1. **Learning Objectives:**

   1. The candidate will understand the key considerations for general insurance actuarial analysis.

**Learning Outcomes:**

(1k) Estimate written, earned and unearned premiums.

**Sources:**


**Commentary on Question:**

The question tests the ability of the candidate to understand certain details of individual insurance policies and to make correct calculations of earned exposures, earned premiums, and written premiums for various policies. The candidate also needs to understand when premiums are not earned evenly throughout a year.

**Solution:**

(a) Calculate earned and written premium for calendar years 2012 and 2013. No Name does not treat multi-year policies as multiple annual policies.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1/2012</td>
<td>Annual</td>
<td>5,000</td>
<td>100%</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>5,000</td>
<td>0</td>
<td>5,000</td>
<td>0</td>
</tr>
<tr>
<td>4/1/2012</td>
<td>Annual</td>
<td>1,000</td>
<td>100%</td>
<td>75%</td>
<td>25%</td>
<td>1,000</td>
<td>0</td>
<td>750</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>7/1/2012</td>
<td>6-month</td>
<td>500</td>
<td>100%</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
<td>500</td>
<td>0</td>
<td>500</td>
<td>0</td>
</tr>
<tr>
<td>10/1/2012</td>
<td>2-year</td>
<td>5,000</td>
<td>100%</td>
<td>12.5%</td>
<td>50%</td>
<td>5,000</td>
<td>0</td>
<td>625</td>
<td>2,500</td>
<td></td>
</tr>
<tr>
<td>1/1/2013</td>
<td>Annual</td>
<td>2,000</td>
<td>100%</td>
<td>0%</td>
<td>100%</td>
<td>0</td>
<td>2,000</td>
<td>0</td>
<td>2,000</td>
<td>0</td>
</tr>
<tr>
<td>7/1/2013</td>
<td>Annual</td>
<td>1,500</td>
<td>100%</td>
<td>0%</td>
<td>50%</td>
<td>0</td>
<td>1,500</td>
<td>0</td>
<td>750</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11,500</td>
<td>3,500</td>
<td>6,875</td>
<td>5,500</td>
<td></td>
</tr>
</tbody>
</table>

(b) Explain how the calculation of written and earned premium might be different if No Name Insurance Company wrote motorcycle policies in a winter climate instead of general liability policies.

**Commentary on Question:**

Candidates need to recognize that the earning pattern would be different but the written premiums would not.
1. Continued

Motorcycles written in a winter climate would typically only have exposure to loss in the spring, summer and fall months. As a result, the insurer might recognize this difference by modifying the even earnings throughout the policy term. Written premium would be unaffected.
2. Learning Objectives:
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

6. The candidate will understand the need for monitoring, documentation, and communication.

Learning Outcomes:
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

(6a) Monitor financial reporting results and pricing changes.

Sources:

Commentary on Question:
This question tests the fundamental understanding of estimating unpaid claims, expected claims and ultimate claims using the expected method and the Bornhuetter Ferguson method. Candidates also need to be able to estimate expected reported claims for an interim period between actuarial analyses using the approach in Friedland Chapter 36.

Solution:
(a) Estimate the unpaid claims for accident year 2013 using the development method with simple all-year average development factors.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12-24</th>
<th>24-36</th>
<th>36-48</th>
<th>48-60</th>
<th>Tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>2.00</td>
<td>1.20</td>
<td>1.06</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>2.20</td>
<td>1.18</td>
<td>1.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>2.00</td>
<td>1.19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>2.20</td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td>Average</td>
<td>2.10</td>
<td>1.19</td>
<td>1.07</td>
<td>1.02</td>
<td>1.00</td>
</tr>
<tr>
<td>CDF</td>
<td>2.727</td>
<td>1.299</td>
<td>1.091</td>
<td>1.020</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Ultimate claims for 2013: 130×2.727 = 355
Paid to date = 75
2013 Unpaid = 355 – 75 = 280
2. Continued

(b) Calculate the accident year 2012 claims expected to be reported in 2014 using the development factors from part (a).

2012 reported claims @ Dec 31, 2013: 242
24-36 month development factor: 1.19
Projected 2012 reported claims @ Dec 31, 2014 (242×1.19): 288
2012 reported claims during the next 12 months (288 – 242): 46

(c) State the two primary assumptions of the development method.

Historical experience is predictive of future experience.
Activity observed to date is relevant for projecting future activity.

(d) Estimate the ultimate claims for accident year 2012 using the Bornhuetter Ferguson method with an expected claim ratio of 60%.

24-ult factor: 1.299
% reported: 1/1.299 = 77%
% unreported: 1 – 77% = 23%
Expected claim ratio: 60%
Unreported: 520×60%×23% = 72
Reported: 242
AY2012 BF Method Ultimate = 72 + 242 = 314

(e) Describe two situations when the Bornhuetter Ferguson method may be preferable to the development method.

Commentary on Question:
Any two of the following situations are acceptable.

- For immature experience periods
- Following the introduction of new GI products when limited or no historical experience is available
- Following entry into a new geographical area for which limited or no historical data exists
- If there have been wide-ranging changes, either internally at the insurer or in the external environment, such that historical relationships and development patterns are not a reliable guide to the future

(f) Compare actual reported claims to expected reported claims for accident year 2012 and comment on the reasonableness of the Bornhuetter Ferguson method.
2. Continued

**Commentary on Question:**

Candidate needs to understand the reasonableness of the inputs for the Bornhuetter Ferguson method can be tested by comparing actual reported to expected reported.

Actual reported for 2012 @ 24 months: 242
24-ult development factor: 1.299
% reported: 1/1.299 = 77%
Earned premium: 520
Expected reported = 520×77%×60% = 240
Difference = 242 – 240 = 2

Expected and actual reported are very close. Therefore it supports the input (development pattern and expected value) for BF method.

(g) Calculate the difference between the actual and expected reported claims from December 31, 2013 through March 31, 2014 for accident year 2013, using linear interpolation of the expected percent reported.

**Commentary on Question:**

Candidates need to follow the approach outlined in section 36.3 of the Friedland text for the solution.

Expected reported at 3/31/2014 = 0.75×0.37 + 0.25×0.77 = 0.47
Actual Difference = 178 – 130 = 48
Expected Difference: $(320 – 130) \times \frac{0.47 – 0.37}{1 – 0.37} = 30.2$
Difference: 48 – 30.2 = 17.8

(h) Identify two questions you might ask in your further investigation based on the results from part (g).

**Commentary on Question:**

*Only two questions are needed to earn credit and there may be others beyond those listed below that are acceptable.*

- Was there a legal decision that affected claims in all years?
- Were claims found that had not been entered properly in the system?
- Is there an expectation that issues have been resolved or is this adverse experience likely to continue for subsequent quarters?
3. **Learning Objectives:**  
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**  
(5h) Calculate deductible factors, increased limits factors, and coinsurance penalties.

**Sources:**  

**Commentary on Question:**  
*This question is testing the candidate’s ability to calculate increased limits factors for capped claims data, as well as how to test for consistency in increased limits factors and why consistency is important.*

**Solution:**  
(a) Calculate the increased limits factors for the 2,000,000 and 3,000,000 policy limits, assuming a 1,000,000 basic limit.

Calculate increased limits factors (ILFs) using the formula:  
\[ \text{ILF}(L) = \frac{\text{Expected Severity}(L)}{\text{Expected Severity}(B)} \]

where \( L \) is the limit for which we are determining the ILF and \( B \) is the basic limit.

