exp (0.119 + 0.4025 Z) – S (0)) = 99%
Pr (A * 1.2214 / 1.1 > S (0) * (exp (0.119 + 0.4025 Z) - 1) = 99%
(ln ( 1 + A * 1.2214 / 1,100,000) - 0.119) / 0.4025 = 2.326
1 + A * 1.2214 / 1,100,000 = exp (0.119 + 0.4025 * 2.326)
1 + A * 1.2214 / 1,100,000 = 2.871943
A = 1,686,780

(c) Recommend and justify the combination of assets instruments (bonds, nonstandard and standard options) needed to statically hedge Product B from issue. Define the exercise prices of the options.

## 14. Continued

**Commentary on Question**:
*Candidates generally did poorly on this part.*
*Common mistake: Very few candidates realize a knock-in put option should be used to hedge the 50% lock-in.*

Zero coupon bond to provide the return of premium
A call option 100% index strike to hedge the increase over the index
A knock-in put option with a barrier and an exercise price equals to 150% of the index level to hedge the 50% lock-in

(d) Compare and contrast Strategy 2 with the static option hedging strategy in part (c) with respect to:

(i) Costs

(ii) Risks

**Commentary on Question**:
*Candidates generally did relatively well on this part. Candidates who answered generally got credit for their answers.*

(i) Costs

Option more expensive, delta hedging in theory cheaper but significant risks could increase the cost
Future needs frequent rebalancing

(ii) Risks

Delta Hedging
Future delta hedging minimal basis risk
The delta of the option is discontinuous at the barrier making conventional hedging very difficult
Market risk – major index move might not get hedged for delta hedging
Model risk - is S&P going to move like the model predictor?
You need to consider hedging Gamma and Rho (not done in delta hedging)
Huge operational

Option Hedging
Liquidity risk, credit risk and operational risk not taken care of with options

## 15. **Learning Objectives:**

3. Derivatives, Pricing and Modeling

**Learning Outcomes:**

(3f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.

(3h) Describe and evaluate equity and interest rate models.

(3i) Contrast commonly used equity and interest rate models.

(3l) Recommend an equity or interest rate model for a given situation.

**Sources:**

Options Futures & Other Derivatives, Hull, J. C., 8th Edition, 2012

- Chapter 28, Interest Rate Derivatives: the Standard Market Models
- Chapter 30, Interest Rate Derivatives: Models of the Short Rate (exclude 30.2 properties of Vasicek and CIR, exclude 30.3 BDT and Black-Karasinski)

**Commentary on Question:**

*This question tests basic foundation of interest rate derivatives in the area of "risk-neutral" valuation concept and its application to determining the "forward swap rate" and the value of the swaption.*

**Solution:**

(a) Verify that each value of $P(0,t)$ given above is correct.

**Commentary on Question**:

*Candidates did well in demonstrating each P(0, t) is correct. A common error was that candidates applied an incorrect approach to verify P(0, t) by using the "expected risk-neutral <u>interest rate</u>" to compute P(0, t). The correct approach was to use the "expected risk-neutral <u>zero-coupon bond price</u>" to derive P(0, t). No credit was given if the validation approach was incorrect even if the final results may have rounded to the correct figure.*

P(0, 1) = exp(-0.04) = 0.9608
P(0, 2) = P(0,1) * (0.4 * exp(-0.045) + 0.6 * exp(-0.035)) = 0.9241

P(1, 3, 0) = exp (-0.045) * (0.4 * exp(-0.05) + 0.6 * exp(-0.04)) = 0.9149
P(1, 3, 1) = exp (-0.035) * (0.4 * exp(-0.04) + 0.6 * exp(-0.03)) = 0.9333
P(0, 3) = exp (-0.04) * (0.4 * P(1,3,0) + 0.6 * P(1,3,1)) = 0.8896

(b) Determine the annual coupon rate for a 3 year bond issued at par at time zero.

**15. Continued**

**Commentary on Question**:
*Candidates did extremely well in this section.*

1 = c * (P(0,1) + P(0,2) + P(0,3)) + P(0,3)
1 = c * (0.9608 + 0.9241 + 0.8896) + 0.8896
c = 3.978%

(c) Demonstrate that the swap rate on a 2-year interest rate swap which starts in 1 year is 3.923%.

