1. **Learning Objectives:**
   1. The candidate will understand the key considerations for general insurance actuarial analysis.

**Learning Outcomes:**
(1) Adjust historical earned premiums to current rate levels.

**Sources:**

**Commentary on Question:**
*This question tests the candidate’s understanding of certain details of individual insurance policies and ability to make correct calculations of earned exposures, earned premium, unearned premium and written premium for various policies. The candidate also needs to understand earned premiums adjusted to current rate level.*

**Solution:**
(a) Calculate the 2012 and 2013 calendar year total earned premiums.

<table>
<thead>
<tr>
<th>Policy 1</th>
<th>Policy 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Renew Date</td>
<td>Written Premium</td>
</tr>
<tr>
<td>April 1/10</td>
<td>900</td>
</tr>
<tr>
<td>April 1/11</td>
<td>900</td>
</tr>
<tr>
<td>April 1/12</td>
<td>900</td>
</tr>
<tr>
<td>April 1/13</td>
<td>900×1.05 = 945</td>
</tr>
</tbody>
</table>

2012 earned premium:
Policy 1 = 900
Policy 2 = 1,200×\(\frac{8}{12}\) + 1,260×\(\frac{4}{12}\) = 1,220
Total = 2,120
1. Continued

2013 earned premium:
Policy 1 = $900 \times \frac{3}{12} + 945 \times \frac{9}{12} = 933.75$
Policy 2 = $1,260 \times \frac{2}{12} = 210$
Total = 1,143.75

(b) Calculate the 2012 calendar year total written premiums.

CY 2012 total written premiums = $900 + 1,260 = 2,160$

(c) Calculate the total unearned premiums as of December 31, 2012.

Total unearned premiums at December 31, 2012
= $900 \times \frac{3}{12} + 1,260 \times \frac{8}{12} = 1,065$

(d) Calculate the 2012 total earned premiums adjusted to current rate level.

2012 on level earned premiums = $945 + 1,260 = 2,205$
2. **Learning Objectives:**
   2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.
   
   3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

**Learning Outcomes:**

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

(3d) Evaluate the estimates of ultimate claims to determine claim liabilities for financial reporting.

**Sources:**


**Commentary on Question:**

*This question is testing the ability to apply the development method of estimating ultimate claims, while adjusting for large claims. The question also tests the ability to estimate the claim liability and the candidate’s understanding of the estimation of the tail factor using Boor’s algebraic method.*

**Solution:**

(a) Calculate the estimated ultimate claims for accident years 2011 and 2013, using the reported development method with the original Bondy method for the tail factor.

**Commentary on Question:**

*There are two approaches that can be used to solve part (a). The text demonstrates an approach where the cumulative reported claims are not restated and no development factors are applied to the large claims. The more commonly applied approach by the candidates is shown here.*

Approach: Remove large claims from cumulative reported claims to calculate development factors.
2. Continued

Cumulative Reported Claims - Excluding Large Claims

<table>
<thead>
<tr>
<th>AY</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>44,200</td>
<td>62,600</td>
<td>79,200</td>
<td>82,000</td>
</tr>
<tr>
<td>2011</td>
<td>42,300</td>
<td>63,600</td>
<td>79,700</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>55,000</td>
<td>78,600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>52,600</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Development factors:

<table>
<thead>
<tr>
<th></th>
<th>12-24</th>
<th>24-36</th>
<th>36-48</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>1.416</td>
<td>1.265</td>
<td>1.035</td>
</tr>
<tr>
<td>2011</td>
<td>1.504</td>
<td>1.253</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>1.429</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tail (Bondy)

| Age-to-age | 1.450 | 1.259 | 1.035 | 1.035 |
| To ultimate| 1.956 | 1.349 | 1.071 | 1.035 |

Ultimate claims:

<table>
<thead>
<tr>
<th>AY</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>79,700×1.071 + 7,200 = 92,559</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>52,600×1.956 + 8,200 = 111,086</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Estimate the accident year 2011 case estimate, IBNR, and total claim liability.

Estimated ultimate claims from part (a): 92,559
Case estimate = Reported – Paid to date = 86,900 – 67,700 = 19,200
IBNR = Ultimate – Reported = 92,559 – 86,900 = 5,659
Total claim liability = Case + IBNR = 19,200 + 5,659 = 24,859

(c) Describe the axiomatic assumption that underlies Boor’s algebraic method for determining paid claims tail factors.

Paid claim and reported claim development estimates are estimating the same quantity.

(d) Calculate the indicated paid claims tail factor for accident year 2011 using Boor’s algebraic method with the results from part (a).

Estimated ultimate claims from part (a) = 92,559
Estimated claims (using paid) at 48 months = 67,700×1.25 = 84,625
Indicated tail factor = \( \frac{92,559}{84,625} \approx 1.094 \)
3. Learning Objectives:
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

Learning Outcomes:
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

Sources:

Commentary on Question:
This question tests the candidate’s ability to project ultimate claims using the development-based frequency-severity method.

Solution:
(a) Calculate the projected ultimate claims for all accident years using the development-based frequency-severity method.

Commentary on Question:
The average of all years is used for the selection of the trended severity at 2013 levels and trended frequency at 2013 levels. Other reasonable selections with explanation are acceptable.

<table>
<thead>
<tr>
<th>AY</th>
<th>EE</th>
<th>Rpt Claims</th>
<th>Rpt Counts</th>
<th>Severity</th>
<th>Freq</th>
<th>Trend</th>
<th>Trended Severity</th>
<th>Trended Freq</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>5,440</td>
<td>333,900</td>
<td>370</td>
<td>902.43</td>
<td>0.0680</td>
<td>1.1025</td>
<td>1.0302</td>
<td>994.93</td>
</tr>
<tr>
<td>2012</td>
<td>5,530</td>
<td>359,900</td>
<td>376</td>
<td>957.18</td>
<td>0.0680</td>
<td>1.0500</td>
<td>1.0150</td>
<td>1,005.04</td>
</tr>
<tr>
<td>2013</td>
<td>5,690</td>
<td>401,600</td>
<td>404</td>
<td>994.06</td>
<td>0.0710</td>
<td>1.0000</td>
<td>1.0000</td>
<td>994.06</td>
</tr>
</tbody>
</table>

Average: 998.01 0.0700

Notes:  
(4) = (2)/(3)  
(5) = (3)/(1)  
(6) & (7) = (1 + trend)\(^{(2013 - AY)}\)  
(8) = (4)(6)  
(9) = (5)(7)

Selected Severity at 2013 level = 998 (based on average)  
Selected Frequency at 2013 level = 0.0700 (based on average)
3. Continued

State two reasons to project ultimate frequencies and severities separately.

Commentary on Question:

Other reasons are possible.

