1. **Learning Objectives:**
   1. The candidate will understand the key considerations for general insurance actuarial analysis.

**Learning Outcomes:**
(1d) Recognize differences in how data are aggregated and segregated.

(1k) Estimate written, earned and unearned premiums.

(1l) Adjust historical earned premiums to current rate levels.

**Sources:**

**Commentary on Question:**
The question is testing the ability of the candidate to understand certain details of individual insurance policies and to make correct calculations of earned exposures, earned premium, and unearned premium at various points in time. The candidate also needs to know how to apply a rate level change correctly.

**Solution:**
(a) Calculate the total earned exposure units and total earned premium in each of the four calendar quarters in 2013.

**Commentary on Question:**
To calculate earned exposure the candidate needs to calculate by policy the number of months in 2013 that each policy was in effect for each quarter, then calculate fractional exposure relative to the twelve months in a year. The earned premium then equals earned exposure multiplied by policy premium. The question asks specifically for earned exposure totals and earned premium totals for each quarter.
There are three insurance policies to consider:

- Policy 14902 has written premium of 1,680, became effective 09/01/2012 and was cancelled 2/28/2013. In 1st quarter 2013 the policy earned two months of exposure (1/6 of a year, 2/3 of a quarter). The earned premium for 1st quarter 2013 is therefore 280 = 1,680 \times (1/6). Due to the policy cancellation in February, all other quarters of 2013 have 0 earned exposure and 0 earned premium.

- Policy 14903 has written premium of 1,680, became effective 12/16/2013 and expired 12/15/2013. In 1st, 2nd and 3rd quarter of 2013 the policy earned 1/4 unit of exposure, with the remaining 5/24 of exposure earned in 4th quarter 2013. The earned premium is as follows:
  - 1st quarter through 3rd quarter 2013, 420 = 1,680 \times 1/4.
  - For 4th quarter 2013, 350 = 1,680 \times 5/24.

- Policy 14904 has written premium = 2,016 (1680 \times 1.2 to adjust for the rate level change effective January 1, 2013). The policy became effective 6/16/2013 and has earned exposure and earned premium as follows for the four quarters of 2013:
  - 1st quarter, 0 earned exposure and 0 earned premium
  - 2nd quarter, 1/24 earned exposure, and earned premium = 84 (2,016 \times 1/24)
  - 3rd quarter and 4th quarter, 1/4 earned exposure and earned premium = 504 (2,016 \times 1/4)

- Totals by quarter of 2013 for the three policies are as follows:
  - 1st quarter: earned exposure = 5/24 (1/6 + 1/4) and earned premium = 700 (280 + 420)
  - 2nd quarter: earned exposure = 7/24 (1/4 + 1/24) and earned premium = 504 (420 + 84)
  - 3rd quarter: earned exposure = 1/2 (1/4 + 1/4) and earned premium = 924 (420 + 504)
  - 4th quarter: earned exposure = 11/24 (5/24 + 1/4) and earned premium = 854 (350 + 504)

(b) Calculate the total unearned premium at December 31, 2012 and December 31, 2013.

**Commentary on Question:**

There are two ways to calculate unearned premium:

1. **By policy**, given the policy details. It is important to note that at year end, the candidate must calculate the full unearned portion even for policies that cancel early the next year.

2. **Using the formula**, \( UEP = WP - EP \). Again care must be taken with the cancellation and also with the years in which written premium earned.
1. Continued

For 2012 unearned premium consider the unearned portion of policies 14902 and 14903:
- Policy 14902 has 8/12 or 2/3 unearned, UEP = 1680 × 2/3 = 1,120
- Policy 14903 has 11.5/12 or 23/24 unearned, UEP = 1680 × 23/24 = 1610
- Total UEP at December 31, 2012 = 2,730 (1,120 + 1,610)

At the end of 2013 only one policy has unearned premium:
- Policy 14904 has 5.5/12 or 11/24 unearned, UEP = 2,016 × 11/24 = 924
- Total UEP at December 31, 2013 = 924
2. Learning Objectives:
   2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

Learning Outcomes:
(2a) Use loss development triangles for investigative testing.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 13 and 19

Commentary on Question:
This question is concerned with identifying potential issues with data triangles, and what adjustments can be made to those triangles prior to estimating ultimate claims.

Solution:
(a) Describe the patterns that would be expected in the triangle of average reported claims (i.e., reported severities) and in the triangle of the ratios of closed to reported counts in a stable environment.

Commentary on Question:
Need to state the expected pattern and also state that the pattern is down each column to get full credit.

Triangle of average reported claims: expect changes down the column (each maturity) to be consistent with inflation.
Triangle of ratios of closed to reported counts: expect values down each column to be approximately the same (level).

(b) Determine whether the above triangle of average case estimates is representative of a stable environment.

Commentary on Question:
It is not necessary to calculate the change in average case reserve triangle to get full credit.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Change in Average Case Reserves</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>2008-2009</td>
<td>4.8%</td>
</tr>
<tr>
<td>2009-2010</td>
<td>11.1%</td>
</tr>
<tr>
<td>2010-2011</td>
<td>3.6%</td>
</tr>
<tr>
<td>2011-2012</td>
<td>14.7%</td>
</tr>
</tbody>
</table>

The triangle is not representative of a stable environment as the changes down each column are not consistent with changes of 9%, especially the most recent diagonal.
2. Continued

(c) Identify two reasons why historical reported claims may not be appropriate for use in a traditional development-based projection method.

**Commentary on Question:**
*Four situations are presented. Any two are acceptable.*

1. There may be a change in the adequacy of case estimates.
2. There may be a change in the rate of settlement of claims.
3. There may be changes due to tort reform.
4. There may be large claims.

(d) Describe an approach, for each of the two reasons identified in (c), that the actuary can implement for use in a development-based projection.

**Commentary on Question:**
*Approach must relate to the reason identified in part (c).*

Reason: change in adequacy of case reserves
   Approach: Use Berquist-Sherman method for change in case reserve adequacy.

Reason: change in the rate of settlement of claims
   Approach: Use Berquist-Sherman method for change in the rate of settlement of claims.

Reason: changes due to tort reform
   Approach: Adjust historical case estimates for the changes due to tort reform so that case reserves are on the same basis pre- and post-tort reform.

Reason: large claims
   Approach: Make an adjustment in the development method, such as removing large claims when calculating development factors and adding the large claims back into the case estimates (thereby assuming the case estimates set by claim management professionals are a better indicator of the ultimate values).
3. **Learning Objectives:**
4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

**Learning Outcomes:**
(4d) Calculate premium trend and apply it to project premiums.

**Sources:**
Fundamentals of General Insurance, Actuarial Analysis, J. Friedland, Chapter 26

**Commentary on Question:**
*Commentary listed underneath question component.*

**Solution:**
(a) Describe the purpose of premium trend and on-level factors.

**Commentary on Question:**
*Reference to both purposes of premium trend factors is needed for full credit.*

Premium trend factors adjust for inflation-sensitive exposure bases and changes in the mix of exposures. On-level factors adjust for rate changes.

(b) Calculate premium trend factors to apply to each of the four years in the experience period.