**Commentary on Question**:
*Candidates did poorly on this section. More than 70% of candidates made no attempt to answer this question. For those who tried, few did correctly with a wide variety of mistakes made.*

1-year forward 2-year swap rate, s, satisfies:
s*P(0,2) + (1+s)*P(0,3) = P(0,1)
s = [P(0,1) – P(0,3)]/[P(0, 2) + P(0,3)]
= [0.9608 – 0.8896] / [0.9241 + 0.8896]
= 3.923%

(d) Calculate the value of the swaption.

**Commentary on Question**:
*Candidates did relatively well on this section. This is a straight-forward application of swaption pricing formula. Many candidates knew the swaption formula to start with and received partial credit. A common mistake was to confuse the option maturity (1 year option) with the tenor of the underlying swap (the underlying is a 2-year swap). As a result, more than half of the candidates used an option maturity of 2 years (T=2) rather than the correct 1 year (T = 1). In addition, many candidates did not calculate A(1,2) correctly. Other common mistakes included A = P(0,1) + P(0,2) or A = P(0,1) + P(0,2) + P(0,3).*

Value = L*A(1,2) * [swap rate * N(d1) – Sk *N(d2)]
L = 10,000,000
A(1,2) = P(0,2) + P(0,3) = 0.9241 + 0.8896 = 1.8137
d1 = (ln (swap rate / Sk) + 1/2 * $\sigma^2$ * T)/$\sigma$*$T^{0.5}$
= (ln (3.923%/4%) + ½ * $0.15^2$ * 1)/0.15*1 = -0.0546
d2 = d1 – 0.15 = -0.2046
N(d1) = N(-0.0546) = 0.47823
N(d2) = N(-0.2046) = 0.41894
Value = 10,000,000 * 1.8137 * (3.923% * 0.47823 – 4% * 0.41894)
= 36,332

**16.** **Learning Objectives:**

1. Modern Corporate Financial Theory

2. Corporate Financial Applications

**Learning Outcomes:**

(1c) Evaluate various profitability measures including IRR, NPV and ROE, etc.

(2e) Apply real options analysis to recommend and evaluate firm decisions on capital utilization.

**Sources:**

Financial Theory and Corporate Policy, Copeland, Weston, Shastri, 4th Edition, 2005

- Chapter 2, Investment Decisions: The Certainty Case
- Chapter 9, Multi-period Capital Budgeting under Uncertainty: Real Options Analysis

**Commentary on Question:**

*.This question was trying to test the candidate's ability to recognize Real Options inherent in a business opportunity, calculate the value of these Options and understand the difference between using Net Present Value and Real Option Analysis.*

*In order to receive maximum points, candidates needed to recognize the value in waiting to see how market demand emerges in order to make decisions on how many staff to hire.*

*Candidates generally did well in evaluating the value of the projects without Real Options and also explaining generally what the value of Real Options is, however, candidates seemed to struggle in evaluating the Real Options inherent in this project.*

**Solution:**

(a) Calculate the Net Present Value of the program if you decide to hire three new staff right now.

**Commentary on Question**:

*Candidates did relatively well on this section. Many candidates were able to calculate the correct NPV. Common mistakes included not recognizing that you can only handle a market demand of 1500 policies (i.e. question mentioned could only hire 3 people at 500 policies per hire) or failing to reflect the variable cost of the new employee's salaries.*

## 16. Continued

| | Present | Yr 1 | Yr 2 |
|---|---|---|---|
| Expected Output | | 300 | 444 |
| Revenue | | 22,500 | 33,333 |
| Variable Cost | | 12,000 | 17,778 |
| Income | | 10,500 | 15,556 |
| Upfront Cost | - 30,000 | | |
| Cashflow | - 30,000 | 10,500 | 15,556 |
| Discount @3% | 1 | 0.97 | 0.94 |
| Present Value | - 30,000 | 10,194 | 14,663 |
| Net Present Value | - 5,143 | | |

(b) Calculate the Net Present Value of the program if you decide to fund it using overtime.

**Commentary on Question**:
*Candidates did well on this section. Most candidates were able to properly calculate the NPV. Common mistakes included failing to recognize that you can only handle 500 policies using overtime or assuming that the first 500 policies were handled by the current staff.*

| | Present | Yr 1 | Yr 2 |
|---|---|---|---|
| Expected Output | | 267 | 333 |
| Revenue | | 20,000 | 25,000 |
| Variable Cost | | 16,000 | 20,000 |
| Income | | 4,000 | 5,000 |
| Upfront Cost | | | |
| Cashflow | | 4,000 | 5,000 |
| Discount @3% | 1 | 0.97 | 0.94 |
| Present Value | - | 3,883 | 4,713 |
| Net Present Value | 8,596 | | |