- Can focus separately on the two primary components driving the ultimate value: frequency and severity
- Supports the analysis of trend

Calculate accident year 2014 projected ultimate claims for the development-based frequency-severity method.

Commentary on Question:

Candidate needs to project severity and frequency with one year of trend to AY 2014 level.

Selected severity at 2013 level = 998
Selected frequency at 2013 level = 0.0700
AY 2014 projected ultimate claims = (998×1.05)(0.0700×1.015)×5,700 = 424,384

<table>
<thead>
<tr>
<th>Ultimate</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AY</td>
<td>Freq</td>
<td>Counts</td>
<td>Severity</td>
</tr>
<tr>
<td>(10)</td>
<td>(11)</td>
<td>(12)</td>
<td>(13)</td>
</tr>
<tr>
<td>2011</td>
<td>0.0679</td>
<td>370</td>
<td>905.22</td>
</tr>
<tr>
<td>2012</td>
<td>0.0690</td>
<td>381</td>
<td>950.48</td>
</tr>
<tr>
<td>2013</td>
<td>0.0700</td>
<td>398</td>
<td>998.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: (10) = 0.0700 / (7)  
(11) = (10)(1)  
(12) = 998 / (6)  
(13) = (11)(12)
4. Learning Objectives:
7. The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

Learning Outcomes:
(7b) Apply catastrophe models to insurance ratemaking, portfolio management, and risk financing.

Sources:

Commentary on Question:
This question tests the candidate’s understanding of best estimates and confidence intervals of catastrophe models.

Solution:
(a) Calculate the combined best estimate for both the probability and the return period.

The values are 0.0081 and 124.5.

(b) Recommend one of the two approaches to calculate the best estimate of the return based on the combined models. Justify your choice.

Results should be consistent; otherwise risk management strategies that employ different measures (probability versus return) will not align.

(c) Calculate the interval using each of the three options.

I: 0.25(0.0022) + 0.50(0.0026) + 0.25(0.0031) = 0.002625
to 0.25(0.0114) + 0.50(0.0109) + 0.25(0.0117) = 0.011225

II: 0.25(0.0022) + 0.50(0.0026) + 0.25(0.0031) = 0.002625
    to 0.25(0.0109) + 0.50(0.0114) + 0.25(0.0117) = 0.011350

III: minimum of 0.0022 to maximum of 0.0117

(d) State the option that produces an interval that can most accurately be described as a 90% confidence interval. Justify your choice.

Only Option I provides a 90% interval as it is the weighted average of such intervals. By varying the weights, Option II distorts the results. Option III is conservative and thus provides more than 90% confidence.
5. Learning Objectives:
3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:
(3e) Evaluate premium liabilities.

Sources:

Commentary on Question:
Commentary listed underneath question component.

Solution:
(a) Calculate the premium liabilities, both gross and net of reinsurance.

Commentary on Question:
Candidate needs to recognize that expected ULAE, maintenance expenses and incentive commissions are calculated from gross unearned premiums for both the gross and net calculations.

<table>
<thead>
<tr>
<th>Line of Business</th>
<th>Unearned Premiums</th>
<th>Expected Claims Ratio</th>
<th>Expected Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross of Reinsurance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Property</td>
<td>350</td>
<td>80%</td>
<td>280</td>
</tr>
<tr>
<td>Liability</td>
<td>510</td>
<td>70%</td>
<td>357</td>
</tr>
<tr>
<td>Total</td>
<td>860</td>
<td></td>
<td>637</td>
</tr>
<tr>
<td>Net of Reinsurance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Property</td>
<td>320</td>
<td>80%</td>
<td>256</td>
</tr>
<tr>
<td>Liability</td>
<td>450</td>
<td>74%</td>
<td>333</td>
</tr>
<tr>
<td>Total</td>
<td>770</td>
<td></td>
<td>589</td>
</tr>
</tbody>
</table>

Expected Claims = (Unearned Premiums)(Expected Claims Ratio)

<table>
<thead>
<tr>
<th></th>
<th>Gross</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Unearned Premiums</td>
<td>860</td>
</tr>
<tr>
<td>2</td>
<td>Expected Claims</td>
<td>637</td>
</tr>
<tr>
<td>3</td>
<td>Expected ULAE [Gross(2) × 11%]</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>Total Exp. Claims &amp; ULAE [(2) + (3)]</td>
<td>707</td>
</tr>
<tr>
<td>5</td>
<td>Maintenance Expenses [Gross(1) × 17% × 25%]</td>
<td>37</td>
</tr>
<tr>
<td>6</td>
<td>Incentive Commissions [Gross(1) × 2.80%]</td>
<td>24</td>
</tr>
<tr>
<td>7</td>
<td>Premium Liabilities [(4) + (5) + (6)]</td>
<td>768</td>
</tr>
</tbody>
</table>
5. Continued

(b) Determine the equity in unearned premiums.

<table>
<thead>
<tr>
<th>Gross</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>860</td>
<td>770</td>
</tr>
</tbody>
</table>

(c) Calculate the maximum deferred policy acquisition expense (DPAE) that your company could record as an asset.

Carried ceded unearned commissions = 40

Max DPAE that can be recorded as an asset = minimum of (40 + 50) & 92 = 90

(d) Explain the premium development component an actuary would consider when determining the premium liability for retrospectively-rated policies.

Actuary would consider the difference, either positive or negative, between the estimated final premium and the premiums collected through the accounting date.
6. **Learning Objectives:**
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5k) Calculate rates for claims-made coverage.

**Sources:**
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34.

**Commentary on Question:**
*This question tests the candidate’s understanding of claims-made ratemaking.*

**Solution:**
(a) Define the following terms in relation to claims-made coverage:

(i) Retroactive date

(ii) Mature claims-made coverage

(iii) Nose or prior acts coverage

(i) Retroactive date for claims-made coverage: date stated in the policy declarations indicating the start of claim occurrences that will be covered under the claims-made policy. Claims with an occurrence date before the retroactive date would not be covered.

(ii) Mature claims-made coverage: coverage which is priced assuming that all claims reported during the coverage period will have occurrence dates after the retroactive date.

(iii) Nose or prior acts coverage: for a new claims-made insured, the new insurer can offer “nose” or “prior acts” coverage for claims with an occurrence date prior to the inception date of the new policy.

(b) Calculate the pure premium for the following policies:

(i) Mature claims-made coverage effective January 1, 2016

(ii) Tail coverage for a first-year claims-made policy effective January 1, 2014
6. Continued

(i) Pure premium for AY 2015, RY 2016 = 235,000 – 100,000 = 135,000
    Pure premium for AY 2016, AY 2016 = 100,000 × 0.9 = 90,000
    Total RY 2016 pure premium = 90,000 + 135,000 = 225,000

(ii) Tail pure premium would cover AY 2014, RY 2015 and would equal
    135,000/0.9 = 150,000.

