INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).
   a) The morning session consists of 9 questions numbered 1 through 9.
   b) The afternoon session consists of 5 questions numbered 10 through 14.

The points for each question are indicated at the beginning of the question. There are no questions in the Morning Session that pertain to the Case Study.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam CFEFD.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d’examen pour la version française.
1. (5 points) Meigs Insurance Company, which has recently had high growth in its whole life business, is evaluating the following alternatives to relieve its increased surplus strain:

   I. Surplus notes
   II. YRT reinsurance
   III. Coinsurance
   IV. Modified coinsurance (mod-co)

Meigs is confident in its asset management department and would like to maintain as much control as possible. Meigs hopes to complete a transaction in three months.

(a) (2 points) Evaluate the appropriateness of each of the four alternatives for Meigs. Support your evaluations.

The mod-co alternative Meigs Insurance Company is considering would begin on 1/1/2015, reinsuring 75% of the whole life business. For the whole life block, you are given:

   Expected reserves as of 1/1/2015  $40
   Expected reserves as of 12/31/2015  $60
   Mod-co interest rate  10%

(b) (1 point) Calculate the mod-co adjustment as of 1/1/2015 and 12/31/2015. Show your work.

(c) (2 points) Describe two advantages and two disadvantages of setting up a trust or escrow account to segment the relevant assets if using the mod-co alternative.
2. (8 points)

(a) (1 point) Contrast no-arbitrage pricing and equilibrium pricing with regard to:

(i) principles used in each methodology

(ii) major applications in financial markets

Consider a one-period pricing model with three assets: bond X, stock Y and commodity Z.

- The interest rate is assumed to be 0%, so bond X sells for 1 at the beginning of the period and pays 1 at the end of the period in all scenarios.

- The price of stock Y is assumed to have the following trinomial evolution:

\[
\begin{array}{c}
1.20 \\
1.00 \\
0.75 \\
\end{array}
\]

- The price of commodity Z is assumed to have the following trinomial evolution:

\[
\begin{array}{c}
1.50 \\
1.10 \\
0.35 \\
\end{array}
\]

(b) (2 points) Calculate the state price vector. Show your work.

c) (1 point) Explain why a state price vector would not exist if the bond interest rate were above 20% per period in all scenarios.

d) (1 point) Calculate the price of a one-period put option written on stock Y with strike price 1.05. Show your work.

(e) (2 points) Construct a replicating portfolio for the put option on stock Y using all three assets. Show your work.
2. Continued

After studying no-arbitrage pricing, an actuarial student sitting for the CFE FD exam says to his boss:

“I think the no-arbitrage pricing is great. We should create multi-period no-arbitrage models to make our pricing job easier for all existing and new products. All we need is to pull out some cash flows from the existing models and then discount them using the appropriate yield curve.”

(f) *(1 point)* Evaluate the student’s statement.
3. **(7 points)**

   (a) **(1 point)**

   (i) Define the Value at Risk (VaR) measure without using formulas.

   (ii) Identify two parametric approaches to calculating the VaR.

   (b) **(1 point)** Describe two examples of how financial firms or traders might exploit the fact that VaR is not subadditive.

Returns on assets A and B are each modeled using a normal distribution with a daily return of 0% and a daily volatility of 1%. Assume that the correlation coefficient between their returns is 0.5.

Portfolio C consists of $600 of asset A and $400 of asset B.

You are given the following values from the cumulative standard normal distribution:

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.9950</th>
<th>0.9975</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_{\alpha}$</td>
<td>2.576</td>
<td>2.807</td>
</tr>
</tbody>
</table>

(c) **(2 points)** Calculate the 10-day 99.5% VaR for each of the following. Show your work.

   (i) $600$ of asset A

   (ii) $400$ of asset B

   (iii) Portfolio C

Assume now that the following VaR equation applies to portfolio C:

$$P \sigma \left( w_A^2 + w_B^2 + 2 \rho w_A w_B \right)^{1/2} z_{\alpha}$$

where

- $P$ is the size of the portfolio,
- $\sigma$ is the daily volatility of each asset,
- $w_A$ and $w_B$ are relative shares of assets A and B in the portfolio,
- $\rho$ is the correlation coefficient, and
- $z_{\alpha}$ is the standard normal percentile at confidence level $\alpha$. 