- For the 2,000,000 limit as follows:
  - First calculate the Expected Severity (Basic Limit) or limited average severity (LAS) using data from all policy limits expressed in millions:
    \[ \text{LAS}(1) = \frac{858 + 629 + (305 \times 1) + 625 + (330 + 32) \times 1}{3,333 + 2,900 + 305 + 3,100 + 330 + 32} = 0.278 \]
  - Next, calculate the LAS for the layer from 1 to 2 using only policy limits data from policies at 2 and 3 limits as follows:
    \[ \text{LAS}(1 \text{ to } 2) = \frac{470 + 533 - (305 + 330) \times 1 + (32 \times 1)}{2,900 + 305 + 3,100 + 330 + 32} = 0.060 \]
  - Then, the \( \text{LAS}(2) = 0.278 + 0.060 = 0.338 \)
- Continuing for the 3,000,000 limit, calculate the LAS for the layer from 2 to 3 using only policy limits data from policies at 3 limits:
  \[ \text{LAS}(2 \text{ to } 3) = \frac{77 - (32 \times 2)}{3,100 + 330 + 32} = 0.004 \]
  - Then, the \( \text{LAS}(3) = 0.338 + 0.004 = 0.342 \)
- The increased limits factors are then calculated as follows:
  - ILF(2) = \( \text{LAS}(2)/\text{LAS}(1) = 0.338/0.278 = 1.216 \)
  - ILF(3) = \( \text{LAS}(3)/\text{LAS}(1) = 0.342/0.278 = 1.230 \)

(b) Determine the range into which a 4,000,000 increased limits factor should fall, considering consistency with the factors determined in part (a).
3. Continued

The incremental increase per million from 3 to 4 should be less than the incremental increase from 2 to 3. Thus, ILF(4) should be greater than 1.230 and less than 1.244 (1.230 + 1.230 – 1.216). Therefore one endpoint of the range is the lowest point or 1.230, and the other endpoint is the highest point or 1.244.

(c) Explain why consistency is important for increased limits factors.

Consistency reflects both a decreasing claim (survival) probability as claim size increases and the practical consideration that the incremental price should decrease as the limit increases.

(d) Explain why it is important to know whether claims have been capped or not in determining increased limits factors.

With uncensored claims data (i.e., claims that have not been capped), there is no consideration of the policy limits at which the policies generating claims have been written. The LAS and ILF can be calculated directly from the distribution considering only the layers involved. However, in practice, claim databases typically show the impact of policy features (deductibles, limits) rather than the full uncensored value of the claims. The use of capped data presents a distortion to the distribution used for calculation. The impact of the distortion can be significant. The method in part (a) shows an example of a calculation using capped data for which policy limits are known. If the policy limits were not known and we only had capped data in total, the resulting calculations would be more inaccurate.
4. **Learning Objectives:**

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**


**Commentary on Question:**

This question tests the mechanics and understanding of the frequency-severity claims closure method.

**Solution:**

(a) Estimate total unpaid claims as of December 31, 2013 using the claims closure method.

<table>
<thead>
<tr>
<th>Incremental Paid Severity</th>
<th>AY 12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>1,290</td>
<td>5,482</td>
<td>15,325</td>
</tr>
<tr>
<td>2012</td>
<td>958</td>
<td>4,084</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>1,794</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Incremental Paid Severity = Incremental Paid Claims ÷ Incremental Closed Counts

<table>
<thead>
<tr>
<th>Incremental Paid Severity Adjusted to 2013 Levels</th>
<th>AY 12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>1,382</td>
<td>5,872</td>
<td>16,417</td>
</tr>
<tr>
<td>2012</td>
<td>992</td>
<td>4,227</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>1,794</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg:</td>
<td>1,389</td>
<td>5,050</td>
<td>16,417</td>
</tr>
</tbody>
</table>

i.e. 4,227 = 4,084×1.035

Complete bottom of incremental paid severity:

<table>
<thead>
<tr>
<th>Incremental Paid Severity</th>
<th>AY 12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>1,290</td>
<td>5,482</td>
<td>15,325</td>
</tr>
<tr>
<td>2012</td>
<td>958</td>
<td>4,084</td>
<td>15,862</td>
</tr>
<tr>
<td>2013</td>
<td>1,794</td>
<td>5,050</td>
<td>16,417</td>
</tr>
</tbody>
</table>

i.e. 15,862 = 16,417÷1.035
4. Continued

Complete bottom of incremental closed counts:

<table>
<thead>
<tr>
<th></th>
<th>AY</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>Ultimate Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td></td>
<td>600</td>
<td>280</td>
<td>120</td>
<td>1,000</td>
</tr>
<tr>
<td>2012</td>
<td></td>
<td>660</td>
<td>308</td>
<td>132</td>
<td>1,100</td>
</tr>
<tr>
<td>2013</td>
<td></td>
<td>720</td>
<td>336</td>
<td>144</td>
<td>1,200</td>
</tr>
</tbody>
</table>

% Closed  | 60.0% | 70.0% | 100.0%

i.e. \(0.7 \times (1,200 - 720) = 336\); \(1,100 - 660 - 308 = 132\)

Complete bottom of incremental paid claims:

<table>
<thead>
<tr>
<th></th>
<th>AY</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>Ultimate Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td></td>
<td>774,000</td>
<td>1,535,000</td>
<td>1,839,000</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td></td>
<td>632,000</td>
<td>1,258,000</td>
<td>2,093,784</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td></td>
<td>1,292,000</td>
<td>1,696,800</td>
<td>2,364,048</td>
<td></td>
</tr>
</tbody>
</table>

i.e. \(336 \times 5,050 = 1,696,800\)

Total Unpaid Claims @ Dec 31, 2013:
- 2012: 2,093,784
- 2013: 1,696,800 + 2,364,048 = 4,060,848

(b) Discuss how the following additional information would affect your estimate in part (a):

(i) New legislation lengthens the statute of limitations.

(ii) The company introduces a new system to accelerate claims processing and settlement.

(i) New legislation lengthens the statute of limitations:
- expect proportion closed to be lower at earlier months of development
- since incremental paid severity is higher at later months of development, unpaid claims estimate should be higher since more claims would be expected to be closed at later maturities.

(ii) Company introduces a new system to accelerate claims processing and settlement:
- expect proportion closed to be higher at earlier months of development
- since incremental paid severity is higher at later months of development, unpaid claims estimate should be lower since more claims would be expected to be closed at earlier maturities.
4. **Continued**

(c) Describe a situation in which a frequency and severity method is preferred to other projection methods.

**Commentary on Question:**

*Any one of the following is acceptable.*

- For immature experience periods
- Following the introduction of new GI products when limited or no historical experience is available
- Following entry into a new geographical area for which limited or no historical data exists
- If there have been wide-ranging changes, either internally at the insurer or in the external environment, such that historical relationships and development patterns are not a reliable guide to the future
5. **Learning Objectives:**
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5j) Perform individual risk rating using standard plans.

**Sources:**

**Commentary on Question:**
*This question tests the graphical understanding of retrospective rating.*

**Solution:**
(a) Describe the insurance savings and insurance charge.

The insurance savings at entry ratio $r$ is the expected amount by which the risk's actual loss falls short of $r$ times the expected loss, divided by the expected loss.

The insurance charge at entry ratio $r$ is the expected amount by which the risk's actual loss exceeds $r$ times the expected loss, divided by the expected loss.