(c) Calculate the step factor for first-year claims-made coverage.

    Consider RY 2016: step factor for first-year claims-made would be
    (100,000×0.90)/225,000 = 0.4.

(d) Explain how a longer claim payment pattern would affect the pricing of occurrence policies for ERR.

    • The longer the claim payment pattern, the more opportunity for investment income (assuming positive investment yield).
    • With the negative trend and longer reporting and payment patterns, occurrence policy pricing would likely decrease.
    • However, there could be more reported claims in total and pricing would therefore go up.
7. **Learning Objectives:**
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**

**Commentary on Question:**
*This question tests the candidate’s understanding of the Berquist-Sherman adjustments when there have been changes to claims settlement rates.*

**Solution:**
(a) Assess whether there is any evidence of changed claims settlement rates.

Calculate disposal ratios (closed count / ultimate):

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>40.0%</td>
<td>80.0%</td>
<td>96.0%</td>
</tr>
<tr>
<td>2012</td>
<td>40.0%</td>
<td>90.0%</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>50.0%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Increase in the latest diagonal indicates a change in the claims settlement rates.

(b) Estimate total unpaid claims using a Berquist-Sherman adjustment to paid claims and simple age-to-age development factors.

Step 1: Select disposal ratios

<table>
<thead>
<tr>
<th>Selected Disposal</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.0%</td>
<td>90.0%</td>
<td>96.0%</td>
<td></td>
</tr>
</tbody>
</table>

Note: last diagonal of table from (a)

Step 2: Calculate adjusted closed counts

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>150</td>
<td>270</td>
<td>288</td>
</tr>
<tr>
<td>2012</td>
<td>165</td>
<td>297</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>160</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: selected disposal ratios × Selected Ultimate Counts

E.g. 165 = 50% × 330
7. Continued

Step 3: Calculate adjusted paid claims

Adjusted Paid Claims

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>16,146</td>
<td>32,892</td>
<td>36,242</td>
</tr>
<tr>
<td>2012</td>
<td>19,416</td>
<td>36,708</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>22,479</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Last diagonal = Actual Paid Claims

Other three figures by substitution in \( y = ae^{bx} \) where \( x \) is adjusted closed count.

\( (19,416 = 8,758 \times e^{(0.004825 \times 165)}) \)

Step 4: Calculate development factors using Adjusted Paid Claims

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>12-24</th>
<th>24-36</th>
<th>36-Ult</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>2.037</td>
<td>1.102</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>1.891</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average 1.964 1.102 1.050
to-Ult 2.273 1.157 1.050

\( (e.g. 2.037 = 32,892 / 16,146) \)

Tail factor = square root of last tail factor \((1.102)^{0.5} = 1.050\)

Step 5: Calculate projected ultimate claims & unpaid claims

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Paid</th>
<th>Factor to Ultimate</th>
<th>Projected Ultimate Claims</th>
<th>Unpaid Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>36,242</td>
<td>1.050</td>
<td>38,054</td>
<td>1,812</td>
</tr>
<tr>
<td>2012</td>
<td>36,708</td>
<td>1.157</td>
<td>42,471</td>
<td>5,763</td>
</tr>
<tr>
<td>2013</td>
<td>22,479</td>
<td>2.273</td>
<td>51,095</td>
<td>28,616</td>
</tr>
</tbody>
</table>

Total 95,429 131,620 36,191

\( (e.g. \) Projected Ultimate Claims: \( 42,471 = 36,708 \times 1.157 \)

Unpaid Claims: \( 5,763 = 42,471 – 36,708 \)

(c) Explain whether you would expect this unpaid claims estimate to be higher or lower than that calculated from the unadjusted paid claims triangle.

Unpaid claims using adjusted paid claims should be lower, since speed up in claim settlement has not been taken into consideration. Berquist-Sherman does take this into consideration, therefore lower future development is expected.
8. Learning Objectives:
   5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:
(5b) Calculate expenses used in ratemaking analyses including expense trending procedures.

Sources:

Commentary on Question:
Commentary listed underneath question component.

Solution:
(a) Explain whether you should include a provision for the cost of the management system in your premium rate calculation.

Commentary on Question:
Candidate can argue for or against including the item, as long as the explanation is provided.

Argument for including: Cost of 5,000,000 is 2.5% of expected claims (20,000×10,000) so this is a significant cost item that should be included.

Argument for not including: New management system presumably would reduce expenses that would cover the cost of the system over the amortization period. Reduction in expense loading not accounted for so premium calculation is fine.

(b) Calculate AIC’s indicated rate per insured for one-year policies effective January 1, 2015.

Annual claim cost per insured (pure premium) = 10,000 (2015 cost level)
Variable expense = 10%
Profit & contingencies = 10%
Gross cost of reinsurance = 2,000 per insured
Expected benefit of reinsurance = 50% of gross cost = 50% × 2,000 = 1,000 (include as fixed expense item)
Need to include provision for management system:
Amortize over 5 years, calculate cost per insured:
The 1,000,000 cost per year for 20,000 architects equals a fixed cost of 50.

Premium = \[
\frac{10,000 + 1,000 + 50}{1 - 0.1 - 0.1} = 13,812.50
\]
(c) Explain the conditions under which AIC can or cannot include the expected policyholder dividend as an expense provision.

**Commentary on Question:**
*Any two of the following are acceptable.*

Policyholder dividends can be included if they are a reasonably expected expense and are associated with the risk transfer. Factors to consider are:

- The company's dividend payment history
- Its current dividend policy or practice
- Whether dividends are related to loss experience
- The capitalization of the company
- Other considerations affecting the payment of dividends
9. **Learning Objectives:**

1. The candidate will understand the key considerations for general insurance actuarial analysis.

4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**

1. Adjust historical earned premiums to current rate levels.

4. Choose trend rates and calculate trend factors for exposures.

5. Calculate loadings for catastrophes and large claims.

5. Demonstrate the use of credibility in ratemaking.

5. Calculate overall rate change indications under the claims ratio and pure premium methods.

**Sources:**


**Commentary on Question:**

*This question tests the candidate’s ability to calculate the indicated average rate, while considering adjustments to earned premium and a loading for non-hurricane weather claims.*

**Solution:**

(a) Calculate the earned premium adjusted to the forecast period for each year.