3. Continued

(d)  (3 points)

(i) Assess the reliability and flexibility of the delVaR approach to estimating incremental VaR (IVaR).

(ii) Calculate the 1-day 99.5% VaR of portfolio C. Show your work.

(iii) Calculate the IVaR of portfolio C at the 99.5% confidence level for a purchase of $10 in asset A using the delVaR approach. Show your work.
4. **(5 points)** The Chief Investment Officer of Mount Hardy Life is concerned that the company’s current method of duration matching may not be sufficient for interest rate risk management and asks you to look into hedging based on principal components analysis (PCA) as an alternative.

An actuarial student at Mount Hardy has performed PCA on daily yield curve data on five maturity points since 2000, with the following results:

<table>
<thead>
<tr>
<th>Principal Component $j$</th>
<th>Eigenvalue $\lambda_j$</th>
<th>Eigenvector $U_{ji}$</th>
<th>Maturity Point $i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3.878</td>
<td>0.39</td>
<td>0.45</td>
</tr>
<tr>
<td>2</td>
<td>1.050</td>
<td>-0.62</td>
<td>-0.44</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>0.34</td>
<td>-0.06</td>
</tr>
<tr>
<td>4</td>
<td>0.005</td>
<td>0.52</td>
<td>-0.53</td>
</tr>
<tr>
<td>5</td>
<td>0.001</td>
<td>0.28</td>
<td>-0.57</td>
</tr>
</tbody>
</table>

(a) **(2 points)** The actuarial student recommends that you retain all five principal components for analysis to avoid discarding valuable information on the underlying data.

(i) Critique her recommendation.

(ii) Propose a reasonable number of principal components to retain. Show your work.

You suggest that Mount Hardy should replace its periodic calculation of duration with a periodic calculation of Mount Hardy’s sensitivity to shocks along each of the retained principal components.

(b) **(2 points)** Describe one advantage and one disadvantage of using PCA instead of duration to measure interest rate risk.
4. Continued

The student suggests that, as additional interest rate data becomes available, it should be added to the existing dataset and the principal components recomputed.

(c) (1 point) Recommend an improvement on the student’s suggestion. Justify your recommendation.
5. (9 points) Linville Inc. is evaluating its capital structure and potential investments as part of its growth strategy.

The following table applies to Linville:

<table>
<thead>
<tr>
<th>Market Value of Equity</th>
<th>$100 million</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Value of Debt</td>
<td>$60 million</td>
</tr>
<tr>
<td>Expected Return on Debt</td>
<td>4%</td>
</tr>
<tr>
<td>Expected Return on Equity</td>
<td>12%</td>
</tr>
<tr>
<td>Corporate Tax Rate</td>
<td>25%</td>
</tr>
</tbody>
</table>

Linville plans to change its capital structure by issuing $20 million of new permanent subordinated debt. The new debt holders require a 6% expected return because of their lower seniority. Linville plans to repurchase equity after issuing the new debt.

(a) (3 points)

(i) Calculate Linville’s weighted average cost of capital before issuing the new debt. Show your work.

(ii) Calculate Linville’s weighted average cost of capital after issuing the new debt and repurchasing $25 million of equity. Show your work.

After issuing the new debt, Linville is considering a two-year project that requires an $8 million initial investment and has the following income projection:

<table>
<thead>
<tr>
<th>(in $ millions)</th>
<th>Year 1</th>
<th>Year 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Profit</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Expenses</td>
<td>(3)</td>
<td>(3)</td>
</tr>
<tr>
<td>Depreciation</td>
<td>(4)</td>
<td>(4)</td>
</tr>
<tr>
<td>EBIT</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Assume the project is of average risk for Linville, taxes are the only market imperfection, and Linville targets a 50% debt-to-value ratio.