(b) Draw a graph with cumulative claim frequency along the x-axis and entry ratio along the y-axis, and identify the areas on the graph corresponding to $\psi(r)$ and $\phi(r)$.
5. Continued

(c) Explain how the graph demonstrates the validity of the fundamental relation above.

\[ \psi(r) = \phi(r) + r \text{ (the lower rectangle)} - 1 \text{ (the area under the curve)} \]

(d) Define \( \psi(r) \) for the limiting case where losses are all equal.

\[ \psi(0) = 0, \ r \leq 1 \ \text{ and } \psi(1) = r - 1, \ r > 1 \]
6. **Learning Objectives:**
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**

**Commentary on Question:**
*This question requires candidates to apply different methods for estimating ultimate salvage.*

**Solution:**
Estimate ultimate salvage for accident year 2013 using two different methods.

**Commentary on Question:**
*Any two of the three methods provided are acceptable. Other methods are possible.*

**Method A: Development of Salvage Claims**
Reported Salvage – Personal Property

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12-24</th>
<th>24-36</th>
<th>36-48</th>
<th>Tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>1.00</td>
<td>0.40</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>3.00</td>
<td>1.70</td>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>3.00</td>
<td>1.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>3.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>3.00</td>
<td>1.70</td>
<td>1.30</td>
<td>1.15</td>
</tr>
<tr>
<td>CDF</td>
<td>7.625</td>
<td>2.542</td>
<td>1.495</td>
<td>1.150</td>
</tr>
</tbody>
</table>

Ultimate salvage for 2013: \(35 \times 3.00 \times 1.70 \times 1.30 \times 1.15 = 35 \times 7.625 = 267\)

**Method B: Ratio of Salvage Reported to Reported Claims Gross of Salvage - Multiplicative**
Reported Claims Gross of Salvage – Personal Property

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12-24</th>
<th>24-36</th>
<th>36-48</th>
<th>Tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>1.00</td>
<td>1.11</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>1.90</td>
<td>1.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>1.90</td>
<td>1.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>1.90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>1.90</td>
<td>1.11</td>
<td>1.01</td>
<td>1.00</td>
</tr>
</tbody>
</table>

2013 Ultimate claims = \(579 \times 1.90 \times 1.11 \times 1.01 = 1,233\)
6. **Continued**

**Ratio of Salvage to Reported Claims Gross of Salvage:**

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>0.042</td>
<td>0.066</td>
<td>0.101</td>
<td>0.130</td>
</tr>
<tr>
<td>2011</td>
<td>0.042</td>
<td>0.066</td>
<td>0.101</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>0.042</td>
<td>0.066</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>0.060</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(i.e. 69 / 1,047 = .066)

**Age-to-Age Factors based on salvage ratios:**

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12-24</th>
<th>24-36</th>
<th>36-48</th>
<th>Tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>1.58</td>
<td>1.53</td>
<td>1.29</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>1.58</td>
<td>1.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>1.58</td>
<td></td>
<td></td>
<td>Tail</td>
</tr>
<tr>
<td>Average</td>
<td>1.58</td>
<td>1.53</td>
<td>1.29</td>
<td>1.15</td>
</tr>
<tr>
<td>CDF</td>
<td>3.586</td>
<td>2.270</td>
<td>1.484</td>
<td>1.150</td>
</tr>
</tbody>
</table>

**Accident Year**

<table>
<thead>
<tr>
<th>Ratio at 31-Dec-13</th>
<th>CDF</th>
<th>Ultimate Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>0.130</td>
<td>1.150</td>
</tr>
<tr>
<td>2011</td>
<td>0.101</td>
<td>1.484</td>
</tr>
<tr>
<td>2012</td>
<td>0.066</td>
<td>2.270</td>
</tr>
<tr>
<td>2013</td>
<td>0.060</td>
<td>3.586</td>
</tr>
</tbody>
</table>

Selected ultimate ratio: 0.150

Estimated salvage = ultimate ratio × ultimate reported gross = 0.15×1,233 = 185

**Method C: Ratio of Salvage Reported to Reported Claims Gross of Salvage - Additive**

2013 Ultimate claims = 579×1.90×1.11×1.01 = 1,233 (same derivation as Method B)

**Ratio of Salvage to Reported Claims Gross of Salvage:** (same as Method B)

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>0.042</td>
<td>0.066</td>
<td>0.101</td>
<td>0.130</td>
</tr>
<tr>
<td>2011</td>
<td>0.042</td>
<td>0.066</td>
<td>0.101</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>0.042</td>
<td>0.066</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>0.060</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(i.e. 69 / 1,047 = .066)
6. Continued

Additive Age-to-Age Factors based on salvage ratios:

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12-24</th>
<th>24-36</th>
<th>36-48</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>0.024</td>
<td>0.035</td>
<td>0.029</td>
</tr>
<tr>
<td>2011</td>
<td>0.024</td>
<td>0.035</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>0.024</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>0.024</td>
<td>0.035</td>
<td>0.029</td>
</tr>
<tr>
<td>CDF</td>
<td>0.108</td>
<td>0.084</td>
<td>0.049</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Ratio at 31-Dec-13</th>
<th>CDF</th>
<th>Ultimate Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>0.130</td>
<td>0.020</td>
<td>0.150</td>
</tr>
<tr>
<td>2011</td>
<td>0.101</td>
<td>0.049</td>
<td>0.150</td>
</tr>
<tr>
<td>2012</td>
<td>0.066</td>
<td>0.084</td>
<td>0.150</td>
</tr>
<tr>
<td>2013</td>
<td>0.060</td>
<td>0.108</td>
<td>0.168</td>
</tr>
</tbody>
</table>

Selected ultimate ratio: 0.150

(2013 seems an outlier, as a result use the 0.15 ratio)

Estimated salvage = ultimate ratio × ultimate reported gross = 0.15×1,233 = 185
7. Learning Objectives:
   3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:
(3a) Estimate unpaid unallocated loss adjustment expenses.

Sources:

Commentary on Question:
This question tests the understanding and the mechanics of estimating unpaid unallocated loss adjustment expenses using several methods.

Solution:
(a) Provide two examples of expense items that are typically unallocated loss adjustment expenses (ULAE) and two examples of expense items that are typically allocated loss adjustment expenses (ALAE).

Commentary on Question:
Any two of the ULAE examples and any two of the ALAE examples are acceptable.

ULAE
- Salaries of claim personnel
- Management/administration cost of claims department
- Cost of facilities related to claims department

ALAE
- Fees for investigation
- Defense attorneys fees
- Cost of medical evaluations
- Cost of expert reviews
- Cost of witnesses
- Cost of record copying

(b) Explain one weakness of the classical paid-to-paid ULAE estimation method using the data from the table above.

Commentary on Question:
Three explanations are given. Any one is acceptable.
7. Continued

- The classical paid-to-paid method is most appropriate for insurers operating in a steady-state environment. In the data the exposure is decreasing, and a ratio calculated on the ratio of paid ULAE to paid claims will tend to understate the true ULAE ratio. We see that the ULAE ratio increases from 9.2% to 11.0% in the given experience period.
- During times of exposure growth, the ULAE ratio is overstated because the numerator is more reactive to the increasing exposures than the denominator.
- During inflationary periods, the classical paid-to-paid method overstates the true ULAE ratio because the influence of inflation is greater on the numerator than it is on the denominator.

(c) Estimate unpaid ULAE as of December 31, 2013 for Simple Insurance Company using the Kittel refinement to the classical paid-to-paid method.