**Commentary on Question:**

*To calculate the on level earned premium, candidate cannot just use the current average premium and multiply by historical exposures, as there may be other distributional changes that cause the average earned premium to be different.*
9. Continued

<table>
<thead>
<tr>
<th>CY</th>
<th>Average Rate Level</th>
<th>On level Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2012</td>
<td>0.7778</td>
<td>0.2222</td>
</tr>
<tr>
<td>2013</td>
<td>0.0556</td>
<td>0.9444</td>
</tr>
</tbody>
</table>

e.g. Area in 2012 at rate level 1.07 = \((8/12)(8/12)(1/2)\) = 0.2222

On level factor = Current rate level (1.07) ÷ Average rate level

Premium trend factors:

<table>
<thead>
<tr>
<th>Experience Period</th>
<th>Average Earned Date</th>
<th>Trending Period in Months</th>
<th>Premium Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>7/1/2011</td>
<td>4/1/2016</td>
<td>57</td>
</tr>
<tr>
<td>2012</td>
<td>7/1/2012</td>
<td>4/1/2016</td>
<td>45</td>
</tr>
<tr>
<td>2013</td>
<td>7/1/2013</td>
<td>4/1/2016</td>
<td>33</td>
</tr>
</tbody>
</table>

e.g. 1.0733 = \(1.015^{(57/12)}\)

(b) Calculate the Territory X loading for non-hurricane weather claims, stated as a claim ratio.
9. Continued

1. Credibility weighted pure premium = 0.45×125 + (1 – 0.45)×185 = 158.00
2. Earned house years (given) = 12,100
3. Expected Non-Hurricane Weather Claims = (1)×(2) = 1,911,800
4. Trended Earned Premiums at Current Rate Level (part (a)) = 15,208,577
5. Loading for Non-Hurricane Weather Expressed as a Claim Ratio = (3) / (4) = 12.57%

(c) Calculate the indicated average rate for Territory X.

**Commentary on Question:**
Candidate needs to calculate the indicated rate to get full credit, not just the indicated change. Also, candidate needs to credibility weight the claim ratios before determining the indicated rate because there are fixed expenses. Credibility weighting the indicated rates would not produce the same answer.

First calculate the weighted average claim ratio:

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Trended EP at Current Rate</th>
<th>Trended Ultimate Claims</th>
<th>Claim Ratio</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>14,578,183</td>
<td>8,455,000</td>
<td>58.0%</td>
<td>20%</td>
</tr>
<tr>
<td>2012</td>
<td>15,027,780</td>
<td>9,092,000</td>
<td>60.5%</td>
<td>30%</td>
</tr>
<tr>
<td>2013</td>
<td>15,208,577</td>
<td>9,049,000</td>
<td>59.5%</td>
<td>50%</td>
</tr>
</tbody>
</table>

Weighted Average Trended Claim Ratio = 59.5%

1. Weighted average trended claim ratio = 59.5%
2. Selected non-hurricane weather claim ratio (part (b)) = 12.57%
3. Total claim ratio including ULAE = (59.5% + 12.57%) × (1 + 8.0%) = 77.84%
4. Permissible claim ratio = [1 – 0.17 – 0.05] / [1 + 0.04/(3)] = 74.19%
5. Credibility = \( \sqrt{\frac{11,900 + 12,100 + 12,100}{40,000}} \) = 95.0%
6. Countrywide claim ratio = 74%
7. Credibility weighted experience claim ratio = 77.65%
8. Indicated rate level change = (7)/(4) – 1 = 4.66%
9. 2013 trended earned premiums at current rate level = 15,208,577
10. 2013 earned house years = 12,100
11. Current average rate = (9) / (10) = 1,256.91
12. Indicated average rate = (11) × [1 + (8)] = 1,315.48
10. **Learning Objectives:**
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**
(5j) Perform individual risk rating using standard plans.

**Sources:**

**Commentary on Question:**
*Commentary listed underneath question component.*

**Solution:**
(a) State two principles underlying prospective experience rating.

1. Credibility of the insured’s experience: the larger the insured, the more reliable the historical claims are expected to be as a predictor of future claims.
2. Frequency is a better predictor of future claims than severity.

(b) Explain how the concepts of stability and responsiveness are important in both experience rating and self-insurance cost allocation systems.

Stability can lead to better planning and use of resources.
Responsiveness is important for equity among insureds and for pricing based on actual risk and consequent claims experience.

(c) State two considerations that ABC likely made in preparing the quote.

**Commentary on Question:**
*Any two of the following are acceptable.*

- The risk characteristics of XYZ – made a subjective assessment that XYZ was a better than average risk
- The experience of XYZ – determined an experience modification factor less than 1.000
- The credibility of XYZ’s experience
- The variability of the experience of XYZ
- The creditworthiness of XYZ

(d) State and support your opinion as to whether XYZ should continue to self-insure or should insure its general liability exposure with ABC.
10. Continued

- XYZ appears to have better than average risk characteristics but worse than average recent experience.
- ABC has presented a wide range from 300,000 minimum to 900,000 maximum premium but has allowed XYZ to pay based on paid losses. Thus, the XYZ could delay premium payment and get some cash flow benefit as it has historically with self-insurance.
- Both endpoints 400,000 and 700,000 of XYZ future loss projection suggest that the future retrospective premium would be in the middle of the range set forth by ABC.
- XYZ has experience with self-insurance and should continue.
11. **Learning Objectives:**
4. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

**Learning Outcomes:**
(4a) Identify the time periods associated with trending procedures.

(4b) Describe the influences on frequency and severity of changes in deductibles, changes in policy limits, and changes in mix of business.

(4c) Choose trend rates and calculate trend factors for claims.

**Sources:**

**Commentary on Question:**
*This question tests the candidate’s understanding of the various influences on frequency and severity trend and the considerations involved in selecting frequency and severity trends. Candidates need to calculate the trend factors and apply trend to pure premiums used in ratemaking.*

**Solution:**
(a) Describe how the following company actions may affect claims trend:

(i) Increasing the minimum deductible from 500 to 1,000:
- expect frequency to decrease as smaller claims are removed
- no fixed relationship between deductible and severity
- overall: expect trend to decrease

(ii) Increasing the minimum policy limit from 200,000 to 500,000:
- no change to frequency
- increase in severity
- overall: expect trend to increase

(iii) Expanding the perils covered for a particular line of business:
- increase in frequency as more perils are now covered
- uncertain whether severity will increase or decrease
- overall: expect trend to increase
11. Continued

(b) Describe the overlap fallacy as it relates to development factors and trend factors.

**Commentary on Question:**
*Candidates do not need to demonstrate that overlap does not exist.*

Overlap fallacy argues that the application of development factors and trend factors to historical claims for the purpose of ratemaking double counts the inflationary forces and thus overstates the projection of future claims.

(c) Recommend the annual frequency trend and severity trend for REV to use. Justify your answers.