(b) (4 points) Calculate the following, showing your work:

(i) The unlevered value of the project

(ii) The levered value of the project

(iii) The value of the interest tax shield generated by the project
5. Continued

Linville has determined that maintaining a constant debt-to-value ratio is difficult and has instead created a fixed debt schedule for the project’s income projection:

<table>
<thead>
<tr>
<th>(in $ millions)</th>
<th>Year 0</th>
<th>Year 1</th>
<th>Year 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Profit</td>
<td>9.00</td>
<td>9.00</td>
<td></td>
</tr>
<tr>
<td>Expenses</td>
<td>(3.00)</td>
<td>(3.00)</td>
<td></td>
</tr>
<tr>
<td>Depreciation</td>
<td>(4.00)</td>
<td>(4.00)</td>
<td></td>
</tr>
<tr>
<td>EBIT</td>
<td>2.00</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>Interest Expense</td>
<td>(0.18)</td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>Pre-tax Income</td>
<td>1.82</td>
<td>1.91</td>
<td></td>
</tr>
<tr>
<td>Income Tax at 25%</td>
<td>(0.46)</td>
<td>(0.48)</td>
<td></td>
</tr>
<tr>
<td>Net Income</td>
<td>1.36</td>
<td>1.43</td>
<td></td>
</tr>
<tr>
<td>Capital Expenditure</td>
<td>(8.00)</td>
<td>4.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Incremental Debt Level</td>
<td>4.00</td>
<td>2.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

(c) (2 points) Calculate the project’s free cash flows to equity (FCFE). Show your work.
6. (6 points) Sliding Rock Assurance is pricing a Guaranteed Minimum Accumulation Benefit (GMAB) rider on its variable annuity product. GMAB guarantees have exposure to both equity and interest rate risk. You are developing the volatility assumption for valuing the GMAB.

(a) (1 point) Contrast implied and historical volatilities.

(b) (1 point) Compare and contrast using equal-weighted and exponentially weighted moving average models for estimating the volatility of equity returns.

The CRO has pointed out that the correlation between equity and interest rate risks will reduce the aggregate capital requirement for this product.

(c) (1 point) Describe three shortcomings with using correlations as a measure of dependency.

You estimate that equity risk requires six units of capital while interest rate risk requires four units of capital. The standalone diversification factors are estimated to be 45% for equity risk and 80% for interest rate risk.

(d) (1 point) Calculate the aggregate diversification factor. Show your work.

Risk professionals at Sliding Rock suggest generating random drivers for equity and interest rate risk separately and using a copula to model the dependency between them.

Assume \( C(u, v) \) denotes a copula, where \( u = F(x) \) and \( v = F(y) \), and \( F(x) \) and \( F(y) \) are the cumulative distribution functions of the random drivers for equity and interest rate respectively.

(e) (2 points) Explain why each of the following formulas is not appropriate for aggregating equity and interest rate risks:

(i) \( C(u, v) = u + v \)

(ii) \( C(u, v) = uv \)

(iii) \( C(u, v) = \min(u, v) \)
7. (8 points) Juney Insurance is based strictly in the Republic of Whank, where investors and firms can trade securities at competitive market prices equal to the present values of future cash flows. Citizens and companies of Whank pay no taxes and incur no transaction costs. In Whank, a company’s financing decisions do not change the cash flows generated by its investments nor do they reveal new information about them. The risk-free rate is 2%.

The following information applies to Juney:

<table>
<thead>
<tr>
<th>Debt-to-equity ratio</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shares outstanding</td>
<td>1.5 million</td>
</tr>
<tr>
<td>Price per share</td>
<td>$42</td>
</tr>
<tr>
<td>Equity cost of capital</td>
<td>12.56%</td>
</tr>
</tbody>
</table>

Juney has an opportunity to purchase an insurance distributor with cash flows of similar risk to Juney’s existing business. The distributor is expected to generate an additional $1 million of free cash flow per year for the foreseeable future and can be purchased for $8.82 million.

Juney is able to issue debt at the risk-free rate. Juney is considering either issuing debt or issuing new equity shares to finance the purchase.

(a) (2 points)

(i) Define the payback rule.

(ii) List two disadvantages of the payback rule compared to the NPV rule.