- Select a ratio of ULAE to claims \((2 \times \text{ULAE})/(\text{CY Paid Claims} + \text{CY Reported Claims})\):
  - The three-year average ratio is 10% but the ratio is increasing from 8.8% in 2011 to 11.6% in 2013. Select 11%.
- Calculate unpaid ULAE:
  - Unpaid ULAE = selected ULAE ratio \(\times ((0.5 \times \text{case estimates}) + \text{IBNR})\)
  - Unpaid ULAE = 0.11\(\times((0.5\times900) + 1000) = 159.50\)

(d) Explain the major steps in determining unpaid ULAE using a count-based method.

- Estimate counts
- Select an average ULAE per weighted count
- Project unpaid ULAE
8. Learning Objectives:
7. The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

Learning Outcomes:
(7b) Apply catastrophe models to insurance ratemaking, portfolio management, and risk financing.

Sources:

Commentary on Question:
This question is concerned with insurance portfolio management in catastrophe models.

Solution:
(a) Show that the two probabilities that total losses exceed 250 were correctly calculated.

With 3 removed:
• Events 3 (0.002) and 5 (0.001) exceed 250. Total is 0.003.

With 4 removed:
• Events 1 (0.003) and 3 (0.002) exceed 250. Total is 0.005.

(b) Recommend which of the two portfolios should be dropped. Justify your choice.

Commentary on Question:
Candidate can receive credit for either choosing portfolio 3 or 4. Full credit requires justification for the choice. Choosing portfolio 3 because it has the greatest reduction in probability is not sufficient for full credit.

The following is a justification for removing portfolio 4.

• Mean – favors removing 4 as more business is retained.
  Note: a smaller mean may look better as less is at stake, but SSIC is giving away business in that case.
• Standard deviation – favors removing 3 as variability is reduced.
• Probability – favors removing 3 as probability of high loss is reduced.
• Removing 3 leaves a 900 loss at 0.001; removing 4 leaves a 600 loss at 0.003. The former is more risky.
• The coefficients of variation are 11.86 and 10.89, favors removing 4. The coefficient of variation is a more reliable risk measure than the standard deviation. Removing portfolio 4 removes a key risk yet allows more business to be retained.
(c) Describe each of the following special issues regarding portfolio risk. For each issue, indicate if it is addressed by the analysis performed in part (b) and then support your answer.

(i) Data quality
(ii) Uncertainty modeling
(iii) Impact of correlation

**Commentary on Question:**
Candidate needs to provide description for each special issue and also for noting how it is addressed.

(i) Data Quality
   - Definition: Inaccuracies in the inventory component will lead to errors in the loss amount for each event.
   - Addressed? No. The calculations assume the values are accurate.

(ii) Uncertainty modeling
   - Definition: Use the full distribution, not just the mean.
   - Addressed? Yes. The full set of relevant probabilities is used.

(iii) Impact of correlation
   - Definition: Highly correlated portfolios can increase risk and thus strategies must account for correlation.
   - Addressed? Yes. While no correlations are calculated, dependencies are reflected.
9. Learning Objectives:
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:
(5k) Calculate rates for claims-made coverage.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34.

Commentary on Question:
This question tests the understanding of claims-made ratemaking, and how to calculate the tail factor for a mature claims-made policy.

Solution:
(a) Define retroactive date for policies written on a claims-made basis.

Claims-made coverage is insurance only for events reported during a policy period, subject to a retroactive date.

The retroactive date is the occurrence date after which coverage is in effect for occurrences reported during the term of a claims-made policy.

(b) Give an example of an insurance product for which claims-made coverage is prevalent and explain why this type of coverage is appropriate for that type of risk.

Example: Professional liability.
Why appropriate? There can be a significant delay between the occurrence of the claim and its reporting to the insurer.

(c) Compare claims-made and occurrence coverage on the following features:

(i) Cost, given that the underlying frequency and severity are increasing.

(ii) Precision in pricing, given sudden unpredictable changes in trend or reporting pattern.

(iii) Opportunity to earn investment income.
9. Continued

(i) Claims-made would cost less.
(ii) Claims-made would be more precise.
(iii) Claims-made would present less opportunity for investment income.

(d) Explain how coverage gaps can occur for insureds purchasing claims-made coverage by providing two examples.

Commentary on Question:
Any two of the following are acceptable.

- When an insured switches from claims-made with one company to claims made with another, there may be unreported occurrences not covered by either policy.
- When an insured with claims-made coverage switches to occurrence, claims reported after the expiration of the claims-made coverage that occurred before the inception of the occurrence coverage are not covered.
- When an insured with claims-made coverage discontinues coverage, there is no coverage for claims reported after the coverage ends.

(e) Calculate the tail factor for a mature claims-made policy given a pure premium of 1,000 for occurrence coverage, a 10% annual pure premium trend, and a claims reporting pattern of 50%, 30%, 20%.

\[
\begin{array}{c|c|c|c}
\text{Lag} & \text{n} & \text{n+1} & \text{n+2} \\
0 & 500.00 & & \\
1 & 272.73 & 300.00 & \\
2 & 165.29 & 181.82 & 200.00 \\
\end{array}
\]

(i.e. \(181.82 = 1,000 \times 0.20 \div 1.1 \))

Tail factor for mature claims-made policy:
\[
\frac{300 + 181.82 + 200}{500 + 272.73 + 165.29} = 0.73
\]
10. **Learning Objectives:**

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**


**Commentary on Question:**

*This question tests the mechanics of the Berquist-Sherman adjustments when there have been changes to case reserves and also the understanding of the Berquist-Sherman adjustments needed when there have been changes to case reserves and settlement rates.*

**Solution:**

(a) Calculate the projected ultimate claims using the Berquist-Sherman method for XYZ Insurer.

#### Step 1: Calculate adjusted average case estimates

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Adjusted Average Case Estimates</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
<td>24</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>6,531</td>
<td>8,048</td>
<td>8,940</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>6,857</td>
<td>8,450</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>7,200</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(i.e. 7,200÷1.05 = 6,857)

#### Step 2: Calculate adjusted reported claims

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Adjusted Reported Claims</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
<td>24</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>1,981,510</td>
<td>3,248,400</td>
<td>3,875,800</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>2,175,680</td>
<td>3,245,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>2,370,000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(i.e. 2,175,680 = 6,857×240 + 530,000)

#### Step 3: Calculate development factors

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12-24</th>
<th>24-36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>1.639</td>
<td>1.193</td>
</tr>
<tr>
<td>2012</td>
<td>1.492</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>1.566</td>
<td>1.193</td>
</tr>
<tr>
<td>CDF</td>
<td>2.229</td>
<td>1.423</td>
</tr>
</tbody>
</table>

(i.e. $1.639 = 3,248,400÷1,981,510$. Bondy method: tail factor = last development factor.)
10. Continued

Step 4: Calculate projected ultimate claims

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Reported Claims</th>
<th>CDF</th>
<th>Projected Ultimate Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>3,875,800</td>
<td>1.193</td>
<td>4,623,829</td>
</tr>
<tr>
<td>2012</td>
<td>3,245,500</td>
<td>1.423</td>
<td>4,618,347</td>
</tr>
<tr>
<td>2013</td>
<td>2,370,000</td>
<td>2.229</td>
<td>5,282,730</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>14,524,906</td>
</tr>
</tbody>
</table>

(b) Explain how you create the reported claims triangle with the Berquist-Sherman adjustments for changes in both case estimates and settlement rates.