**Commentary on Question:**
*Direct comparison of the average limits differentials in the experience period and forecast period is also acceptable. Some adjustment for the anomalous data in 2012 is needed for full credit.*

Average limits differentials: 2010—53% × 1.00 + 47% × 1.50 = 1.235, 2011—50% × 1.00 + 50% × 1.50 = 1.250, 2012—65% × 1.00 + 35% × 1.50 = 1.175, 2013—48% × 1.00 + 52% × 1.50 = 1.260, 2015—45% × 1.00 + 55% × 1.50 = 1.275. Excluding 2012, which is an outlier, the annual trend in the average limits differential appears to be around 1%. Combining this with the 5% annual trend in the inflation-sensitive exposure base of sales revenue, the premium trend factors are 1.05^5 × 1.01^5 = 1.341 for 2010, 1.05^4 × 1.01^4 = 1.265 for 2011, and 1.05^2 × 1.01^2 = 1.125 for 2013. 2012 will be disregarded in the rate analysis, so a premium trend factor will not be needed for this year.

(c) Assign weights to each year in the experience period and describe the rationale for such weights.

**Commentary on Question:**
*Identifying 2012 as an outlier and giving it reduced or zero weight is required for full credit. Anything reasonable for the other three years is acceptable.*
3. **Continued**

Experience from 2012 will be disregarded and the other three years will be given equal weight. The company appears to have written a large block of business for 2012 only. Its limits profile is clearly different from the rest of the company’s book of business, and other characteristics may be different as well. There is nothing in the information provided for the other three years that indicates that any of those years should be weighted more than the others.

(d) Explain how the premium trend factors would be affected by the following:

(i) A 10% rate increase that is implemented at the beginning of 2014.

(ii) The introduction of a loyalty discount program at the beginning of 2015.

**Commentary on Question:**

*Suggesting that the introduction of a loyalty discount program could be considered a rate change that would not affect premium trend factors also received full credit.*

Premium trend factors are not affected by rate changes. The loyalty discount program would decrease premiums, so the premium trend factors would decrease.
4. Learning Objectives:
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

Learning Outcomes:
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 18

Commentary on Question:
The question is testing the mechanics of the Cape Cod method of estimating ultimate claims, situations that would cause a difference in the estimate of ultimate claims when using paid versus reported claims, and how to incorporate professional judgment in the Cape Cod method. The candidate also needs to understand the major difference between the Bornhuetter Ferguson and Cape Cod projection methods.

Solution:
(a) Calculate the projected ultimate claims using the Cape Cod method with paid claims.

Commentary on Question:
The overall adjusted expected claim ratio needs to be applied to each accident year to estimate the expected paid claims, instead of determining a claim ratio for each accident year separately.

(b) Explain two situations that could result in such a difference in Cape Cod projections based on paid and reported claims.

Commentary on Question:
Four situations are presented. Any two are acceptable.
4. Continued

1. Change in the adequacy of case reserves
2. Change in the settlement rates resulting in higher or lower paid claims than in past
3. Unusual reporting of large claims that are not yet paid
4. Change in environment (internal or external) that is reflected in case estimates but not yet seen in paid claims that lag the reporting of claims

(c) Describe how the actuary can incorporate professional judgment in the Cape Cod method.

**Commentary on Question:**
*Four are presented. Any two are acceptable.*

Possible answers include:
- Length of the experience period chosen
- Selecting the decay factor to use for the generalized Cape Cod method
- Trend selection used
- Development pattern selected

(d) Explain the major difference between the Bornhuetter Ferguson and Cape Cod projection methods.

Determination of expected claims differs between the 2 methods:
- BF method requires an *a priori* estimate
- Cape Cod method is based on earned premium, claims, development pattern, and trend
5. **Learning Objectives:**
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5b) Calculate expenses used in ratemaking analyses.
(5f) Calculate overall rate change indications under the claims ratio and pure premium methods.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 29 and 31

**Commentary on Question:**
*This question tests the ability to allocate fixed expenses by coverage and calculate overall rate indications. The candidate needs to be able to explain reasons why allocating fixed expenses to all coverages is more appropriate.*

**Solution:**
(a) Determine the revised indicated rate change for both the mandatory coverage and optional coverage by allocating all fixed expenses to the mandatory coverage.

**Commentary on Question:**
*There is not enough information provided in the question to calculate indications using a pure premium approach. Therefore, need to use the claim ratio approach.*

<table>
<thead>
<tr>
<th></th>
<th>Mandatory Coverage</th>
<th>Optional Coverage</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Trended earned premium at current rate level (000) [given]</td>
<td>3,260</td>
<td>2,190</td>
<td>5,450</td>
</tr>
<tr>
<td>2 Fixed expenses as a ratio to premium at current rate level [given]</td>
<td>11.5%</td>
<td>11.5%</td>
<td></td>
</tr>
<tr>
<td>3 Fixed expenses - amount [(1) × (2)]</td>
<td>374.90</td>
<td>251.85</td>
<td>626.75</td>
</tr>
<tr>
<td>4 Total amount of fixed expenses allocated to line</td>
<td>626.75</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>5 Fixed expenses as a ratio to premium at current rate level [(4)/(1)]</td>
<td>19.23%</td>
<td>0.00%</td>
<td></td>
</tr>
<tr>
<td>6 Variable expenses as a % of premium [given]</td>
<td>10.0%</td>
<td>10.0%</td>
<td></td>
</tr>
<tr>
<td>7 Profit and contingencies as a % of premium [given]</td>
<td>5.0%</td>
<td>5.0%</td>
<td></td>
</tr>
<tr>
<td>8 Claim ratio including ULAE [(ult. claims) × (1 + ULAE ratio) / (1)]</td>
<td>74.54%</td>
<td>83.59%</td>
<td></td>
</tr>
<tr>
<td>9 PCR [1 - (6) - (7)]/[1 + (5)/(8)]</td>
<td>67.57%</td>
<td>85.00%</td>
<td></td>
</tr>
<tr>
<td>10 Indicated Rate Change [(8)/(9) - 1]</td>
<td>10.31%</td>
<td>–1.66%</td>
<td></td>
</tr>
</tbody>
</table>
5. Continued

(b) Draft a response to the regulator outlining why you believe your expense allocation method is more appropriate.

- Since fixed expenses include such expenses as office expenses and staff salary, such expenses should be shared by all policies equally regardless of the coverage(s) purchased.
- If expenses are allocated to the mandatory coverage only, customers who want to purchase policies with mandatory coverage only would select against our company due to an uncompetitive premium.
6. **Learning Objectives:**
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 19

**Commentary on Question:**
*This question is concerned with determining adjusted closed counts from observed data using the Berquist-Sherman method, and using an exponential formula to estimate adjusted paid claims.*

**Solution:**
(a) Describe two situations where Berquist-Sherman methods are most commonly implemented.

Two situations where Berquist-Sherman methods are most commonly used are…

When there has been a significant change in the adequacy of case estimates in the most recent periods; and

When there has been a significant change in the rate of settlement of claims in the most recent periods.

(b) Recommend disposal ratios for each maturity age.