(iii) Describe two general situations when the payback rule is useful despite its disadvantages.
7. Continued

(b) **(2 points)** Calculate the equity cost of capital in each of the following scenarios. Show your work.

(i) Juney issues $8.82 million of new equity and uses the proceeds to purchase the distributor.

(ii) Juney issues $8.82 million of debt and uses the proceeds to purchase the distributor.

(c) **(1 point)** Explain, using Modigliani-Miller Proposition I, why Juney is indifferent to the two financing options in part (b).

(d) **(1 point)**

(i) Calculate the value of the acquisition. Show your work.

(ii) Recommend whether Juney should purchase the distributor.

Now assume transaction and monitoring costs exist. Juney determines it must now hold $50 million of risk capital to insure the value of the firm’s net assets relative to the risk-free rate.

The deadweight cost to distribute surplus to shareholders is now 20% of surplus. Assume that Juney’s profits are normally distributed and that Juney can buy a guarantee to avoid the costs of adverse selection and moral hazard for a 30% spread above its fair value.

Select values from the cumulative normal distribution are provided:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\Phi(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>0.5</td>
</tr>
<tr>
<td>0.2533</td>
<td>0.6</td>
</tr>
<tr>
<td>0.5244</td>
<td>0.7</td>
</tr>
<tr>
<td>0.8416</td>
<td>0.8</td>
</tr>
<tr>
<td>1.2816</td>
<td>0.9</td>
</tr>
</tbody>
</table>

(e) **(2 points)** Calculate the optimal cash cushion required for Juney to minimize their deadweight cost of risk capital. Show your work.
8. (7 points) You are helping Looking Glass Corporation calculate its market value margins (MVMs) for non-hedgeable risks in its life business.

Looking Glass has an inforce single premium term life insurance block. You are given:

<table>
<thead>
<tr>
<th>Remaining term of the block</th>
<th>3 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swap rate (flat)</td>
<td>4.5%</td>
</tr>
<tr>
<td>Cost of capital rate for non-hedgeable risks</td>
<td>3.5%</td>
</tr>
</tbody>
</table>

The present value (PV) of future claims will be used as the basis for the solvency capital requirement (SCR) projection. Your actuarial student has provided the following:

<table>
<thead>
<tr>
<th>Time ( (t) )</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV( (t) ) of future claims</td>
<td>47,312</td>
<td>31,978</td>
<td>16,214</td>
</tr>
<tr>
<td>SCR( (t) )</td>
<td>5,860</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) (1 point) Define market-consistent value of liabilities (MVL).

(b) (2 points) Describe the advantages of the market cost of capital (MCoC) approach over the percentile approach for calculating MVMs for non-hedgeable risks in terms of:

(i) Implementation
(ii) Reflection of risk
(iii) Transparency
(iv) Solvency in run-off mode

(c) (2 points) Calculate the MVM for the non-hedgeable risks of the term block using the MCoC approach. Show your work.

(d) (1 point) Looking Glass is considering offering single premium whole life insurance, investing the proceeds in long-term corporate bonds.

(i) List two examples of financial non-hedgeable risks with this product.
(ii) List two examples of non-financial non-hedgeable risks with this product.
8. Continued

Looking Glass’s CEO states: “The MCoC approach isn’t appropriate because the SCR calculation is needed to value the MVM, but the MVM should be included in our SCR!”

(e) (1 point) Explain to the CEO why you agree or disagree with her point.

9. (5 points) Turtleback Financial has hired you to help evaluate a proposed move from its parametric approach for calculating financial risk measures to a non-parametric approach. Turtleback’s data is heavily weighted to recent time periods, and the company is not interested in investing in any new software packages.

(a) (1 point) List four advantages for Turtleback of using a non-parametric approach.

You propose the following non-parametric historical simulation (HS) approaches:

I. Basic HS
II. Bootstrapped HS
III. HS using Non-Parametric Density Estimation

(b) (3 points)

(i) Describe each of the three approaches.

(ii) Outline general advantages and disadvantages of each approach.

(c) (1 point) Recommend the most appropriate non-parametric HS approach for Turtleback. Justify your recommendation.

**END OF EXAMINATION**

Morning Session