First, determine adjusted open counts:

\[ \text{Adjusted Open Counts} = (\text{Original reported counts}) - (\text{Closed counts adjusted as part of the adjustment for settlement rates}) \]

Second, adjusted reported claims:

\[ \text{Adjusted Reported Claims} = (\text{Adjusted Open Counts}) \times (\text{Adjusted Average Case Estimates}) + (\text{Adjusted Paid Claims}) \]
11. Learning Objectives:
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

Learning Outcomes:
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

Sources:

Commentary on Question:
The question tests the mechanics of the Cape Cod method of estimating ultimate claims. Candidates also need to understand the key components of the actuarial control cycle and how the actuarial control cycle can be illustrated by the estimation of trend rates used in the reserving and ratemaking processes.

Solution:
(a) Estimate the ultimate claims for this line of business using the Cape Cod method.

Commentary on Question:
Candidate must use the used-up exposures in determining the pure premium based on adjusted claims to get full credit. Column (8) in the table below only determines the expected claims. Ultimate claims (column 11) are determined by adding the reported claims (column 5) to the expected unreported claims (column 10).

- Cumulative development factors:

<table>
<thead>
<tr>
<th>Development Period</th>
<th>Age-to-Age Development Factor</th>
<th>Age-to-Ultimate Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-24</td>
<td>1.50</td>
<td>2.302</td>
</tr>
<tr>
<td>24-36</td>
<td>1.28</td>
<td>1.534</td>
</tr>
<tr>
<td>36-48</td>
<td>1.13</td>
<td>1.199</td>
</tr>
<tr>
<td>48-60</td>
<td>1.04</td>
<td>1.061</td>
</tr>
<tr>
<td>Tail factor</td>
<td>1.02</td>
<td>1.020</td>
</tr>
</tbody>
</table>

Pure Premium Trend: $1.022 \times 1.045 - 1 = 6.8\%$
11. Continued

Ultimate claims:

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>CDF</th>
<th>Expected % Reported</th>
<th>Expected Earned Exposure</th>
<th>Used-Up Earned Exposure</th>
<th>Reported Claims</th>
<th>Pure Premium Trend @6.8%</th>
<th>Adjusted Reported Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>1.19</td>
<td>83.4%</td>
<td>5,580</td>
<td>4,655</td>
<td>702</td>
<td>1.141</td>
<td>801</td>
</tr>
<tr>
<td></td>
<td>1.53</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>4</td>
<td>65.2%</td>
<td>5,670</td>
<td>3,695</td>
<td>545</td>
<td>1.068</td>
<td>582</td>
</tr>
<tr>
<td></td>
<td>2.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>2</td>
<td>43.4%</td>
<td>5,460</td>
<td>2,372</td>
<td>515</td>
<td>1.000</td>
<td>515</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>10,723</td>
<td>1,762</td>
<td></td>
<td></td>
<td>1,898</td>
</tr>
</tbody>
</table>

(A) Pure Premium based on Adjusted Claims: \( \frac{1,000 \times \text{Total}(7)}{\text{Total}(4)} \) = 177

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Expected Claims</th>
<th>Expected % Unreported</th>
<th>Expected Unreported</th>
<th>Projected Ultimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>866</td>
<td>16.6%</td>
<td>143</td>
<td>845</td>
</tr>
<tr>
<td>2012</td>
<td>940</td>
<td>34.8%</td>
<td>327</td>
<td>872</td>
</tr>
<tr>
<td>2013</td>
<td>966</td>
<td>56.6%</td>
<td>547</td>
<td>1,062</td>
</tr>
<tr>
<td>Total</td>
<td>2,772</td>
<td>1,017</td>
<td></td>
<td>2,779</td>
</tr>
</tbody>
</table>

(b) Identify the three key components of the actuarial control cycle and illustrate with the selection and use of trend rates in reserving and ratemaking.

Three components of the control cycle:
- Define the problem
- Design the solution
- Monitor the results

Actuaries require an estimate of ultimate claims for pricing; but actuaries projecting ultimate claims require an estimate of trend, which is typically derived during the pricing analysis.

The circular nature of these requirements and the information sharing are important aspects of the actuarial control cycle.
12. Learning Objectives:
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

Learning Outcomes:
(2d) Identify the various changing conditions that affect the determination of ultimate claims.

Sources:

Commentary on Question:
This question tests the understanding of how various methods of estimating ultimate claims are affected by changing conditions.

Solution:
(a) Explain whether you expect the estimate of ultimate claims to understate, overstate or be similar to actual ultimate claims for book of business 1, for the following methods:

(i) The expected method

(ii) The development method on reported claims

(iii) The Bornhuetter Ferguson method on reported claims

Commentary on Question:
Candidates need to state whether the estimate of ultimate claims would understate, overstate or be similar to actual claims, as well as provide the explanation.

(i) Expected method:
- Expected to understate claims under this scenario.
- Reason: This method is responsive to a change in volume but is not responsive to a change in the overall performance of claims.

(ii) Development method:
- Expect this method would be similar to actual ultimate claims.
- Reason: The deteriorating claims would be accounted for with the development method.

(iii) Bornhuetter Ferguson method:
- The projected ultimate claims from the BF method will be understated.
- Reason: The estimate of ultimate claims = actual observed experience + expected unobserved (“undeveloped”) experience. Therefore, the projected ultimate claims from the BF method will be understated if claims are deteriorating without an explicit increase in the expected claim ratio.
12. Continued

(b) Explain whether you expect the estimate of ultimate claims to understate, overstate or be similar to actual ultimate claims for book of business 2, for the following methods:

(i) The expected method
(ii) The development method on reported claims
(iii) The Bornhuetter Ferguson method on reported claims

Commentary on Question:
Candidates need to state whether the estimate of ultimate claims would understate, overstate or be similar to actual claims, as well as provide the explanation.

(i) Expected method:
   • Expect this method would be similar to actual ultimate claims.
   • Reason: This method is not affected by a change in case estimates.

(ii) Development method:
   • Expect the development method to overstate actual ultimate claims.
   • Reason: A higher proportion of ultimate claims are now reported earlier than in the past, and lower cumulative development factors would be required to project the reported claims to an ultimate basis.

(iii) Bornhuetter Ferguson method:
   • Expect the BF method to overstate actual ultimate claims.
   • Reason: (similar to development method) A higher proportion of ultimate claims are now reported earlier than in the past, and lower cumulative development factors would be required to project the unreported claims.
13. Learning Objectives:
1. The candidate will understand the key considerations for general insurance actuarial analysis.

Learning Outcomes:
(11) Adjust historical earned premiums to current rate levels.

Sources:

Commentary on Question:
This question tests the fundamental understanding of how to adjust premiums to current rate level.

Solution:
(a) State the key assumption that underlies the parallelogram method.

The key assumption is that exposures are uniformly distributed over time.

(b) Calculate the on-level factor to be used to adjust calendar year 2011 earned premium to current rates.

Figure for 2011:
13.  Continued

- Area for B = 0.5×0.5×.05 = 0.125
- Area for A = 1 – 0.125 = 0.875
- Weighted average rate level for 2011 = (1.0×0.875) + (1.07×0.125) = 1.00875
- New discount impact needs to be included in rate level relative value: impact is a decrease in overall premium of 20%×10% = 2%
- Current rate level relative value = 1×1.07×0.98×0.97 = 1.017142
- On-level factor = 1.017142÷1.00875 = 1.00832

(c) Explain how you would recognize a state-mandated change in minimum policy limits in the on-level calculation

For a state-mandated change in minimum policy limits, the average premium would increase or decrease to reflect such a change but there is also the expectation that claims would increase or decrease as well as policyholders would receive more or less coverage. As a result, the change should have no effect on the on-level calculation.
14. **Learning Objectives:**
1. The candidate will understand the key considerations for general insurance actuarial analysis.