The best recommended disposal ratios under the Berquist-Sherman method are the disposal ratios of closed counts at the current point in time to the selected ultimate counts. The following table shows calculation of ratios across the data set, with the bottom row indicating the recommended selected disposal ratios.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Ratio Closed Counts to Selected Ultimate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>2009</td>
<td>0.68</td>
</tr>
<tr>
<td>2010</td>
<td>0.67</td>
</tr>
<tr>
<td>2011</td>
<td>0.68</td>
</tr>
<tr>
<td>2012</td>
<td>0.60</td>
</tr>
<tr>
<td>Selected</td>
<td>0.60</td>
</tr>
</tbody>
</table>
6. Continued

(c) Calculate the development triangle of adjusted closed counts using your recommended ratios from (b).

Adjusted closed counts are determined by using the above selected disposal ratios and applying them to ultimate claims. This leads to the following development triangle of adjusted closed counts.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Adjusted Closed Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>2009</td>
<td>1,570</td>
</tr>
<tr>
<td>2010</td>
<td>1,680</td>
</tr>
<tr>
<td>2011</td>
<td>1,763</td>
</tr>
<tr>
<td>2012</td>
<td>1,860</td>
</tr>
</tbody>
</table>

For example, the adjusted closed counts for Accident Year 2011 at month 12 would be 2,938 of ultimate counts, multiplied by the 0.60 disposal ratio to arrive at 1,763 of adjusted closed counts.

(d) Calculate adjusted paid claims at December 31, 2011 for accident year 2009 using the information above and the adjusted closed count triangle.

Adjusted paid claims are determined as a function of the entry in the adjusted closed count table for Accident Year 2009 at 36 months, which is 2,355.

\[2,345.11 e^{0.00047 \times 2,355} = 7,094\]
7. Learning Objectives:
7. The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

Learning Outcomes:
(7a) Describe the structure of catastrophe models.

Sources:
Catastrophe Modeling: A New Approach to Managing Risk, Grossi, P. and Kunreuther, H., Sections 4.2 – 4.4

Commentary on Question:
This question is concerned with types of uncertainty in catastrophe models.

Solution:
(a) Provide an example of each type of uncertainty with regard to earthquake models and explain why each example reflects that type of uncertainty.

Commentary on Question:
There are several examples of each type of uncertainty. Only one is needed to earn credit and there may be others beyond those listed below that are also creditworthy.

In earthquake models aleatory uncertainty is reflected in the probability distribution of the frequency of earthquakes, the probability distribution of the location, magnitude and direction of earthquakes, and the probability distribution of the amount of damage caused, given the above. In each case the error is due to the randomness reflected in the probability distribution.

Regarding epistemic uncertainty, for earthquake losses there is limited data on which to base an exceedance probability curve. There may also be limited or poor data on construction quality, building values and repair costs. There may be incomplete information regarding the topography or soil composition. As a result, the models are likely to be wrong.

(b) Describe which of these types of uncertainty can be reduced by collecting more data, and illustrate your response using your example from part (a).

Commentary on Question:
The response should relate to the example in part (a). The solution below responds to the items listed in that solution.
Epistemic uncertainty can be reduced by collecting more data. For exceedance probability curves, parameter estimates can be improved with more data. For construction quality, building values, repair costs, topography or soil composition, more data will lead to more accurate models of how the exposures are distributed among various levels of that factor.

(c) Explain how logic trees can be used to reflect epistemic uncertainty in the construction of exceedance probability curves.

**Commentary on Question:**
*For this question it is necessary to cover both how trees are constructed and how they are used to reflect uncertainty.*

Construct a set of assumptions, where each set covers all the parameters of the model. For a given set of assumptions, run the model and develop an EP curve. Weight each assumption set with subjective probabilities. An average EP curve can then be constructed using the weights. In addition, looking at the fifth and ninety-fifth percentiles of EP curves (using the probabilities and ordering the curves from worst to most favorable) provides a ninety percent confidence interval for the true curve.


8. Learning Objectives:
3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:
(3a) Estimate unpaid unallocated loss adjustment expenses.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 22

Commentary on Question:
This question tests the estimation of unallocated loss adjustment expenses. Candidates need to know the application of the classical paid-to-paid method, the Kittel refinement, and the Mango-Allen smoothing adjustment.

Solution:
(a) Describe two of the key assumptions of the classical paid-to-paid method.

Commentary on Question:
Only two assumptions are needed for full credit.

1. Payments for ULAE are proportional to payments for claims.
2. The timing of payments for ULAE follows the timing of payments for claims.
3. The insurer’s relationship of paid ULAE to paid claims has achieved a steady state such that the paid-to-paid ratio is a reasonable approximation of the relationship between ultimate ULAE and ultimate claims.
4. The historical relationship between paid ULAE and paid claims represents the relationship expected between future ULAE and future claim payments.
5. The ULAE associated with open and pure IBNR claims are proportional to the case estimates and IBNR claims.

(b) Estimate unpaid ULAE as of December 31, 2012 using the classical paid-to-paid method and a multiplier of 50%. Justify any selections.

Commentary on Question:
Any selected ULAE ratio is acceptable as long as a reasonable justification is provided.

ULAE Ratio = (500 + 800 + 1,000) / (7,000 + 7,000 + 8,000) = 0.105
Unpaid ULAE = 0.105 × 3,000 + 0.105 × 0.5 × 5,000 = 578

The data is not credible enough to assume a trend in the ULAE ratio, so a weighted average was selected.
8. Continued

(c) Describe the Kittel refinement to the classical paid-to-paid method and the weakness it is designed to address.

The Kittel refinement substitutes the average of paid and reported claims for paid claims in the denominator of the formula for the estimated ULAE ratio.

It is designed to take into account the fact that ULAE is associated with the reporting of claims, in addition to the payment of claims.

(d) Explain the weakness of the classical paid-to-paid method that the Mango-Allen smoothing adjustment is designed to address and identify a situation in which it would be useful.

Commentary on Question:
*Identification of only one situation is needed for full credit.*

By using expected claims instead of actual claims in the denominator of the formula for the estimated ULAE ratio, it takes some of the volatility out of the estimate.

It is useful with long-tail lines, changing exposure volumes, new insurers, and for situations where distortions are present due to large claims or sparsity of claims.

(e) Estimate unpaid ULAE as of December 31, 2012 using the classical paid-to-paid method, a multiplier of 50%, and the Mango-Allen smoothing adjustment.

ULAE Ratio = (500 + 800 + 1,000) / (5,000 + 8,000 + 10,000) = 0.100
Unpaid ULAE = 0.100 × 3,000 + 0.100 × 0.5 × 5,000 = 550

(f) Compare the significance of ULAE for reinsurers to that of primary insurers and explain the reason for any difference.

ULAE is less significant for reinsurers than for primary insurers.

Primary insurers typically retain ULAE and do not cede it to reinsurers, and reinsurers have a limited role in the claim-handling process.
9. Learning Objectives:
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:
(5g) Calculate risk classification changes and territorial changes.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 32

Commentary on Question:
This question tests the candidate’s understanding of classification ratemaking. It requires the candidate to incorporate credibility and also understand distributional bias.

Solution:
(a) Calculate the indicated class relativities for the risk characteristic color of car.