Frequency trend selection:
- Change in policy limits would affect severity and not frequency, therefore industry trend is preferred
- Not much difference in industry trend for the two different trend periods, suggesting that the reform didn’t have much of an effect on frequency
- Selection: 0.6%

Severity trend selection:
- Industry experience has not been adjusted for reform, therefore should give more consideration to REV experience. Also, the difference in industry trends for the two periods supports this conclusion.
- Company experience is similar for the two periods, but higher with better fit post-reform, therefore select post-reform indicated trend
- Selection: 4.8%

(d) Calculate the pure premium trend factor that will be applied to 2012 experience for the ratemaking analysis.

Trend period:
- 2012 experience period: average accident date = July 1, 2012
- 2015 future rating period: Rates effective Jan 1, 2015 for 1 year, all 6-month policies: average accident date = Oct 1, 2015
- trend period = July 1, 2012 to Oct 1, 2015 = 3.25 years

Pure premium trend: 1.006×1.048

Trend factor = \( (1.006 \times 1.048)^{3.25} = 1.187 \)
12. Learning Objectives:
2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

Learning Outcomes:
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

Sources:

Commentary on Question:
This question tests the candidate’s ability to estimate ultimate claims using the Cape Cod method.

Solution:
(a) Calculate the used-up on-level earned premiums for each accident year shown.

Used-up on-level earned premium is the earned premium multiplied by the on-level factor and divided by the age-to-ultimate development factor (or multiplied by 1/age-to-ultimate development factor):

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Earned Premium</th>
<th>On-Level Factors</th>
<th>On-Level Earned Premium</th>
<th>Age-to-Ult Dev Factor</th>
<th>Expected % Paid</th>
<th>Used-Up Earned Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>20,000</td>
<td>1.035</td>
<td>20,700</td>
<td>4.550</td>
<td>22.0%</td>
<td>4,549</td>
</tr>
<tr>
<td>2012</td>
<td>22,000</td>
<td>1.020</td>
<td>22,440</td>
<td>8.750</td>
<td>11.4%</td>
<td>2,565</td>
</tr>
<tr>
<td>2013</td>
<td>23,000</td>
<td>1.000</td>
<td>23,000</td>
<td>25.500</td>
<td>3.9%</td>
<td>902</td>
</tr>
<tr>
<td>Total</td>
<td>8,016</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Calculate the adjusted expected claims ratio.

Adjusted paid claims amount is the actual paid claims multiplied by the trend factor and multiplied by the tort factor.
12. Continued

(c) Calculate the expected claims for each accident year.

Expected claims for each year are equal to the on-level earned premiums multiplied by the adjusted expected claims ratio and divided by the trend and tort reform adjustments:

\[
(11) = (A) \times (3) / [(8)(9)]
\]

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Expected Claims</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)=(7)(8)(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>19,170</td>
<td>4,100</td>
<td>1.051</td>
<td>0.900</td>
<td>19,057</td>
</tr>
<tr>
<td>2012</td>
<td>19,178</td>
<td>1,900</td>
<td>1.025</td>
<td>1.000</td>
<td>18,886</td>
</tr>
<tr>
<td>2013</td>
<td>20,148</td>
<td>1,200</td>
<td>1.000</td>
<td>1.000</td>
<td>20,558</td>
</tr>
<tr>
<td>Total</td>
<td>58,496</td>
<td>7,200</td>
<td></td>
<td></td>
<td>58,501</td>
</tr>
</tbody>
</table>

(A) Adjusted Expected Claims Ratio: 87.6%

\[
\text{Adjusted Expected Claims Ratio} = \frac{\text{sum}(10)}{\text{sum}(6)}
\]

(d) Calculate the projected ultimate claims for each accident year.

Projected ultimate claims are the actual paid claims plus the expected claims multiplied by the % unpaid:

\[
(13) = (7) + (11)(12)
\]

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Projected Ultimate Claims</th>
<th>(7)</th>
<th>(11)</th>
<th>(12) = 1 - (5)</th>
<th>(13) = (7) + (11)(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>19,057</td>
<td>4,100</td>
<td>19,170</td>
<td>78.0%</td>
<td>19,057</td>
</tr>
<tr>
<td>2012</td>
<td>18,886</td>
<td>1,900</td>
<td>19,178</td>
<td>88.6%</td>
<td>18,886</td>
</tr>
<tr>
<td>2013</td>
<td>20,558</td>
<td>1,200</td>
<td>20,148</td>
<td>96.1%</td>
<td>20,558</td>
</tr>
<tr>
<td>Total</td>
<td>58,501</td>
<td>7,200</td>
<td>58,496</td>
<td></td>
<td>58,501</td>
</tr>
</tbody>
</table>
12. Continued

(e) Explain the challenge reinsurance actuaries face when using the Cape Cod method to estimate ultimate claims.

Unobserved claims relies on the on-level earned premium by year. For reinsurance actuaries, detailed information about premium rate changes is not always available.
13. Learning Objectives:
   2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.
   5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:
(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

(5f) Calculate overall rate change indications under the claims ratio and pure premium methods.

Sources:

Commentary on Question:
This question tests the candidate’s ability to estimate the expected claim ratio and expected claims. This question also tests the understanding of the effect of a tort reform adjustment on the ratemaking analysis.

Solution:
(a) Calculate and select the 2013 expected claim ratio to be used to calculate expected claims.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Earned Premiums</th>
<th>Projected Ultimate Claims from Development Method</th>
<th>Trend Factor</th>
<th>Tort Reform Premium On-Level Factors</th>
<th>Trended On-Level Claim Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>8,500</td>
<td>8,390</td>
<td>1.067</td>
<td>0.80</td>
<td>1.08</td>
</tr>
<tr>
<td>2011</td>
<td>7,480</td>
<td>6,990</td>
<td>1.044</td>
<td>0.80</td>
<td>1.04</td>
</tr>
<tr>
<td>2012</td>
<td>8,890</td>
<td>7,080</td>
<td>1.022</td>
<td>1.00</td>
<td>1.03</td>
</tr>
<tr>
<td>2013</td>
<td>9,150</td>
<td>6,950</td>
<td>1.000</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

All years average: 77%
Selected Expected Claim Ratio: 77% 

Notes: (3) = (1 + 2.2%)^{2013-AY}
(4): adjustment to 2010 & 2011; 1 – 20% = 80%
(6) = [(2) × (3) × (4)] / [(1) × (5)]
13. Continued

Selection: no significant indication that any of the years are outliers, therefore select average.

(b) Calculate the 2012 expected claims.

Claim ratio at 2012 cost level = \(77\% \times 1.03 / (1.022 \times 1.00) = 77.60\%
\)

2012 expected claims = \(8,890 \times 77.60\% = 6,899\)

(c) Explain whether a ratemaking analysis would overstate or understate the true rate change required if the above tort reform adjustment were not made when estimating each of the following:

(i) Ultimate claims

(ii) Annual claim trend

(i) Ultimate claims: Without the adjustment, ultimate claims for AYs before 2012 would be overstated, thus overstating the true rate change required.