**Learning Outcomes:**
(1j) Create a claims development triangle from claims transaction data.

**Sources:**

**Commentary on Question:**
*This question tests the fundamental understanding of how claims triangles are constructed.*

**Solution:**
Restate the four data triangles to include the transactions from the subsequently provided claim transaction data.

**Commentary on Question:**
*Summarizing the claim data for each claim is helpful in determining the changes necessary to the four data triangles.*

Summary of cumulative claim data for Claim #1: Accident Year = 2010, reported in 2010, closed in 2012.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Paid to date</td>
<td>50</td>
<td>150</td>
<td>210</td>
<td>210</td>
</tr>
<tr>
<td>Case estimate</td>
<td>150</td>
<td>60</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Reported claims</td>
<td>200</td>
<td>210</td>
<td>210</td>
<td>210</td>
</tr>
<tr>
<td>Reported count</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Closed count</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Summary of cumulative claim data for Claim #2: Accident Year = 2010, reported in 2011, closed in 2011.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Paid to date</td>
<td>0</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Case estimate</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Reported claims</td>
<td>0</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Reported count</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Closed count</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
14. Continued

Summary of cumulative claim data for Claim #3: Accident Year = 2011, reported in 2012, still open at December 31, 2013.

<table>
<thead>
<tr>
<th>Data Type</th>
<th>Dec 31, 2011</th>
<th>Dec 31, 2012</th>
<th>Dec 31, 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paid to date</td>
<td>0</td>
<td>140</td>
<td>140</td>
</tr>
<tr>
<td>Case estimate</td>
<td>0</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Reported claims</td>
<td>0</td>
<td>290</td>
<td>290</td>
</tr>
<tr>
<td>Reported count</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Closed count</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Restated triangles (Note: only accident years 2010 and 2011 have changes):  

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Cumulative Paid Claims</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>2,200</td>
<td>4,750</td>
<td>6,780</td>
<td>7,950</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>2,460</td>
<td>5,330</td>
<td>7,590</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>2,370</td>
<td>4,890</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>3,260</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(i.e. 4,750 = 4,510 + 150 + 90)

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Cumulative Reported Claims</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>7,210</td>
<td>8,810</td>
<td>9,860</td>
<td>10,190</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>8,320</td>
<td>10,630</td>
<td>11,440</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>9,610</td>
<td>11,620</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>9,620</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(i.e. 10,630 = 10,340 + 290)

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Cumulative Closed Counts</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>22</td>
<td>40</td>
<td>56</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>20</td>
<td>44</td>
<td>62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>25</td>
<td>47</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>29</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Cumulative Reported Counts</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>77</td>
<td>81</td>
<td>84</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>82</td>
<td>94</td>
<td>95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>98</td>
<td>108</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>90</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
15. **Learning Objectives:**

4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

**Learning Outcomes:**

(4d) Calculate premium trend and apply it to project premiums.

**Sources:**


**Commentary on Question:**

This question tests the fundamental trend adjustments to premium.

**Solution:**

(a) Explain the purpose of premium trend adjustments

The purpose of premium trend adjustments is to adjust premium from historical periods to a future rating period. Adjustments account for inflation-sensitive exposures and changes in mix of business and rating characteristics.

(b) Calculate and select an annual trend due to the shift in policy limits.

<table>
<thead>
<tr>
<th>Calendar Year Experience Period</th>
<th>Weighted Average Increased Limits Factor</th>
<th>Annual Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>0.24×0.90 + 0.52×1.00 + 0.24×1.15 = 1.012</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>0.22×0.90 + 0.52×1.00 + 0.26×1.15 = 1.017</td>
<td>0.49%</td>
</tr>
<tr>
<td>2013</td>
<td>0.20×0.90 + 0.52×1.00 + 0.28×1.15 = 1.022</td>
<td>0.49%</td>
</tr>
</tbody>
</table>

Selected trend 0.49%

(c) Calculate the trend factor to be used for 2012 earned premium using the annual trend selected in part (b).

**Commentary on Question:**

The provided solution interpolates between the two dates. It is also an acceptable solution to interpolate between two trend factors.

Average earned date in experience period = 7/1/2012
Average earned date in forecast period for annual policies = 9/1/2015
Average earned date in forecast period for 6-month policies = 6/1/2015
Average earned date in forecast period: 67% of 9/1/2015 & 33% of 6/1/2015 = 8/1/2015
Trending period in years = 7/1/2012 to 8/1/2015 = 3 + 1/12 = 3.083 years
Premium trend factor = 1.0049^{3.083} = 1.015
16. Learning Objectives:
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:
(5g) Calculate risk classification changes and territorial changes.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 32.

Commentary on Question:
This question tests the candidate’s understanding of classification ratemaking. It requires the candidate to incorporate credibility and also understand the inputs to a minimum bias calculation.

Solution:
(a) Calculate the indicated rating relativities using the pure premium one-way analysis procedure. The indicated relativities should be shown so that the base territory A has no change.

<table>
<thead>
<tr>
<th>Territory</th>
<th>Exposures</th>
<th>Written</th>
<th>Trended</th>
<th>Indicated</th>
<th>Ultimate</th>
<th>Credibility</th>
<th>Complement</th>
<th>PP Relativity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ult PP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Actual</td>
<td>Rebalanced</td>
</tr>
<tr>
<td>A</td>
<td>15,200</td>
<td>450.00</td>
<td>0.949</td>
<td>1.200</td>
<td>100.0%</td>
<td>1.000</td>
<td>0.988</td>
<td>0.949</td>
</tr>
<tr>
<td>B</td>
<td>12,400</td>
<td>475.00</td>
<td>1.002</td>
<td>729</td>
<td>90.0%</td>
<td>0.950</td>
<td>0.939</td>
<td>0.996</td>
</tr>
<tr>
<td>C</td>
<td>10,700</td>
<td>507.00</td>
<td>1.070</td>
<td>635</td>
<td>84.0%</td>
<td>1.100</td>
<td>1.087</td>
<td>1.072</td>
</tr>
<tr>
<td>Total</td>
<td>38,300</td>
<td>474.02</td>
<td>1.000</td>
<td>2,564</td>
<td>1.012</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
(4) = (3) / (3)_{total}
(6) = \sqrt{(15/900)}; max of 1.0
(7)_{total} = \sum((2)\times(7)) / (2)_{total}
(8) = (7) / (7)_{total}
(9) = (4)(6) + [1-(6)](8)
(10) = (9) / (9)_{A}

(b) Calculate the first set of age-of-home factors using the minimum bias procedure. Use the existing territory relativities as inputs to the calculation.

Commentary on Question:
This part of the question follows the approach outlined in section 32.10.3 of the Friedland text.
16. Continued

Total Expected Claims:
Age of Home:
0-15 years: $7,600 \times 390 + 4,960 \times 400 + 6,420 \times 461 = 7,907,620$
16+ years: $7,600 \times 510 + 7,440 \times 525 + 4,280 \times 576 = 10,247,280$

First iteration of values for Age of Home:
0-15 years:

\[
\frac{7,907,620}{(7,600 \times 1.00 + 4,960 \times 0.95 + 6,420 \times 1.10)} = 0.861 \times 474.02
\]

16+ years:

\[
\frac{10,247,280}{(7,600 \times 1.00 + 7,440 \times 0.95 + 4,280 \times 1.10)} = 1.116 \times 474.02
\]

(c) Explain why you expect the rating factors for territory calculated using the one-way procedure to be the same or different than the rating factors for territory calculated using the minimum bias procedure. In your response, give two reasons.