Commentary on Question:
Preliminary claim ratio indicated relativity (column 5) must be calculated relative to the total to be on the same basis as the complement, so that the credibility weighted indicated relativities are calculated properly.

<table>
<thead>
<tr>
<th>Car Color</th>
<th>EP @ Average Curr Rate</th>
<th>Tended Ultimate Claims</th>
<th>Wtd Avg Claim Ratio</th>
<th>Claim Ratio Indicated</th>
<th>Ultimate Counts</th>
<th>Credibility</th>
<th>Complement</th>
<th>Credibility-Weighted Ind. Rel.</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>568,000</td>
<td>449,000</td>
<td>79.0%</td>
<td>1.044</td>
<td>775</td>
<td>71.0%</td>
<td>1.000</td>
<td>1.031</td>
</tr>
<tr>
<td>Black</td>
<td>780,000</td>
<td>606,000</td>
<td>77.7%</td>
<td>1.026</td>
<td>935</td>
<td>78.0%</td>
<td>1.000</td>
<td>1.020</td>
</tr>
<tr>
<td>Other</td>
<td>1,150,000</td>
<td>837,000</td>
<td>72.8%</td>
<td>0.961</td>
<td>1,635</td>
<td>100.0%</td>
<td>1.000</td>
<td>0.961</td>
</tr>
<tr>
<td>Total</td>
<td>2,498,000</td>
<td>1,892,000</td>
<td>75.7%</td>
<td>1.000</td>
<td>3,345</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: (4) = (3) / (2)
(5)_i = (4)_i / (4)_TOTAL
(7) = Squareroot{(6) / 1,537}; max of 1.0
(9) = (5)(7) + [1-(7)](8)

(b) Explain what is implied by a complement of credibility of 1.

Complement of 1 implies that the balance of credibility assumes there is no difference for car color for ratemaking.

(c) Determine the color of car classification relativities and the revised base rate assuming a base class of other, given that the current base rate is 475.
9. Continued

Commentary on Question:
Not adjusting the base rate would introduce an overall increase of 3.6%. As a result, need to divide the base rate by 1.036.

<table>
<thead>
<tr>
<th>Car Color</th>
<th>EP @ Average Curr Rate</th>
<th>Credibility-Weighted Ind. Rel.</th>
<th>Relativity to Base</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>568,000</td>
<td>1.031</td>
<td>1.073</td>
</tr>
<tr>
<td>Black</td>
<td>780,000</td>
<td>1.020</td>
<td>1.062</td>
</tr>
<tr>
<td>Other</td>
<td>1,150,000</td>
<td>0.961</td>
<td>1.000</td>
</tr>
<tr>
<td>Total</td>
<td>2,498,000</td>
<td></td>
<td>1.036</td>
</tr>
</tbody>
</table>

(4) Current Base Rate: 475.00
(5) Revised Base Rate: 458.58
{(4) / (3)total}

(d) Describe how you could check the risk characteristic color of car for distributional bias relative to another risk characteristic such as territory.

Check to see if the distribution of each territory within each car color is the same. There is distributional bias if they are not consistent by car color.
10. Learning Objectives:
   3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:
(3e) Evaluate premium liabilities.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 24

Commentary on Question:
*Commentary listed underneath question component.*

Solution:
(a) Select expected claim ratios for each line of business, gross and net of reinsurance, that will be used in the determination of premium liabilities. Justify each selection.

Commentary on Question:
*Any reasonable selections which take into account the 1-in-100 year nature of the hurricane that affected the 2012 property experience are acceptable.*

Property gross of reinsurance—55% (average of 60% in 2011 and 50% in 2013, unusual catastrophe year of 2012 will be ignored)

Property net of reinsurance—55% (average of 55% in 2011 and 55% in 2013, unusual catastrophe year of 2012 will be ignored)

Liability gross of reinsurance—90% (average of 2011-13)

Liability net of reinsurance—95% (average of 2011-13)

(b) Calculate the net premium liabilities for Acme as of December 31, 2013 given the selected expected claim ratios in (a) and the information provided.

Commentary on Question:
*Inclusion of all three terms in the formula is needed for full credit.*

Property portion of net premium liabilities—\(100,000 \times n\) (expected claims) + 0.10 \(\times 120,000 \times g\) (ULAE) + 0.05 \(\times 120,000\) (maintenance expenses), where g and n are gross and net expected claim ratios from part (a). With g and n both 55%, this yields 67,600. (Commission of 15% has already been paid and so is not included.)

Liability portion of net premium liabilities—same formula as property. With g equal to 90% and n equal to 95%, this yields 111,800.

\[67,600 + 111,800 = 179,400.\]
10. Continued

(c) Explain the purpose of a premium deficiency reserve.

Commentary on Question:
A one sentence explanation is sufficient for full credit.

A premium deficiency reserve is a liability to account for any excess of net premium liabilities over the unearned premium reserve.

(d) Determine, based on your calculations in (b), Acme’s premium deficiency reserve as of December 31, 2013.

Commentary on Question:
Any reasonable expected claim ratio selections lead to the conclusion that a premium deficiency reserve is not needed.

No premium deficiency reserve is needed because the total company net unearned premium of 200,000 exceeds the net premium liabilities of 179,400.

(e) Explain how premium liabilities for Acme would change if some of the liability policies are written using sales as an exposure base that are subject to audit following the end of the policy period.

Commentary on Question:
A one sentence explanation is sufficient for full credit.

Premium liabilities would be adjusted up or down, depending on whether aggregate audit premiums are expected to be negative or positive, respectively.
11. Learning Objectives:
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

Learning Outcomes:
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 14, 16, and 17

Commentary on Question:
This question tests the ability to estimate ultimate claims using the development method, the expected method, the Bornhuetter Ferguson method, and the Benktander method. The candidate needs to understand how to check the reasonableness of the inputs for the Bornhuetter Ferguson method as well as select estimates of ultimate claims.

Solution:
(a) Develop an estimate of ultimate claims for 2008 and 2012 using the following methods:

(i) Development method

(ii) Expected method

(iii) Bornhuetter Ferguson method

(iv) Benktander method, one iteration

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Actual Reported Claims (1)</th>
<th>A Priori Expected Claims (2)</th>
<th>Reported CDF (3)</th>
<th>Development Method (4) = (1)x(3)</th>
<th>Expected Method (5) = (2)</th>
<th>BF Method (6) = (1) + (2)x[1 - 1/(3)]</th>
<th>Benktander Method (7) = (1) + (6)x[1 - 1/(3)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>57,800</td>
<td>62,000</td>
<td>1.00</td>
<td>57,800</td>
<td>62,000</td>
<td>57,800</td>
<td>57,800</td>
</tr>
<tr>
<td>2012</td>
<td>19,300</td>
<td>52,100</td>
<td>2.00</td>
<td>38,600</td>
<td>52,100</td>
<td>45,350</td>
<td>41,975</td>
</tr>
</tbody>
</table>

(b) Evaluate the reasonableness of the inputs for the Bornhuetter Ferguson method.