(ii) Annual claim trend: Without the adjustment, ultimate claims for AYs before 2012 would be overstated, thus resulting in a lower estimate of claim trend. The lower claim trend would understate the true rate change required.
14. Learning Objectives:
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:
(5g) Calculate risk classification changes and territorial changes.

Sources:
Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 32.

Commentary on Question:
Commentary listed underneath question component.

Solution:
(a) Determine if the exposures exhibit distributional bias. Support your conclusion.

Commentary on Question:
Only one inconsistency needs to be found to demonstrate distributional bias.

Male/Female ratios are not consistent across the 3 locations (7/2, 2/6, 3/5), and the urban/suburban/rural ratios are not consistent across the 2 genders (7/2, 2/6 and 7/3, 2/5 and 2/3, 6/5). As a result, there is distributional bias.

(b) Calculate the rebalanced pure premiums using one-way analysis relativities for each rating variable.

Commentary on Question:
See table 32.19 in text for reference.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Location</th>
<th>Exp.</th>
<th>One-Way Relativities</th>
<th>Gender</th>
<th>Location</th>
<th>PP</th>
<th>Rebal PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>U</td>
<td>7</td>
<td>1.223 1.417 58.44</td>
<td>M</td>
<td>U</td>
<td>58.44</td>
<td>56.86</td>
</tr>
<tr>
<td>M</td>
<td>S</td>
<td>2</td>
<td>1.223 0.882 36.37</td>
<td>M</td>
<td>S</td>
<td>35.38</td>
<td>35.38</td>
</tr>
<tr>
<td>M</td>
<td>R</td>
<td>3</td>
<td>1.223 0.649 26.76</td>
<td>M</td>
<td>R</td>
<td>26.03</td>
<td>26.03</td>
</tr>
<tr>
<td>F</td>
<td>U</td>
<td>2</td>
<td>0.794 1.417 37.94</td>
<td>F</td>
<td>U</td>
<td>36.91</td>
<td>36.91</td>
</tr>
<tr>
<td>F</td>
<td>S</td>
<td>6</td>
<td>0.794 0.882 23.61</td>
<td>F</td>
<td>S</td>
<td>22.97</td>
<td>22.97</td>
</tr>
<tr>
<td>F</td>
<td>R</td>
<td>5</td>
<td>0.794 0.649 17.38</td>
<td>F</td>
<td>R</td>
<td>16.91</td>
<td>16.91</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td>34.66</td>
<td>33.72</td>
</tr>
</tbody>
</table>

Notes: Male relativity: 41.25 / 33.72 = 1.223
Urban relativity: 47.78 / 33.72 = 1.417
Male/Urban PP: 33.72×1.223×1.417 = 58.44
Total PP (34.66) weighted average using exposures as weights.
Rebalanced Male/Urban: 58.44×33.72/34.66 = 56.86
14. Continued

(c) Calculate the revised relativities for gender that result from a single iteration of the minimum bias method.

Total Expected Claims:
- Male: $7 \times 50 + 2 \times 35 + 3 \times 25 = 495$
- Female: $2 \times 40 + 6 \times 28 + 5 \times 20 = 348$

New gender factors - 1st iteration (use one-way location factors as starting point: 47.78/33.72 = 1.417, 29.75/33.72 = 0.882, 21.88/33.72 = 0.649):

$$g_m = \frac{495}{7 \times 1.417 + 2 \times 0.882 + 3 \times 0.649} \times 33.72 = 1.077$$

$$g_f = \frac{348}{2 \times 1.417 + 6 \times 0.882 + 5 \times 0.649} \times 33.72 = 0.908$$

(d) Describe the conditions under which the minimum bias method reproduces the trended ultimate pure premiums and demonstrate that the conditions are met in this case.

The observed pure premiums must be independent for the minimum bias method to reproduce them. One way to verify independence is to observe the ratios. By gender they are 50/40, 35/28, and 25/20, all equal to 1.25. By location they are 35/50 = 28/40 = 0.7, 25/50 = 20/40 = 0.5, and 25/35 = 20/28 = 0.714.

An alternative is to point out that by definition, if the pure premiums can be reproduced by a multiplicative formula, they are independent.
15. Learning Objectives:
5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:
(5h) Calculate deductible factors, increased limits factors, and coinsurance penalties.

Sources:

Commentary on Question:
This questions tests the candidate’s understanding of coinsurance.

Solution:
(a) Explain the benefit to the insured of purchasing a policy with an amount of insurance of 90,000.

If a policyholder purchases a policy of 90,000, the 10,000 claim layer from 80,000 to 90,000 will be covered.

(b) Calculate the expected claim cost for policies with each of the following amounts of insurance:

(i) 70,000

(ii) 90,000

The expected claim cost of 520 is equal to the frequency $f$ multiplied by the average of the two claim sizes, adjusted for coinsurance and policy limit:
- $520 = f \times (50,000 + 80,000)/2$
- solving for $f = 0.008$

For the 70,000 amount of insurance policy:
- Amount paid for 50,000 claim = min[7/8×50,000 ; 70,000] = 43,750
- Amount paid for 90,000 claim = min[7/8×90,000 ; 70,000] = 70,000
- Expected claim cost = $f \times$ (average of claims after coinsurance) = $0.008 \times (43,750 + 70,000)/2 = 455$

For the 90,000 amount of insurance policy:
- Expected claim cost = $f \times$ (average of claims after coinsurance) = $0.008 \times (50,000 + 90,000)/2 = 560$

(c) Calculate both the coinsurance penalty percentage and the amount retained by the insured for a claim of 50,000.
15. Continued

Coinsurance reduces the claim paid to the insured from 50,000 to 37,500 = 50,000 \times \frac{60,000}{80,000} and the coinsurance penalty percentage is 25\% = 1 - \frac{60,000}{80,000}.

Subtracting the 1,500 deductible further reduces the claim to 36,000 (37,500 - 1,500), and the amount retained by the insured equals 14,000 = 50,000 - 36,000.
16. **Learning Objectives:**

2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**

(2d) Explain the effect of changing conditions on the projection methods cited in (b).

**Sources:**


**Commentary on Question:**

This question tests the candidate’s ability to estimate the change in ultimate claims and IBNR estimates under a mix of business changing condition.

**Solution:**

(a) Calculate ultimate claims for accident year 2013 using both the historical and actual business mix.