The values are expected to be different between the two procedures. Reasons:

- There is distributional bias (the distribution of age of home in each territory is not the same).
- Credibility is considered in the one-way procedure and is not considered in the minimum bias procedure.
17. **Learning Objectives:**
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**
(2a) Use loss development triangles for investigative testing.

**Sources:**

**Commentary on Question:**
*This question is concerned with identifying potential issues with data triangles and what diagnostic tests can be used on data triangles.*

**Solution:**
(a) State two observations about the pattern of the ratios of paid claims to reported claims in the above table.

- Ratio for the first half of each year is always significantly higher than the second half in the first 6 months development.
- The ratio for the most recent diagonal is significantly lower.

(b) Explain a possible cause of each observation from part (a).

**Commentary on Question:**
The observation and possible cause need to be correctly identified. For example, the ratio for the first half of each year always being significantly higher than the second half is the observation. The possible cause of this observation is seasonality.

- First half of year ratio higher implies seasonality.
- Most recent diagonal lower suggests possible reserve strengthening or possibly change in the settlement pattern.

(c) State two other diagnostics you would review to confirm your observations from part (a), and describe the patterns you expect to see for each diagnostic.

**Commentary on Question:**
*Other diagnostics, not provided below, are acceptable. Candidates need to state the diagnostic and then describe the pattern that is expected.*

*Average reported claims:*
In a stable environment, expect values to be relatively consistent at each maturity age, with changes down each column (from accident year to accident year) limited to the rate of trend only.
17. Continued

*Ratios of closed to reported counts:*
Used in conjunction with the ratios of paid to reported claims. Expect a similar pattern to exist between the ratios of closed to reported counts and the ratios of paid to reported claims in a stable environment.

(d) State two examples of actions that could result in shifts in the average reported claims.

**Commentary on Question:**
*Five are provided; any two are acceptable.*

- New procedures for the payment of claims such as direct deposit to a claimant's bank account instead of issuance of checks.
- New philosophies about establishment of case estimates such as explicit consideration of S&S recoveries.
- Changes in the distribution of policy limits purchased by insureds.
- Changes in the use of partial settlements or ex gratia payments.
- Shifts in the attitude toward defense of questionable claim files.
18. **Learning Objectives:**
3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

**Learning Outcomes:**
(3a) Estimate unpaid unallocated loss adjustment expenses.
(3c) Evaluate the estimates of ultimate claims to determine claim liabilities for financial reporting.
(3d) Describe components of premium liabilities.
(3e) Evaluate premium liabilities.

**Sources:**

**Commentary on Question:**
This question tests the estimation of unpaid claims, unpaid unallocated loss adjustment expenses and premium liabilities.

**Solution:**
(a) Calculate total unpaid claims as of December 31, 2013, including unpaid ULAE.

\[
\begin{align*}
\text{Case estimates } [88,300 - 72,400] &= 15,900 \\
\text{Indicated IBNR } [\text{total ultimate } - 88,300] &= 13,850 \\
\text{ULAE } &\left[ (8\% \times 2) + (8\% \times 50\% \times 1) \right] = 1,744 \\
\text{Total unpaid claims } &\left( 1 + 2 + 3 \right) = 31,494
\end{align*}
\]

(b) State two points the actuary should consider when selecting claim ratios to be used for calculating premium liabilities.

In selecting claim ratios, the actuary should consider:
- Any recent actions internally at the insurer, such as recent rate changes, and
- Any external factors (such as trend) that could influence the claim experience over the next year.

(c) Calculate the premium liabilities as of December 31, 2013.

**Commentary on Question:**
Premium liabilities are determined using the expected claims from the unearned premiums and not the unpaid claims determined in part (a).
18. Continued

First need to selected claim ratio and general expense ratio:

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Claim Ratio</th>
<th>General Expense Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>72.0%</td>
<td>14.50%</td>
</tr>
<tr>
<td>2012</td>
<td>73.0%</td>
<td>14.29%</td>
</tr>
<tr>
<td>2013</td>
<td>71.0%</td>
<td>14.40%</td>
</tr>
<tr>
<td>Total</td>
<td>72.0%</td>
<td>14.40%</td>
</tr>
<tr>
<td>Selected:</td>
<td>72.0%</td>
<td>14.40%</td>
</tr>
</tbody>
</table>

Unearned premiums: 32,600
Expected claim ratio: 72%
Expected claims = 32,600×0.72 = 23,472
ULAE ratio: 8.0%
Expected ULAE = 23,472×0.08 = 1,878
Total expected claims and LAE = 23,472 + 1,878 = 25,350
Selected maintenance expense ratio = 0.144×0.250 = 3.6%
Maintenance expenses = 32,600×0.036 = 1,174
Total claims and expenses (Premium Liabilities) = 25,350 + 1,174 = 26,523
Equity in unearned premium = 32,600 – 26,523 = 6,077

(d) Determine either the premium deficiency reserve or the equity in the unearned premium.

Equity in unearned premium = Unearned premium – Premium liabilities
= 32,600 – 26,523 = 6,077
Since it is positive, there is no premium deficiency.
19. **Learning Objectives:**

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

7. The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

**Learning Outcomes:**

(5d) Calculate loadings for catastrophes and large claims.

(7a) Describe the structure of catastrophe models.

**Sources:**


**Commentary on Question:**

*This question tests the understanding of the structure of catastrophe models, as well as establishing claim loadings.*

**Solution:**

(a) Describe each of the components.

- Hazard module: The module assigns probabilities of an event by location. It also provides a model for the severity and propagation of an event.
- Inventory module: The nature of the buildings in the area as a detailed census. Construction type is the most important characteristic.
- Vulnerability module: Estimates the level of building damage expected for differing severities of event.

(b) Indicate similarities (if any) and differences (if any) between the CommCo and HomeCo implementations of an earthquake model for each component. Justify each of your answers.

- Hazard module:
  - No difference. The events are identical regardless of the insurance coverage.
- Inventory module:
  - Completely different. One is an inventory of homes and the other of businesses.
19. Continued

Vulnerability module:
- There will be similarities as the module is based on construction types and other building characteristics.

(c) Compare the analyses required to establish the claims loading for hurricane and non-hurricane weather claims for HomeCo.

Non-hurricane weather claims loading:
- Analyze historical claims data to estimate loading

Hurricane claims loading:
- Rely on estimates from a catastrophe model (i.e. simulation model).
- Models simulate the event, translate into a damage ratio & damage ratios are applied to current or projected amounts of insurance and produce the expected catastrophe loss estimate.
20. **Learning Objectives:**

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**

(5i) Calculate rates for large accounts.

**Sources:**


**Commentary on Question:**

This question tests the understanding of experience rating plans.

**Solution:**

(a) Define the following forms of general insurance rating:

(i) Manual

(ii) Schedule

(iii) Prospective experience

(iv) Retrospective experience

(i) Manual rating: Deriving a premium solely from multiplying the exposures by the manual rate and any applicable rating factors.

(ii) Schedule rating: A program in which manual rates are adjusted, either upward (debts) or downward (credits), to reflect the insured’s risk characteristics such as its safety program, financial strength, and overall management capabilities. The judgmental rating factors are used to distinguish a specific insured from the average insured in its class using characteristics that are not already recognized in the rating process.