Commentary on Question:
Only 2008 & 2012 calculations required for full credit.
11. Continued

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Actual Reported Claims (1)</th>
<th>A Priori Expected Claims (2)</th>
<th>Reported CDF (3)</th>
<th>Expected % Developed (4) = 1/(3)</th>
<th>Expected Claims Developed (5) = (2)x(4)</th>
<th>Difference Actual vs. Expected (6) = (1)-(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>57,800</td>
<td>62,000</td>
<td>1.00</td>
<td>100.0%</td>
<td>62,000</td>
<td>(4,200)</td>
</tr>
<tr>
<td>2009</td>
<td>53,100</td>
<td>59,500</td>
<td>1.05</td>
<td>95.2%</td>
<td>56,667</td>
<td>(3,567)</td>
</tr>
<tr>
<td>2010</td>
<td>25,200</td>
<td>51,000</td>
<td>1.10</td>
<td>90.9%</td>
<td>46,364</td>
<td>(21,164)</td>
</tr>
<tr>
<td>2011</td>
<td>20,600</td>
<td>49,000</td>
<td>1.50</td>
<td>66.7%</td>
<td>32,667</td>
<td>(12,067)</td>
</tr>
<tr>
<td>2012</td>
<td>19,300</td>
<td>52,100</td>
<td>2.00</td>
<td>50.0%</td>
<td>26,050</td>
<td>(6,750)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>47,747</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conclusion: Inputs are not reasonable.
Reason: Actual is significantly lower than expected for both years (all years as well), suggesting that the inputs are not reasonable and further investigation is recommended.

(c) Select estimates of ultimate claims for 2008 and 2012 and justify your selections.

**Commentary on Question:**
*Due to the significant difference in actual versus expected claims, the A Priori estimate would be inappropriate as a selection.*

2008:
- Select 57,800.
- Reason: Claims are fully developed for 2008 (reported CDF = 1.0) and there is no evidence that further change will occur.

2012:
- Select BF estimate of 45,350.
- Reason: Immature so select a blended method.
12. **Learning Objectives:**
   1. The candidate will understand the key considerations for general insurance actuarial analysis.

**Learning Outcomes:**

(1g) Identify trend adjustments and describe the relationship between trend and loss development.

**Sources:**

**Commentary on Question:**
*No calculations are required for full credit because the trend factors can be read directly off the graph.*

**Solution:**

(a) Illustrate this inflationary trend effect on the layer from 0 to 50 and the layer from 50 to 100 using a graph with cumulative claim frequency along the $x$-axis and claim size along the $y$-axis.
12. **Continued**

(b) Determine the inflationary trend factors that apply to each of the following layers using the graph from (a):

(i) 0 to 50

(ii) 50 to 100

(iii) 0 to 100

Trend factor for 0 to 50 layer is \((C+D)/D = 7/6 = 1.167\).

Trend factor for 50 to 100 layer is \((A+B)/B = 5/2 = 2.500\).

Trend factor for 0 to 100 is \((A+B+C+D)/(B+D) = 3/2 = 1.500\).
13. **Learning Objectives:**
   5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5k) Calculate rates for claims-made coverage.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34

**Commentary on Question:**
*Commentary listed underneath question component.*

**Solution:**
(a) Calculate the expected claims on an occurrence policy sold in 2013.

**Commentary on Question:**
*A matrix approach similar to that shown in the text leading to an approximate answer would also receive full credit.*

\[
500 \times (0.2)^0 \times (1.10)^0 + 500 \times (0.2)^1 \times (1.10)^1 + 500 \times (0.2)^2 \times (1.10)^2 + \ldots = \\
500/(1 – 0.2 \times 1.10) = 641
\]

(b) Calculate the second-year claims-made step factor.

**Commentary on Question:**
*An equation is sufficient for full credit.*

\[
(500 + 100)/625 = 0.96
\]

(c) Calculate the second-year claims-made tail factor.

**Commentary on Question:**
*A matrix approach similar to that shown in the text leading to an approximate answer would also receive full credit.*

\[
((500 \times 0.2 \times 1.10 + 100 \times 0.2 \times 1.10)/(1 – 0.2 \times 1.10))/(500 + 100) = 0.28
\]

(d) Identify the gap in coverage that can arise when changing from claims-made coverage with one insurer to claims-made coverage with a different insurer.

**Commentary on Question:**
*A one sentence explanation is sufficient for full credit.*
13. Continued

Claims occurring prior to the retroactive date of the new policy will not be covered.

(e) Explain how the gap in (d) can be addressed.

Commentary on Question:

*Only one solution for the gap is needed for full credit.*

Tail coverage may be purchased from the former insurer, or prior acts (or nose) coverage may be purchased from the new insurer, with a retroactive date no later than the retroactive date that applied with the former insurer.
14. Learning Objectives:
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

Learning Outcomes:
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 14

Commentary on Question:
This question tests the ability to estimate ultimate claims using the development method. Candidates need to know the criteria to consider when selecting age-to-age development factors, and also adjust the estimate when there is a large claim.

Solution:
(a) Calculate three alternative average age-to-age factors for the interval 12-24, based on:

(i) Volume weighted 3-year

(ii) Medial 5×1

(iii) Geometric 3-year

Volume weighted 3-year:
\[
\frac{10,081+7,541+7,201}{8,366+6,380+5,962} = \frac{24,823}{20,708} = 1.199
\]

Medial 5×1: (average most recent 5, excluding high & low) average
\[
(1.190,1.197,1.205) = 1.197
\]

Geometric 3-year: \[(1.205\times1.182\times1.208)^{\frac{1}{3}} = 1.198\]

(b) List three considerations in selecting age-to-age factors.

Commentary on Question:
Nine considerations are presented. Any three are acceptable.

- Volume and credibility of insurer's experience
- Stability of factors at each maturity interval
- Any discernible trends when comparing short-term to long-term averages
- Number of recent factors that are greater than or less than the various averages
- Factors preceding or following the interval
- Any known changes that affect future development
- Influence of large claims
14. Continued

- Relevance of other information such as industry benchmarks
- Selected factors from prior actuarial work

(c) Calculate the percentage of incremental reported claims expected between 48 and 60 months.

**Commentary on Question:**
*Only the calculation of the 48-ultimate and 60-ultimate values are needed to get full credit.*

|     | 12-24 | 24-36 | 36-48 | 48-60 | 60-72 | 72-84 | 84-Ult
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>1.198</td>
<td>1.121</td>
<td>1.058</td>
<td>1.042</td>
<td>1.033</td>
<td>1.023</td>
<td>1.023</td>
</tr>
<tr>
<td>(2)</td>
<td>1.601</td>
<td>1.336</td>
<td>1.192</td>
<td>1.126</td>
<td>1.081</td>
<td>1.047</td>
<td>1.023</td>
</tr>
<tr>
<td>(3)</td>
<td>62.5%</td>
<td>74.8%</td>
<td>83.9%</td>
<td>88.8%</td>
<td>92.5%</td>
<td>95.6%</td>
<td>97.8%</td>
</tr>
</tbody>
</table>

Incremental % reported between 48 & 60 months: 92.5% - 88.8% = 3.7%

(d) Calculate the ultimate claims for this accident year both with and without a large claim adjustment.

Need 12-Ult factor: from part (c), 12-Ult factor = 1.601

With NO adjustment: \[10,190 \times 1.601 = 16,314\]

WITH adjustment: \[(10,190 - 4,000) \times 1.601 + 4,000 = 13,910\]

(e) State an assumption underlying each approach in (d).