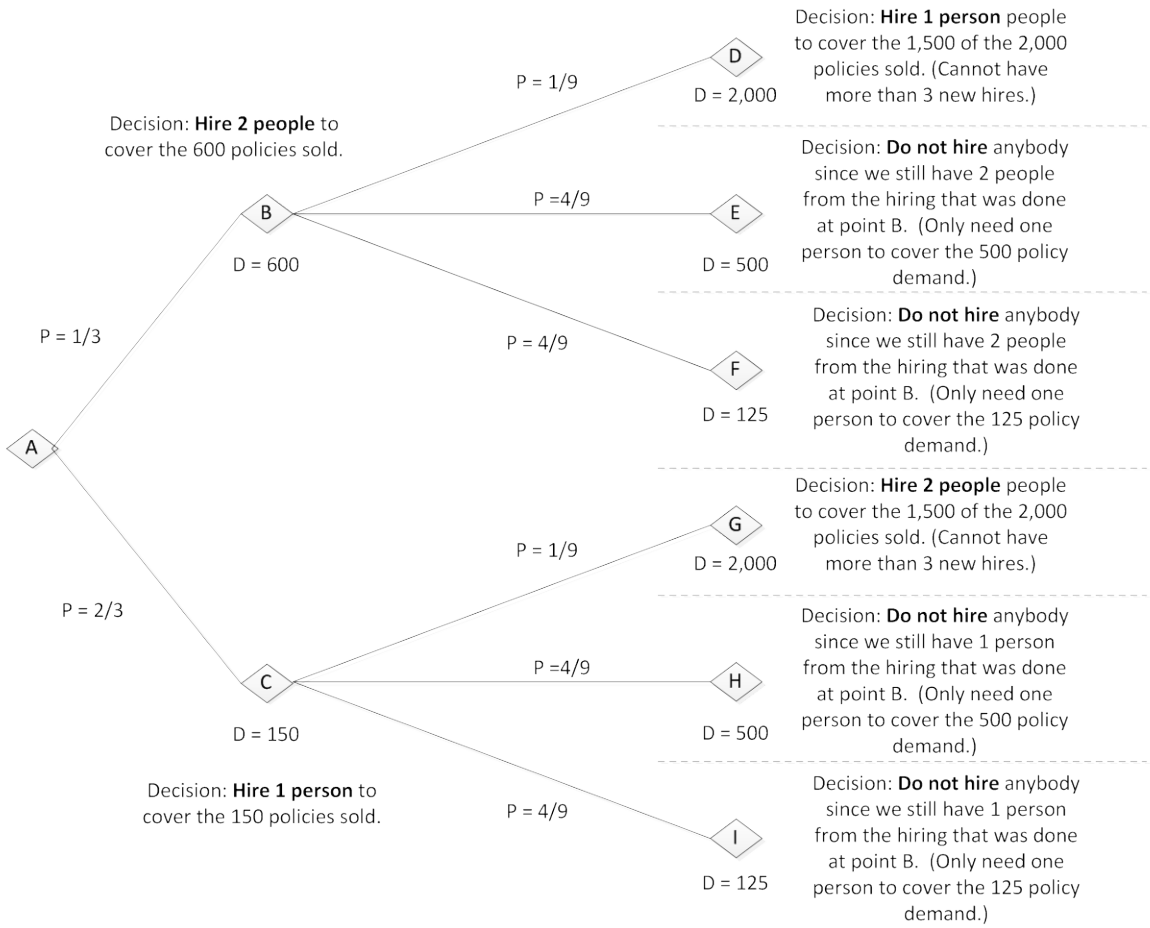
(c) Show that a higher Net Present Value can be obtained using Real Options Analysis.

**Commentary on Question**:
*Candidates did relatively poorly on this section. Many candidates made hiring decisions at time T=2 but failed to take into consideration hiring decisions from the prior period (T=1). Candidates that did well used clear explanations of their proposed hiring and/or diagrams showing the various projection paths.*

**16.** **Continued**

*The question asked to calculate the NPV by hiring no more than three people, but meet all market demand – this is not possible when each hire can only serve 500 policies. Candidates who made an assumption either way (i.e. only satisfied 1500, or hired a fourth person) were not penalized.*

First we determine when we need to hire people to meet demand and must keep in mind that we are not permitted to hire more than three people. The situation is shown in the figure below:



## 16. Continued

The following table summarizes the Revenue, Hiring costs and maintenance costs:

| Node | Year | Demand Met | Prob. | # of new hirees | Total Hiring Cost | Revenue | Maintenance Cost ($40 / policy) | Net Cash Flow |
|---|---|---|---|---|---|---|---|---|
| B | 1 | 600 | 0.333 | 2 | 20,000 | 45,000 | 24,000 | 1,000 |
| C | 1 | 150 | 0.667 | 1 | 10,000 | 11,250 | 6,000 | (4,750) |
| D | 2 | 1500 | 0.037 | 1 | 10,000 | 112,500 | 60,000 | 42,500 |
| E | 2 | 500 | 0.148 | 0 | - | 37,500 | 20,000 | 17,500 |
| F | 2 | 125 | 0.148 | 0 | - | 9,375 | 5,000 | 4,375 |
| G | 2 | 1500 | 0.074 | 2 | 20,000 | 112,500 | 60,000 | 32,500 |
| H | 2 | 500 | 0.296 | 0 | - | 37,500 | 20,000 | 17,500 |
| I | 2 | 125 | 0.296 | 0 | - | 9,375 | 5,000 | 4,375 |

The table below calculates the expected cash flow and discounts these back to get the net present value.

| Node | Year | Prob. | Net Cash Flow | Expected Net Cash Flow | Discount Factor | NPV |
|---|---|---|---|---|---|---|
| B | 1 | 0.333 | 1,000 | 333 | 0.971 | 324 |
| C | 1 | 0.667 | (4,750) | (3,167) | 0.971 | (3,074) |
| D | 2 | 0.037 | 42,500 | 1,574 | 0.943 | 1,484 |
| E | 2 | 0.148 | 17,500 | 2,593 | 0.943 | 2,444 |
| F | 2 | 0.148 | 4,375 | 648 | 0.943 | 611 |
| G | 2 | 0.074 | 32,500 | 2,407 | 0.943 | 2,269 |
| H | 2 | 0.296 | 17,500 | 5,185 | 0.943 | 4,888 |
| I | 2 | 0.296 | 4,375 | 1,296 | 0.943 | 1,222 |
| | | | | | **Total =** | **10,166** |

(d) Explain why Real Options Analysis gives a higher value.

**Commentary on Question**:
*Candidates did well on this section.*

Real Options gives a higher value because it allows for flexibility in making future decisions, whereas NPV assumes all decisions need to be made at time zero. For example, with this business opportunity, you can hire staff in the future once market demand is known, rather than hiring them at time zero. This option is valuable, which should be reflected in the Real Options project value.

**17.** **Learning Objectives:**
3. Derivatives, Pricing and Modeling

**Learning Outcomes:**

(3f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.

(3g) Identify limitations of each option pricing technique.

(3h) Describe and evaluate equity and interest rate models.

(3i) Contrast commonly used equity and interest rate models.

**Sources:**
Options Futures & Other Derivatives, Hull, J. C., $8^{th}$ Edition, 2012

- Chapter 30, Interest Rate Derivatives: Models of the Short Rate (exclude 30.2 properties of Vasicek and CIR, exclude 30.3 BDT and Black-Karasinski)

**Commentary on Question:**
*This question tested the candidate's knowledge of and ability to use the Ho-Lee and Hull-White interest rate models. Candidates did reasonably well comparing and contrasting the two models, but for the most part struggled with the rest of the question.*

**Solution:**

(a) Compare and contrast the Hull-White and Ho-Lee interest rate models.

**Commentary on Question**:
*Candidates did well on this section. Candidates received credit for correctly identifying the similarities and differences between the models. A common mistake was to not identify both models as arbitrage free.*

Similarities of Ho-Lee and Hull-White Models:

- Both are no-arbitrage models
- For both, the drift is dependent on time
- Both have constant volatility (sigma)
- For both, the current term structure of interest rates is an input.
- Both maintain analytical tractability

Differences of Ho-Lee and Hull-White Models:
Ho-Lee Model:

- $dr = \theta(t)\, dt + \sigma\, dz$
- Implies that rates are equally variable at all times

## 17. Continued

Hull-White Model:

- $dr = [\theta(t) - ar]\, dt + \sigma\, dz$
- Incorporates mean reversion
- Allows richer description of volatility environment

(b) Solve for the Hull-White volatility such that the caplet price is equal under both models and explain why the volatility parameters between models are different.