(iii) Prospective experience rating: Rating where the premium depends on historical experience prior to the policy period and a rating formula.

(iv) Retrospective experience rating: Rating where the premium charged in a policy period depends upon the claims experience in the policy period and a rating formula. There is usually a deposit premium with the final premium subject to a minimum and maximum.
20. Continued

(b) Explain why retrospective experience rating is typically not appropriate for insureds with small premium size or poor claims experience.

Commentary on Question:
A brief sentence covering each is sufficient for full credit.

Insureds with small premium size are likely to have variable claims experience and one large loss may result in a maximum premium.

Insureds with poor claims experience will pay greater than average premium and could have losses resulting in a maximum premium.

(c) Determine allocation percentages for Centre and Exurb based on the agreed allocation procedure after the change in claims definition described above, given the additional information below:

<table>
<thead>
<tr>
<th>Pool Participant</th>
<th>Population Estimate</th>
<th>Ultimate Claims (Including ALAE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre</td>
<td>435,600</td>
<td>550,000</td>
</tr>
<tr>
<td>Exurb</td>
<td>250,000</td>
<td>700,000</td>
</tr>
<tr>
<td>Total</td>
<td>685,600</td>
<td>1,250,000</td>
</tr>
</tbody>
</table>

Calculate the credibility for both participants:
- Centre: \((435,600/1,000,000)^{0.5} = 0.66\)
- Exurb: \((250,000/1,000,000)^{0.5} = 0.50\)

Calculate the experience modification for each participant using the formula experience modification factor,
\[\frac{(% \text{claims} / % \text{exposure}) \times \text{credibility}}{1 - \text{credibility}}\]
- Centre: \(\frac{(550,000 \div 1,250,000) \times 0.66 + 0.34 = 0.797}{(435,600 \div 685,600)}\)
- Exurb: \(\frac{(700,000 \div 1,250,000) \times 0.50 + 0.50 = 1.268}{(250,000 \div 685,600)}\)

Calculate the percentage allocated claims for Centre using the formula,
\[\frac{\text{experience mod Centre} \times \% \text{exposure for Centre}}{\text{((Experience mod Centre} \times \% \text{exposure Centre}) + (\text{Experience mod Exurb} \times \% \text{Exposure Exurb}))}\]
- Centre: \(\frac{0.797 \times 0.635}{(0.797 \times 0.635) + (1.268 \times 0.365)} = 52.2\%\)
20.  **Continued**

Calculate the allocation for Exurb by subtracting the portion for Centre from 100%:
- Exurb: 100% – 52.2% = 47.8%

(d) Explain the importance of understanding the distribution of ALAE in the allocation of retained claims and expenses to participants in self-insurance pools.

The distribution of ALAE for participants may be different from the distribution of claims excluding ALAE. Thus, the calculated experience modification factors can be different depending on the treatment of ALAE in the definition of claims.
21. **Learning Objectives:**
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5b) Calculate expenses used in ratemaking analyses.
(5f) Calculate overall rate change indications under the claims ratio and pure premium methods.

**Sources:**

**Commentary on Question:**
This question tests the allocation of fixed and variable expenses and the calculation of overall rate indications. The candidate needs to be able to explain the importance of recognizing the appropriate split of fixed and variable expenses.

**Solution:**
(a) Explain the importance of recognizing the appropriate split between fixed and variable expenses in ratemaking.

**Commentary on Question:**
Any of the following points are acceptable.

- U.S. Standards specifically require actuaries to consider whether expenses should be split into fixed and variable components.
- Rates based solely on variable expenses can lead to inadequate expense provisions for those insureds with a relatively low premium and excessive provisions for those insureds with an exceptionally high premium.
- On an aggregate basis, using an all-variable expense approach will cause the fixed expense provision to be overstated/(understated) when the pricing indicates an increase/(decrease) in the rates.

(b) Calculate the indicated rate under the initial assumption for the split of general and other acquisition expenses.

Initial indicated rate:
\[
\frac{350 \times 1.09 + 100}{1 - 0.16 - 0.06 - 0.05} = 659.59
\]

(c) Calculate the revised fixed and variable general and other acquisition expenses.
21. Continued

Variable portion of general & other acquisition (initial) = \(6\%\) of premium = 0.06 \(\times\) \(125,000,000\) = \(7,500,000\)
Variable was initially assumed to be \(1/3\) of total general & other acquisition, therefore total general & other acquisition = \(3 \times 7,500,000 = 22,500,000\)

Revised split: variable = \(2/3\), fixed = \(1/3\):
Variable = \(2 \times\) previous = 12%
Fixed = \(\frac{22,500,000 \times 1/3}{250,000}\) = 30 per exposure

Commentary on Question:
Candidate needs to recognize that the initial total fixed expense per exposure (100) includes fixed expenses other than general and other acquisition expenses, in this case, 40 = 100 – 60.

Original fixed general & other acquisition expense = 30 \(\times\) 2 = 60 per exposure
Revised total fixed expense = 30 + (100 – 60) = 70 per exposure
Revised indicated rate = \(\frac{350 \times 1.09 + 70}{1 - 0.16 - 0.12 - 0.05}\) = 673.88
22. **Learning Objectives:**

4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

**Learning Outcomes:**

(4b) Calculate loss trend and apply it to project ultimate claims.

**Sources:**


**Commentary on Question:**

*This question tests the claim trend calculation and how portfolio changes can affect both frequency and severity.*

**Solution:**

(a) Explain how portfolio changes in policy deductibles can affect both frequency and severity.

- **Frequency:** Shifts in a portfolio toward higher deductibles will tend to decrease frequency, and shifts in a portfolio toward lower deductibles will tend to increase frequency.
- **Severity:** There is no fixed relationship between the level of deductible and the severity.

(b) Describe two options to consider when experience is not fully credible for trending.

**Commentary on Question:**

*Three are provided. Any two are acceptable.*

- Rely on industry data for a similar line of business in a similar jurisdiction.
- Combine the insurer’s experience in specific states or provinces with the experience of a larger region.
- Combine the insurer’s experience with that of other insurers in a group under common ownership.

(c) Calculate the pure premium trend factors for each year in the experience period.

\[
\text{Pure premium trend} = (1 - .012)(1 + .058) - 1 = 4.53\%.
\]

*Average accident dates for experience period: average accident date for each accident year, or July 1 each year.*

*Average accident date for forecast period: average accident date for policies written between June 1, 2014 and May 31, 2015 = March 1, 2015.*
22. Continued

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Experience Period</th>
<th>Forecast Period</th>
<th>Trending Period in Months</th>
<th>PP Trend Factor @4.53%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>7/1/2012</td>
<td>3/1/2015</td>
<td>32</td>
<td>1.125</td>
</tr>
<tr>
<td>2013</td>
<td>7/1/2013</td>
<td>3/1/2015</td>
<td>20</td>
<td>1.077</td>
</tr>
</tbody>
</table>

i.e., \(1.0453^{32/12} = 1.125\)

(d) Explain how the trend factors calculated in part (c) would be calculated if you are pricing a single large policy that renews on June 1, 2014 for a two-year term.

Since this is only one policy, the average accident date for the forecast period is the average of the period that runs from June 1, 2014 through May 31, 2016, or June 1, 2015. The average accident dates for the experience period are unchanged. The resulting trending periods in months will be 35 and 23, for 2012 and 2013, respectively.