With NO adjustment:
Assumption is that the 4,000 claim is:
(i) Not unusual, and
(ii) Is representative of historical claims experience.

WITH adjustment:
Assumption is:
(i) Case estimate developed by claim adjuster is the best estimate of unusual claim, and
(ii) An actuarial-based estimate is not appropriate.
15. **Learning Objectives:**
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5j) Perform individual risk rating using standard plans.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 35

**Commentary on Question:**
*Commentary listed underneath question component.*

**Solution:**
(a) State the two basic principles on which prospective experience rating plans are based.

**Commentary on Question:**
*A brief sentence covering each principle is sufficient for full credit.*

The larger the insured, the more reliable the historical claims will be as a predictor of future claims. Frequency is a better predictor of future claims than severity.

(b) Propose an experience rating formula and explain how it takes into account these two basic principles.

**Commentary on Question:**
*Any formula is acceptable as long as an explanation is provided to show how the formula takes into account the two principles.*

\[
\frac{\text{Actual Primary Claims}}{\text{Expected Primary Claims}} \times Z + (1-Z)
\]

Larger insureds are given greater credibility and the use of primary claims emphasizes frequency over severity.

(c) Recommend whether, from WC Plumbing’s perspective, it would be a good candidate for retrospective rating and justify your recommendation.

**Commentary on Question:**
*The recommendation needs to reference both reasons to obtain full credit.*

A retrospective rating plan is not appropriate for this company for two reasons. The substantial operating loss in the latest year indicates that the company has financial problems, and the fact that the company had only three claims probably indicates that it is not large enough for a retrospective rating plan to be advisable.
16. **Learning Objectives:**

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**

(5b) Calculate expenses used in ratemaking analyses.

(5f) Calculate overall rate change indications under the claims ratio and pure premium methods.

**Sources:**

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 29 and 31

**Commentary on Question:**

*This question tests the selection of a fixed expense ratio to be used for ratemaking and the resulting indication. The candidate needs to understand how various changes can lead to distortions when selecting a fixed expense percentage applied to a projected average premium for ratemaking.*

**Solution:**

(a) Select a fixed and variable expense ratio as a percentage of direct earned premiums to be used for ratemaking purposes assuming that historically 30% of general and other acquisition expenses are considered to be fixed expenses. Justify your selection.

**Commentary on Question:**

Candidate needs to recognize that 2009 (possibly 2008) is an anomalous year and should be excluded from the selection of the expense ratios.

<table>
<thead>
<tr>
<th>Calendar Year</th>
<th>General and Other Acquisition Expenses</th>
<th>Direct Earned Premiums</th>
<th>(3) = (1) / (2) Total General &amp; Other Acq. Expenses %</th>
<th>(3) × 30% Fixed Expense Percentage</th>
<th>(4) × 70% Variable Expense Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>108,000</td>
<td>691,000</td>
<td>15.63%</td>
<td>4.69%</td>
<td>10.94%</td>
</tr>
<tr>
<td>2009</td>
<td>138,000</td>
<td>725,000</td>
<td>19.03%</td>
<td>5.71%</td>
<td>13.32%</td>
</tr>
<tr>
<td>2010</td>
<td>115,000</td>
<td>770,000</td>
<td>14.94%</td>
<td>4.48%</td>
<td>10.45%</td>
</tr>
<tr>
<td>2011</td>
<td>126,000</td>
<td>834,000</td>
<td>15.11%</td>
<td>4.53%</td>
<td>10.58%</td>
</tr>
<tr>
<td>2012</td>
<td>130,000</td>
<td>866,000</td>
<td>15.01%</td>
<td>4.50%</td>
<td>10.51%</td>
</tr>
<tr>
<td>Total</td>
<td>617,000</td>
<td>3,886,000</td>
<td>15.88%</td>
<td>4.76%</td>
<td>11.11%</td>
</tr>
</tbody>
</table>

Average all years excluding 2009: 15.17% 4.55% 10.62%

Average of 2010, 2011 & 2012 only: 15.02% 4.51% 10.51%

Selected ratios: 4.55% 10.60%
16. Continued

(b) Calculate the indicated rate and indicated rate change given the selected fixed expense ratio from (a).

Commentary on Question:
The question is set up to solve using the pure premium method, although enough information is provided so that the claim ratio method could also be used.

Pure Premium Method:
1. Weighted Average Trended Pure Premium: 476.00
2. ULAE as a Ratio to Claims: 8.0%
3. Selected Fixed Expense Ratio as a % of Premium: 4.55%
4. Total Fixed Expenses \((3) \times 895,000\): 40,722.50
5. Fixed Expenses per Vehicle \([4 / 1,150]\): 35.41
6. Commissions: 12.0%
7. Premium Taxes: 2.0%
8. Licenses: 1.0%
9. Selected General Variable Expenses as a % of Premium: 10.60%
10. Total Variable Expenses [sum of (6) through (9)]: 25.6%
11. Profit and Contingencies Factor: 3.0%
12. Indicated Rate \([\frac{(1) \times (1 + (2)) + (5)}{1 – (10) – (11)}]\): 769.60
13. Current Rate \([895,000 / 1,150]\): 778.26
14. Indicated Rate Change \([\frac{(12)}{(13)} – 1]\): -1.11%

(c) Explain how each of these situations can affect the level of fixed expenses in a ratemaking analysis and recommend a solution for each to avoid potential distortion.

Commentary on Question:
Candidate needs to explain how each situation can affect the level of fixed expenses and also recommend a solution to get full credit.

Distortions caused by recent rate changes:
- How the level of fixed expenses is affected: A recent rate increase will mean the historical ratio will be applied to a higher current premium resulting in too high a provision for fixed expenses.
- Solution: Adjust historical premium to on level in the expense analysis.

Distortions caused by shifts in mix of business:
- How the level of fixed expenses is affected: Changes in mix of business will have a different effect on premiums and expenses such that the expenses estimated for ratemaking are either overstated or understated.
- Solution: Adjust historical premium and expenses with trend in the expense analysis.
17. **Learning Objectives:**

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 15

**Commentary on Question:**

*This question is concerned with projecting ultimate claims through frequency-severity methods, and recognizing seasonality.*

**Solution:**

Project ultimate claims for Spring 2013 and Summer 2013 using a frequency-severity method given the information above, and justify all selections.

