Solution #1

(a) $NC_x = B(x) q_y \; \ddot{a}_{65:\overline{5}|}^{(12)} v^{(65-y)} \frac{D_y}{D_x}$

$ER\ NC_x = NC_x - EE$ Contributions

<u>Group J Employees</u>

$$NC = 30 \times \left[(0.9)^{\wedge(35-30)} \times \max \left\{ 2\% \times \$40{,}000 \times \ddot{a}_{65:\overline{5}|}^{(12)} \times v^{(65-30)},\ 2 \times 4\% \times \$40{,}000 \right\} + (0.1) \times 2 \times 4\% \times \$40{,}000 \times \left\{ 1 + (0.9) + (0.9)^{\wedge(32-30)} + (0.9)^{\wedge(33-30)} + (0.9)^{\wedge(34-30)} \right\} \right]$$

$$= 30 \times \left[0.59 \times \max\{800 \times 10.4 \times 0.110,\ \$3{,}200\} + (0.1) \times \$3{,}200 \times \ddot{a}_{\overline{5}|11.11\%} \right]$$

$$= 30 \times [1{,}888 + 1{,}312] = \$96{,}000$$

EE Cont. $= 30 \times 4\% \times 40{,}000 = \$48{,}000$

ER NC $= NC -$ EE Cont. $= \$48{,}000$

<u>Group K Employees</u>

$$NC = 30 \times \max\left[2\% \times 60{,}000 \times \ddot{a}_{65:\overline{5}|}^{(12)} \times v^{(65-50)}, 2 \times 4\% \times 60{,}000 \right]$$

$$= 30 \times \max[1{,}200 \times 10.4 \times 0.3888,\ 4{,}800]$$

$$= 30 \times [4{,}852.56] = \$145{,}577$$

EE Cont. $= 30 \times 4\% \times 60{,}000 = \$72{,}000$

ER NC $=$ NC $-$ EE Cont. $= \$73{,}577$

<u>Total</u>

ER NC = \$48,000 + \$73,577 = \$121,577

Solution #1 – Continued

(b) $AL_x = \sum B(x) q_y \ \ddot{a}_{65}^{(12)} v^{(65-y)} \dfrac{D_y}{D_x}$

Group J Employees

$$AL = 24 \times \Big[(0.9)^{\wedge(35-31)} \times \max\Big\{ 800 \times \ddot{a}_{65:\overline{5}|}{}^{(12)} \times v^{(65-31)}, 2 \times 1600 \times 1.08 \Big\}$$
$$+(0.1)\ 3200 \times 1.08 \times \ddot{a}_{\overline{4}|11.11\%} \Big]$$
$$= 24 \times [0.656 \times \max\{800 \times 10.4 \times 0.118, 3200 \times 1.08\}$$
$$+(0.1)\ 3200 \times 1.08 \times 3.44]$$
$$= 24 \times [3200 \times 1.08] = \$82{,}944$$

Refunds = $6 \times 3200 \times 1.08 = \$20{,}736$

Group K Employees

$$AL = 29 \times \max\Big\{ 1200 \times \ddot{a}_{65:\overline{5}|}{}^{(12)} \times v^{(65-51)}, 2 \times 2400 \times 1.08 \Big\}$$
$$= 29 \times \max\{1200 \times 10.4 \times 0.414, 5184\}$$
$$= 29 \times [5184] = \$150{,}336$$

Refund = $1 \times 4800 \times 1.08 = \$5{,}184$

Total

$$AL = \$82{,}944 + \$150{,}336 = \$233{,}280$$

Assets

Total EE contributions = \$48,000 + \$72,000 = \$120,000

$$\text{Assets} = (\text{Assets}_{2001} + \text{Contributions}) \times 1.08 - \text{Refunds}$$
$$= (0 + 121{,}577 + 120{,}000) \times 1.08 - (20{,}736 + 5{,}184)$$
$$= \$234{,}983$$

Solution #1 – Continued

(c) Gains

$$\text{Investments} = \text{Assets}_{2002} - (\text{Assets}_{2001} + \text{Contributions}) \times 1.065 + \text{Refunds}$$
$$= 234{,}983 - (0 + 121{,}577 + 120{,}000) \times 1.065 + 25{,}920$$
$$= 3623$$

$$\text{Term'n}\,(J) = (AL_{2001} + NC_{2001}) \times 1.065 - AL_{2002} - \text{Refunds}$$
$$= (0 + 96{,}000) \times 1.065 - 82{,}944 - 20{,}736$$
$$= (1440)$$

$$\text{Death}\,(K) = (\text{AL}_{2001} + NC_{2001}) \times 1.065 - AL_{2002} - \text{Refunds}$$
$$= (0 + 145{,}577) \times 1.065 - 150{,}336 - 5{,}184$$
$$= (480)$$

Solution #2

(a) NC for Basic Plan using PUC:

$$NC = 2000 \times v^{65-50} \times \ddot{a}_{65}^{(12)}$$
$$= 2000 \times (1.08)^{-15} \times 9.0$$
$$= \$5,674$$