**Commentary on Question**:
*Candidates did relatively well on this section. Candidates were to recognize that $\sigma_p$ for both the Ho-Lee and Hull-White models would be the same for the caplet price to be the same under both models. In order to solve this problem, the value of $\alpha = 0.05$ was to be recognized as being the input to use as the assumed value of the parameter a in the Hull-White formula. While not intended to confuse, the committee felt this is was a reasonable assumption to make and we scored this accordingly.*

T=5; s=6; $\alpha$=a=0.05; Ho-Lee $\sigma$=0.013; ATM Strike = 3.50%

Ho-Lee: $\sigma_p$ = $\sigma$*(s-T)*sqrt(T) = 0.013*(6-5)*sqrt(5) = 0.02907

Hull-White: $\sigma_p$ = $\sigma$/a * (1-exp(-a*(s-T))) * sqrt((1-exp(-2*a*T))/(2*a)) =>
0.02907 = $\sigma$/0.05*(1-exp(-0.05*(6-5)))*sqrt((1-exp(-2*0.05*5))/(2*0.05)) =>
0.01502

(c) Calculate the volatility of the caplet by taking the limit as the expiration approaches infinity. Recommend and justify which model is most suitable based on the volatility derived from both models.

**Commentary on Question**:
*Candidates did relatively poorly on this section. To get full credit, candidates needed to identify that the Ho-Lee model is unable to limit the growth of the volatility as the maturity (S) went to infinity while the Hull-White model is able to limit the growth. Partial credit was awarded for providing a well-reasoned recommendation. A common mistake was to have the time (T) go to infinity.*

Ho-Lee Model: The limit as s goes to infinity of $\sigma$*(S-T)*sqrt(T) is infinity.

Hull-White Model: The limit as s goes to infinity of
$\sigma$/a * (1-exp(-a*(s-T))) * sqrt((1-exp(-2*a*T))/(2*a))
= $\sigma$/a * sqrt((1-exp(-2*a*T))/(2*a)) * Lim(1-exp(-a*(s-T)))
= $\sigma$/a * sqrt((1-exp(-2*a*T))/(2*a)).

## 17. Continued

The Hull-White model is more reasonable as it is capable of limiting the growth of the bond volatility as the maturity increases.

(d) Calculate the spot rates for a zero coupon bond that matures at year 6 for each model and each of the three simulations.

**Commentary on Question**:
*Candidates did poorly on this section. Very few candidates attempted to answer this section. Those candidates that did received partial credit for correctly identifying the appropriate formulas and values and for correctly setting up the calculations.*

T=6; t=5; F(0,t) = 0.035; P(0,t) = 0.8521; P(0,T) = 0.8228

Ho-Lee Model:
$A(t,T) = \exp( \ln(P(0,T)/P(0,t)) + (T-t)*F(0,t) – (1/2*\sigma^2)*t*(T-t)^2) =$
$\exp(\ln(.8228/.8521)+(6-5)*0.035-(1/2*0.013^2)*5*(6-5)^2) = 0.999587$
$P(t,T) = A(t,T)*\exp(-r(t)*(T-t))$

For r(t) = 2.5% - spot rate = -ln(P(t,T))/(T-t) = 0.025413
For r(t) = 3.5% - spot rate = -ln(P(t,T))/(T-t) = 0.035413
For r(t) = 4.5% - spot rate = -ln(P(t,T))/(T-t) = 0.045413

Hull-White Model:
$B(t,T) = (1 – \exp(-a*(T-t)))/a = (1 – \exp(0.05*(6-5)))/0.05 = 0.97541$
$\ln(A(t,T)) = \ln(P(0,T)/P(0,t)) + B(t,T)*F(0,t) – (1/4*a^3)* \sigma^2*(\exp(-a*T) – \exp(-a*t))^2*(\exp(2*a*t)-1) = -0.001274$
$P(t,T) = \exp(-\ln(A(t,T))-r(t)*B(t,T))$

For r(t) = 2.5% - spot rate = P(t,T) = 0.025659
For r(t) = 3.5% - spot rate = P(t,T) = 0.035413
For r(t) = 4.5% - spot rate = P(t,T) = 0.045167

(e) Interpret the results from part (d) in regards to the sensitivity between short rate movements and spot zero coupon bond movements and explain the difference in model results.

**Commentary on Question**:
*Candidates did poorly on this section. Very few candidates attempted to answer this section. Those candidates that did received partial credit for providing either a justified interpretation or a justified explanation.*

## 17. **Continued**

The Ho-Lee model shows 100% correlation.
The Hull-White model shows a strong positive correlation but is not 100%.
The inclusion of the mean reversion factor in the Hull-White model limits the correlation.