The first step is to project ultimate counts. It is clear from the data that the age-to-age factors vary by the accident period of Summer, Fall and Spring, so count development should recognize this. Development of claims without the recognition of seasonality would not receive full credit for this question. The following shows the development of age-to-ultimate factors:

**Average Age-to-Age factors:**

<table>
<thead>
<tr>
<th>Accident Period</th>
<th>4-8</th>
<th>8-12</th>
<th>12-16</th>
<th>16-20</th>
<th>20-24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer</td>
<td>2.01</td>
<td>1.44</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Fall</td>
<td>3.01</td>
<td>2.46</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Spring</td>
<td>1.60</td>
<td>1.83</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Selected Age-to-Age factors:**

<table>
<thead>
<tr>
<th>Accident Period</th>
<th>4-8</th>
<th>8-12</th>
<th>12-Ult</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer</td>
<td>2.00</td>
<td>1.45</td>
<td>1.00</td>
</tr>
<tr>
<td>Fall</td>
<td>3.00</td>
<td>2.50</td>
<td>1.00</td>
</tr>
<tr>
<td>Spring</td>
<td>1.60</td>
<td>1.83</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Calculated Age-to-Ultimate factors:**

<table>
<thead>
<tr>
<th>Accident Period</th>
<th>4-Ult</th>
<th>8-Ult</th>
<th>12-Ult</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer</td>
<td>2.90</td>
<td>1.45</td>
<td>1.00</td>
</tr>
<tr>
<td>Fall</td>
<td>7.50</td>
<td>2.50</td>
<td>1.00</td>
</tr>
<tr>
<td>Spring</td>
<td>2.93</td>
<td>1.83</td>
<td>1.00</td>
</tr>
</tbody>
</table>
17. Continued

The next step would be to arrive at an overall claim frequency by comparing ultimate counts to exposures.

<table>
<thead>
<tr>
<th>Accident Period</th>
<th>Reported Accident Counts</th>
<th>Age-to-Ult Development Factor</th>
<th>Ultimate Counts</th>
<th>Exposures</th>
<th>Indicated Ultimate Frequency</th>
<th>Trend at 0% Frequency</th>
<th>Tended Frequency</th>
<th>Selected Ultimate Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer 2010</td>
<td>7,079</td>
<td>1.00</td>
<td>7,079</td>
<td>250,000</td>
<td>2.83%</td>
<td>1.00</td>
<td>2.83%</td>
<td>7,250</td>
</tr>
<tr>
<td>Fall 2010</td>
<td>10,287</td>
<td>1.00</td>
<td>10,287</td>
<td>350,000</td>
<td>2.94%</td>
<td>1.00</td>
<td>2.94%</td>
<td>10,150</td>
</tr>
<tr>
<td>Spring 2011</td>
<td>8,672</td>
<td>1.00</td>
<td>8,672</td>
<td>300,000</td>
<td>2.89%</td>
<td>1.00</td>
<td>2.89%</td>
<td>8,700</td>
</tr>
<tr>
<td>Summer 2011</td>
<td>7,614</td>
<td>1.00</td>
<td>7,614</td>
<td>262,500</td>
<td>2.90%</td>
<td>1.00</td>
<td>2.90%</td>
<td>7,613</td>
</tr>
<tr>
<td>Fall 2011</td>
<td>10,696</td>
<td>1.00</td>
<td>10,696</td>
<td>367,500</td>
<td>2.91%</td>
<td>1.00</td>
<td>2.91%</td>
<td>10,658</td>
</tr>
<tr>
<td>Spring 2012</td>
<td>9,293</td>
<td>1.00</td>
<td>9,293</td>
<td>315,000</td>
<td>2.95%</td>
<td>1.00</td>
<td>2.95%</td>
<td>9,135</td>
</tr>
<tr>
<td>Summer 2012</td>
<td>7,912</td>
<td>1.00</td>
<td>7,912</td>
<td>270,000</td>
<td>2.93%</td>
<td>1.00</td>
<td>2.93%</td>
<td>7,830</td>
</tr>
<tr>
<td>Fall 2012</td>
<td>11,339</td>
<td>1.00</td>
<td>11,339</td>
<td>378,000</td>
<td>3.00%</td>
<td>1.00</td>
<td>3.00%</td>
<td>10,962</td>
</tr>
<tr>
<td>Spring 2013</td>
<td>5,163</td>
<td>1.83</td>
<td>9,448</td>
<td>324,000</td>
<td>2.92%</td>
<td>1.00</td>
<td>2.92%</td>
<td>9,396</td>
</tr>
<tr>
<td>Summer 2013</td>
<td>2,761</td>
<td>2.90</td>
<td>8,007</td>
<td>277,500</td>
<td>2.89%</td>
<td>1.00</td>
<td>2.89%</td>
<td>8,048</td>
</tr>
</tbody>
</table>

Average - All years: 2.92%
Selected Frequency: 2.9%

Finally, to arrive at ultimate claims for the two accident periods, selected ultimate counts need to be multiplied by ultimate severities. Using the 2.90% selected frequency, selected ultimate counts are determined by multiplying exposures by 2.90%. The following table derives the projected ultimate claims:

<table>
<thead>
<tr>
<th>Accident Period</th>
<th>Selected Ultimate Counts (8)</th>
<th>Ultimate Severity (9)</th>
<th>Projected Ultimate Claims (10)=(8)(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring 2013</td>
<td>9,396</td>
<td>2,460</td>
<td>23,114,160</td>
</tr>
<tr>
<td>Summer 2013</td>
<td>8,048</td>
<td>2,480</td>
<td>19,959,040</td>
</tr>
</tbody>
</table>
18. **Learning Objectives:**
7. The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

**Learning Outcomes:**
(7b) Apply catastrophe models to insurance ratemaking, portfolio management, and risk financing.

**Sources:**
Catastrophe Modeling: A New Approach to Managing Risk, Grossi, P. and Kunreuther, H., Section 7.3

**Commentary on Question:**
Commentary listed underneath question component.

**Solution:**
(a) Define basis risk and describe how it can occur in this context.

**Commentary on Question:**
The solution quotes the definition from the text. The most important item is to note that the risk is the potential mismatch between the hedging and actual cash flows.

The text definition is “Basis risk arises in derivative products as a result of uncertainty associated with ability of the cash flows of the hedging instrument to exactly offset the cash flows from the instrument being hedged.” In the case of hurricane catastrophe hedging, the payment to the insurance company may not be a function of the actual losses incurred but rather a function of other items that correlate with actual losses but do not exactly match them.

(b) State if there is basis risk for BCI with each of these securitization types. If there is no basis risk, explain why not, and if there is basis risk, explain how that securitization type creates basis risk.

**Commentary on Question:**
Where there is basis risk it is necessary to explain how the risk is created by that securitization type. Just stating it creates a mismatch is not sufficient.

(i) Indemnity-based securitization – Losses are paid based on the actual company losses. There is no basis risk because by definition the indemnity matches the losses.

(ii) Index-based transaction – Because the index is based on average industry losses rather than BCI’s losses, there is a possibility of basis risk.

(iii) Parametric indices – With the payment depending on the physical parameters of the loss event, basis risk arises when the actual damages and resultant losses do not correlate with the parameters.
18. Continued

(iv) Notional portfolio – With payments based on a hypothetical portfolio of policies rather than the company’s actual portfolio of policies, there can be basis risk.

(c) Indicate an action BCI can take to reduce basis risk for two of the securitization types that you identified as having basis risk.

Commentary on Question:
*The action should be something that is within BCI’s control. The model solution responds to all three cases; candidates need to only provide two responses.*

(ii) Before issuing such a security, a company should verify that its losses are highly correlated with industry losses. This can be done by reviewing past results and comparing the company’s exposure to the industry average.

(iii) The risk can be reduced by having a model that accurately links the event to the company’s losses. Then triggers can be set that maximize the correlation.