<table>
<thead>
<tr>
<th>Historical Premium Mix</th>
<th>2013 Premium</th>
<th>Ultimate Claim Ratio</th>
<th>Ultimate Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group A</td>
<td>500</td>
<td>60%</td>
<td>300</td>
</tr>
<tr>
<td>Group B</td>
<td>500</td>
<td>40%</td>
<td>200</td>
</tr>
<tr>
<td>Total</td>
<td>1,000</td>
<td></td>
<td><strong>500</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Actual 2013 Premium Mix</th>
<th>2013 Premium</th>
<th>Ultimate Claim Ratio</th>
<th>Ultimate Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group A</td>
<td>700</td>
<td>60%</td>
<td>420</td>
</tr>
<tr>
<td>Group B</td>
<td>300</td>
<td>40%</td>
<td>120</td>
</tr>
<tr>
<td>Total</td>
<td>1,000</td>
<td></td>
<td><strong>540</strong></td>
</tr>
</tbody>
</table>

(b) Calculate 2013 IBNR claim liabilities for accident year 2013 using the actual business mix.

2013 reported claims at 12 months = ultimate claims × percent reported at 12 months = (420×40%) + (120×50%) = 228

2013 IBNR claim liabilities = ultimate claims – reported claims at 12 months = 540 – 228 = 312
16. Continued

(c) Calculate the percentage over- or under-estimate in 2013 IBNR claim liabilities for accident year 2013 under the following methods if the historical business mix is assumed:

(i) Expected Claims

Ultimate claims = 500 (historical mix)
IBNR = ultimate claims – reported claims at 12 months = 500 – 228 = 272
Compared to actual: 40 lower than actual, or understatement of 13%

(ii) Reported Development

2013 Reported claims at 12 months using historical mix = ultimate claims × percent reported at 12 months = (300×40%) + (200×50%) = 220
(Historical) percent reported at 12 months = 220 / 500 = 44%
Implied 12 months to ultimate development factor = 1/0.44 = 2.273
Ultimate claims = 228 × 2.273 = 518
IBNR = 518 – 228 = 290
Compared to actual: 22 lower than actual, or understatement of 7%

(iii) Reported Bornhuetter Ferguson

(Historical) percent reported at 12 months = 220 / 500 = 44%
Implied future development = 56%
Expected ultimate claims (from (i)) = 500
IBNR = 500 × 56% = 280
Compared to actual: 32 lower than actual, or understatement of 10%

Commentary on Question:
Key to solving this part is to recognize that IBNR = Ultimate claims – Reported claims at 12 months, and that the reported claims at 12 months (228) is the same for (i), (ii), and (iii). Calculate ultimate claims using the historical mix for each of (i), (ii), and (iii) and subtract 228 to get the IBNR, and compare to actual IBNR of 312 from part (b).
17. **Learning Objectives:**

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

**Learning Outcomes:**

(5h) Calculate deductible factors, increased limits factors, and coinsurance penalties.

**Sources:**


**Commentary on Question:**

*This question tests the candidate’s understanding of increased limits factors.*

**Solution:**

(a) Explain the consistency test for increased limits factors.

The consistency test for increased limits factors is that the increased limits factor must increase at a decreasing rate as the limit increases.

(b) Illustrate your answer to part (a) by means of a graph.

Increased limit factor at \( k_2 \) (relative to \( k_1 \)) = \( \frac{A + B}{A} \). With layers of equal width, the expected payment in any layer is less than that in a preceding layer. If the layers are of different widths, this property still holds between the layers for the expected payment per unit of coverage.
17. Continued

(c) Calculate the trend factor in the layer 20 to 60 resulting from a 50% inflationary trend.

Reading from the graph, the area of the original layer is $24 = (40/2)(0.80 + 0.40)$, post-inflation is $29.33 = (40/2)(0.8667 + 0.6000)$, so trend in layer = $1.222 \times (29.33/24)$, or 22.2%.

(d) Explain why there is a difference between the effect of trend on claims capped by the basic limit and the effect of trend on claims excess of the basic limit.

The 0-20 layer, for example, only gets a small portion of trend, $4\% = \{(20/2)\times(0.8667+1)\}/\{(20/2)\times(0.8+1)\} - 1$. The portion excess of 20 gets trend of $76\% = \{(130/2)\times0.8667\}/\{(80/2)\times0.8\} - 1$. Thus, there is a leveraged effect because of the extra area that inflation brings into the diagram for the excess portion. The retained portion, or basic limits portion, is less because the retention does not inflate.
18. **Learning Objectives:**
3. The candidate will understand financial reporting of claim liabilities and premium liabilities.

**Learning Outcomes:**
(3b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

**Sources:**

**Commentary on Question:**
*This question tests the Mango and Allen smoothing adjustment to calculating unallocated loss adjustment expenses.*

**Solution:**
(a) State three circumstances where the Mango and Allen adjustment is particularly valuable.

**Commentary on Question:**
*Any three of the following are acceptable.*

- Long-tail lines of business
- Changing exposure volume
- When large claims result in significant distortions to the calendar year paid and reported claims from year to year
- Where there are few claims paid or reported per year with great variability in the average claim value (i.e., low frequency and highly variable severity)
- Relatively new insurer who does not have a significant volume of credible paid or reported claims

(b) Calculate the expected reported claims in calendar years 2012 and 2013.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Selected Ultimate Claims</th>
<th>Reported CDF</th>
<th>Cumulative % reported (1 / Reported CDF)</th>
<th>Incremental % reported</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>24</td>
<td>36</td>
</tr>
<tr>
<td>2010</td>
<td>8,750</td>
<td>1.923</td>
<td>52%</td>
<td>52%</td>
</tr>
<tr>
<td>2011</td>
<td>8,920</td>
<td>1.205</td>
<td>83%</td>
<td>31%</td>
</tr>
<tr>
<td>2012</td>
<td>9,040</td>
<td>1.000</td>
<td>100%</td>
<td>17%</td>
</tr>
</tbody>
</table>

Projected in Calendar Year: 8,954 (e.g. 8,750 × 0.17 = 1,488)
18. Continued

(c) Select the ULAE ratio using the Mango and Allen smoothing adjustment based on the average of expected paid and expected reported claims. Justify your selection.

<table>
<thead>
<tr>
<th>Calendar Year</th>
<th>Paid ULAE</th>
<th>Expected Paid Claims</th>
<th>Expected Reported Claims</th>
<th>Ratio of Paid ULAE to Average of Expected Paid &amp; Reported Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>828</td>
<td>8,860</td>
<td>8,954</td>
<td>9.30%</td>
</tr>
<tr>
<td>2013</td>
<td>808</td>
<td>9,022</td>
<td>9,128</td>
<td>8.90%</td>
</tr>
<tr>
<td>Total</td>
<td>1,636</td>
<td>17,882</td>
<td>18,082</td>
<td>9.10%</td>
</tr>
</tbody>
</table>

Selected ratio: 8.9%
There is a significant difference between 2012 and 2013. Put more weight on most recent year as there has been something that has changed in the paid ULAE.
19. **Learning Objectives:**
   2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

   6. The candidate will understand the need for monitoring results.

**Learning Outcomes:**
(2f) Evaluate and justify selections of ultimate values based on the methods cited in (b).

(6b) Analyze actual claims experience relative to expectations.

**Sources:**

**Commentary on Question:**
This question tests the selection of ultimate claims using various diagnostic values. Candidates also need to be able to estimate expected reported claims for an interim period between actuarial analyses using the approach from Friedland Chapter 36.

**Solution:**
(a) Select the ultimate claims for each accident year. Justify each selection using the information from the diagnostic tables.

**Commentary on Question:**
An approach is to evaluate each of the 3 methods to determine if each is reasonable for any of the accident years. Observing that some estimates are smaller or larger than other estimates received no credit. Candidate needs to identify whether the pattern is reasonable or not for each method.

Observations from diagnostics:
Development method:
- Highly leveraged factors for paid approach and most recent years for reported, therefore should rely on a different method for most recent periods.
- Problem with reported as ultimate claim ratios increase in more recent years significantly more than annual claims trend.

Cape Cod:
- Good method for immature periods.
- Problem with reported as ultimate claim ratios increase in more recent years significantly more than annual claims trend.
19. **Continued**

Frequency-severity method:
- Ultimate pure premiums are increasing consistent with claims trend.

Recommend using frequency-severity method for all accident years.

(b) Calculate the expected reported claims from December 31, 2013 through March 31, 2014 for accident year 2013 using the selections from part (a).

Step 1: Calculate expected percent reported at December 31, 2013 for AY 2012 & AY 2013:
- Implicit development factors = Ultimate (reported, development method) / Reported claims
- Expected percent reported at December 31, 2013 = 1 / Implicit development factor
  = Reported / Ultimate
  = 2012: 5,200 / 11,600 = 44.8%
  = 2013: 4,100 / 13,900 = 29.5%

Step 2: Interpolate to determine expected percent reported at March 31, 2014:
0.75 × 0.295 + 0.25 × 0.448 = 0.333

Step 3: Expected reported between December 31, 2013 and March 31, 2014:

\[
(8,600 - 4,100) \times \frac{0.333 - 0.295}{1 - 0.295} = 243
\]
20. **Learning Objectives:**

5. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

7. The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

**Learning Outcomes:**

(5d) Calculate loadings for catastrophes and large claims.

(7b) Apply catastrophe models to insurance ratemaking, portfolio management, and risk financing.

**Sources:**


**Commentary on Question:**

*Candidates who attempted this question did reasonably well. For Parts (a) and (b) most candidates identified the components/attributes but often provided incomplete definitions. For Part (c) most all candidates earned full marks.*

**Solution:**

(a) Describe each of the three components and indicate how each may be determined for Cool Breeze.

Average Annual Loss (AAL) – Expected value from the exceedence curve.

Risk Load – Can be based on the standard deviation calculated from the curve, though any standard risk load method can be stated.

Expense Load – Includes fees, taxes, profits, commissions, loss adjustment expenses.

(b) Describe each of the two attributes.

Structure – Construction (masonry is superior to wood for hurricanes), building codes, and occupancy are the key features.

Location – Distance from the coast.

(c) Define post-event inflation and provide an example that relates to hurricane losses.

Post-event inflation is a sudden and temporary increase in the cost of materials, services, and labor due to increased demand following a catastrophe. For example, shortages in materials due to destruction of warehouses, retail outlets, and roads.
21. **Learning Objectives:**
   
   2. The candidate will understand how to calculate projected ultimate claims and claims-related expenses.

**Learning Outcomes:**

(2b) Estimate ultimate claims using various methods: development method, expected method, Bornhuetter Ferguson method, Cape Cod method, frequency-severity methods, Berquist-Sherman methods.

**Sources:**


**Commentary on Question:**

*This question tests the understanding of estimating ultimate claims using the expected method and the Bornhuetter Ferguson method.*

**Solution:**

(a) Calculate ultimate claims for accident years 2011, 2012 and 2013 using the Bornhuetter Ferguson method.

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Actual Reported Claims at Dec. 31, 2013</th>
<th>Expected Claims</th>
<th>Cumulative Development Factors</th>
<th>Expected % Unreported</th>
<th>Expected Unreported Claims</th>
<th>BF Estimated Ultimate Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>5,830</td>
<td>9,540</td>
<td>1.64</td>
<td>39.0%</td>
<td>3,721</td>
<td>9,551</td>
</tr>
<tr>
<td>2012</td>
<td>4,650</td>
<td>9,750</td>
<td>2.27</td>
<td>55.9%</td>
<td>5,450</td>
<td>10,100</td>
</tr>
<tr>
<td>2013</td>
<td>3,270</td>
<td>10,100</td>
<td>3.85</td>
<td>74.0%</td>
<td>7,474</td>
<td>10,744</td>
</tr>
</tbody>
</table>

Notes: 

(4) = 1 – 1/(3)

(5) = (2)(4)

(6) = (1) + (5)

(b) Assess the reasonableness of the inputs for the Bornhuetter Ferguson method using this data.

**Commentary on Question:**

*The reasonableness of the inputs for the Bornhuetter Ferguson method can be tested by comparing actual reported to expected reported.*
21. Continued

<table>
<thead>
<tr>
<th>Accident Year</th>
<th>Actual Reported Claims at Dec. 31, 2013</th>
<th>Expected Claims</th>
<th>Cumulative Development Factors</th>
<th>Expected % Reported</th>
<th>Expected vs. Expected</th>
<th>Actual vs. Expected</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>5,830</td>
<td>9,540</td>
<td>1.64</td>
<td>61.0%</td>
<td>5,819</td>
<td>11</td>
</tr>
<tr>
<td>2012</td>
<td>4,650</td>
<td>9,750</td>
<td>2.27</td>
<td>44.1%</td>
<td>4,300</td>
<td>350</td>
</tr>
<tr>
<td>2013</td>
<td>3,270</td>
<td>10,100</td>
<td>3.85</td>
<td>26.0%</td>
<td>2,626</td>
<td>644</td>
</tr>
</tbody>
</table>

Notes: (7) = 1/(3)  
(8) = (2)(7)  
(9) = (1) − (8)

Assessment of the inputs: Actual reported consistently higher than expected so inputs are not reasonable.

(c) Critique the interpretation of the Bornhuetter Ferguson method as a credibility weighting of the claim development and expected methods.

The description of the Bornhuetter Ferguson method as a credibility weighted method has been criticized as there are conceptual problems when the cumulative development factor is less than 1. A cumulative development factor of less than 1 results in an assigned Z greater than 100%, which by the definition of credibility is not possible.

(d) Explain why the expected method is likely more appropriate than the development method for analyzing a new line of business.

Expected method requires an *a priori* estimate that is not necessarily based on actual experience. Development method uses prior experience which is not available for a new line of business.