(iv) The risk is minimized when the notional portfolio closely resembles the actual portfolio.
19. **Learning Objectives:**
6. The candidate will understand the need for monitoring, documentation, and communication.

**Learning Outcomes:**
(6a) Monitor financial reporting results and pricing changes.

**Sources:**
*Fundamentals of General Insurance Actuarial Practice*, J. Friedland, Chapter 36

**Commentary on Question:**
*Commentary listed underneath question component.*

**Solution:**
(a) Show that Tony’s calculation for accident year 2010 is correct, based on the methodology he used.

**Commentary on Question:**
The description of Tony’s methodology allows for the use of several interpolation formulas, but regardless of the specific formula used, the result should be the same. The model solution presents two possible approaches.

April 30 is 1/3 of the way through the year so the interpolated percent reported is $(2/3)(96.2) + (1/3)(98.1) = 96.833$. The expected count is $0.96833(2800) = 2711$. Note that this is a simplification of the formula in the book that can only be used when the percent reported is the ratio of observed to selected. Using the book’s formula, the solution is:

$$2694 + \frac{2800 - 2694}{1 - 0.962}(0.96833 - 0.962) = 2711.$$

(b) Write a brief note to Tony explaining why it may have been more appropriate to use the cumulative development factors from one specific method for this task.

**Commentary on Question:**
The answer to this question is independent of the specific situation faced by Tony. The answer should be to the general situation.

The final selection of ultimate counts may depend upon factors that are specific to the prior experience. Values for specific accident years may be adjusted to reflect changing circumstances either in the external environment or in company processes. As a result, the selected CDFs for adjacent accident years may not align in a reasonable manner. It may even be the case that the earlier accident year has a smaller CDF. Thus the interpolation could produce values that lead to a smaller number of expected reported counts. Using the likely smoother pattern from a formulaic method lessens the likelihood of such a distortion.
19. Continued

(c) Provide two questions you would pose to your company colleagues in an investigation of this observation.

Commentary on Question:
There are many questions that could be asked. The model solution lists several, but there are others that could receive credit. To qualify, the question must refer to something that can affect some accident years without affecting others.

- Has there been a change in the external environment (such as tort reform) that is delaying claim closing?
- Has there been a change in the internal systems of the company that affects only claims from the past two years?
- Has there been a change in the type of business written?
- Has there been a change in underwriting processes?
- Has there been a significant error in the reporting process?
- Has there been a significant loss event that distorted the patterns?
20. **Learning Objectives:**

1. The candidate will understand the key considerations for general insurance actuarial analysis.

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**

(1b) Identify different types of data used for actuarial analysis.

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 4 and 16

**Commentary on Question:**

*Commentary listed underneath question component.*

**Solution:**

(a) Describe two situations for which the expected method would be a preferred approach for projecting ultimate claims.

**Commentary on Question:**

*Four situations are presented. Any two are acceptable.*

- Introduction of new products
- Entry into a new geographical area where there is no historical data
- There have been significant changes internal to the insurer or external, such that existing relationships and patterns are not a reliable guide to the future
- There are immature accident years, particularly for long-tailed business

(b) Define exposure base and leading indicator.

An **exposure base** is a measure that is known or accurately estimated in advance and that varies directly with the quantity being estimated. A **leading indicator** is not known in advance, but is directly correlated with the quantity being measured.

(c) Describe two desirable characteristics of exposures for actuarial work.

**Commentary on Question:**

*Six characteristics are presented. Any two are acceptable.*
20. Continued

- It should accurately reflect the overall exposure to loss
- It should be simple to compile
- It should not be subject to manipulation
- It should accurately reflect differences in exposure to loss
- It should consider any pre-existing exposure base established within the industry
- A leading indicator is preferred
- A leading indicator should require few adjustments
- It should use the latest information

(d) Calculate the expected claims for 2008 using the expected method with the following approaches:

(i) Expected claim ratio

(ii) Pure premium

The following initial calculations precede the two approaches:
- The premium on-level factor for 2008 is \(1 - 0.1(0.4) = 0.96\)
- The trend factor is \(1.024^4 = 1.0995\)
- The tort reform factor for 2008 is 0.7

The calculation for the expected claim ratio approach is:

\[
\frac{(\text{selected expected claim ratio}_{2012}) \times (\text{premium on-level factor}_{2008})}{(\text{trend factor}_{2008}) \times (\text{tort reform factor}_{2008})} \times (2008 \text{ earned premium})
\]

\[
= \frac{0.75 \times 0.96}{1.0995 \times 0.7} \times 24,540,000 = 22,956,928
\]

The calculation for the pure premium approach is:

\[
\frac{(\text{selected pure premium}_{2012})}{(\text{trend factor}_{2008}) \times (\text{tort reform factor}_{2008})} \times (2008 \text{ earned vehicles})
\]

\[
= \frac{220}{1.0995 \times 0.7} \times 87,600 = 25,039,953
\]
21. **Learning Objectives:**
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5h) Calculate deductible factors, increased limits factors, and coinsurance penalties.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 33

**Commentary on Question:**
The question deals with calculating deductible factors and explaining the underlying assumptions for a calculation based on claims and counts by layer. Candidates need to understand how to test deductible factors for consistency.

**Solution:**
(a) Calculate the indicated deductible factors for deductibles of 250 and 750 relative to a base of zero deductible.

- To calculate the 250 deductible factor, first calculate the claims eliminated by layer:
  - Claims in the 0–250 layer = 30,000 eliminated completely
  - In the next two layers the first 250 of each claim is eliminated = 250 × (300+100) = 100,000
  - Indicated deductible factor = 1−(total claims eliminated/total claims) = 1−(130,000/330,000) = 0.606

- To calculate the 750 deductible factor, first calculate the claims eliminated by layer:
  - Claims in the first and second layers = 180,000 (30,000+150,000) eliminated completely
  - In the next layer the first 750 of each claim is eliminated = 750×100 = 75,000
  - Indicated deductible factor = 1−(total claims eliminated/total claims) = 1−(180,000+75,000)/330,000 = 0.227

(b) State two assumptions that you needed to make in using the information above to perform the calculation in part (a).

**Commentary on Question:**
Listed below are several assumptions. Any two are acceptable for full credit.

- The frequency distribution is the same for each deductible considered.
- Expenses apply as a percentage of premiums for the calculated deductibles.
21. **Continued**

- Counts and claims are at ultimate values, trended appropriately for the forecast period.
- The given information (which may be based on industry data or historical company data) is reflective of future experience (i.e., no significant changes in either the internal or external environment).
- The information given on counts and claims treats ALAE in a manner consistent with the product to be priced/sold in the future.
- Data provided is ground-up and uncensored.
- Changing deductible will have no impact on insured behavior.

(c) Determine the range into which the deductible factor for a 500 deductible must fall in order to be consistent with the deductible factors calculated in part (a) and explain your reasoning.

The selected deductible factor needs to be between the 250 and the 750 deductible factors and needs to be less than the average of the two factors (i.e., \( 0.227 < F(500) < 0.606 \) and \( F(500) < (0.227+0.606)/2 = 0.417 \)) to be consistent with decreasing marginal change in deductible factors. Thus, \( 0.227 < F(500) < 0.417 \).