(b) NC for Supplemental Plan suing EAN:

$$NC = \frac{PVFB_{40}}{PVFS_{40}} \times \frac{S_{50}}{S_{40}}$$

$$PVFB_{40} = \left[2\% \times \text{Salary} \times (1+SS)^{64-x} - 2000\right] \times 25 \times v^{65-40} \times \ddot{a}_{65}^{(12)}$$
$$= \left[2\% \times 300,000 \times (1.05)^{14} - 2000\right] \times 25 \times (1.06)^{-25} \times 11.0$$
$$= 9880 \times 64.07$$
$$= 633,050$$

$$PVFS_{40} = \ddot{a}_{\overline{25}|(1.06/1.05)-1} = 22.3643$$

$$NC = \frac{633050}{22.3643} \times 1.05^{10}$$
$$= 28306 \times 105^{10}$$
$$= \$46,108$$

Solution #3

(a) Member A:

$$AL@\text{Jan. }1,2001 = B_{(x)}\left[\sum_{t=x}^{y} v^{t-n}\ {}_{t-x}P_2\left(q_t^{ret}\{1-0.05(60-t)\}\ddot{a}_t^{(12)}\right)\right]$$

$$= 20\times 12\times 25\left[0.1\times 0.85\times 10+\frac{0.9\times 0.1\times 0.9\times 9}{1.07}+\frac{0.9^2\times 0.1\times 0.95\times 8}{1.07^2}+\frac{0.9^3\times 7}{1.07^3}\right]$$

$$= 6000[6.2346]$$

$$= 37{,}407$$

$$NC@\text{ Jan. }1,2001 = \Delta B(x)\left[\sum_{t=x+1}^{y} v^{t-x}\ {}_{t-x}P_x\left(q_t^{ret}\{1-0.05(60-t)\}\ddot{a}_t^{(12)}\right)\right]$$

$$= 20\times 12\left[\frac{0.9\times 0.1\times 0.9\times 9}{1.07}+\frac{0.9^2\times 0.1\times 0.95\times 8}{1.07^2}+\frac{0.9^3\times 7}{1.07^3}\right]$$

$$= 240[5.3846]$$

$$= \$1{,}292.30$$

Member B:

$$AL@1/1/2001 = 20\times 12\times 29\left[0.1\times 0.9\times 9+\frac{0.9\times 0.1\times 0.95\times 8}{1.07}+\frac{0.9^2\times 7}{1.07^2}\right]$$

$$= 6960[6.4016]$$

$$= 44{,}555$$

$$NC@1/1/2001 = 20\times 12\left[\frac{0.9\times 0.1\times 0.95\times 8}{1.07}+\frac{0.9^2\times 7}{1.07^2}\right]$$

$$= 240\times 5.5916$$

$$= \$1{,}341.98$$

$$UL@\ 1/1/2001 = AL - \text{Fund}$$

$$= [37{,}407+44{,}555]-100{,}000$$

$$= -18{,}038 \text{ (Surplus)}$$

Solution #3 – Continued

(b)

$$AL^{A}_{1/1/2002} = 20 \times 12 \times 26\left[0.1 \times 0.9 \times 9\frac{0.9 \times 0.1 \times 0.95 \times 8}{1.07} + \frac{0.9^2 \times 7}{1.07^2}\right]$$
$$= 6240[6.4016]$$
$$= \$39{,}946$$

$$AL^{B}_{1/1/2001} = B(x) \times \text{Reduction factoı} \times \ddot{a}^{(12)}_{59}$$
$$= 20 \times 12 \times 30[1 - 0.05(60-59)] \times \left[\ddot{a}^{(12)}_{59} = 8.0\right]$$
$$= 7200(0.95)(8.0)$$
$$= \$54{,}720$$

$$AL^{C}_{1/1/2002} = \frac{20 \times 12 \times 10}{1.07^{12}}\left[0.1 \times 10.85 + \frac{0.9 \times 0.1 \times 0.9 \times 9}{1.07} + \frac{0.9^2 \times 0.1 \times 0.95 \times 8}{1.07^2} + \frac{0.9^3 \times 7}{1.07^3}\right]$$
$$= \frac{20 \times 12 \times 10}{1.07^{12}}[6.2346]$$
$$= \$6{,}644$$

$$UL_{1/1/2002} = [39{,}946 + 54{,}720 + 6644] - 112{,}000 = -10{,}690$$

Solution #3 – Continued

(c) Gain/Loss:

New Member (past service):

$$\begin{aligned} \text{Gain} &= \text{Transfer-In} - AL \\ &= 10{,}000 - 6{,}644 \\ &= 3{,}356 \end{aligned}$$

Retirement:

- Member Who Retired:
  $\text{Loss} = AL \text{ as retired} - \widetilde{AL} \text{ as active}$
  $= 54{,}720 - [44{,}555 + 1{,}342] \times 1.07 = 5610$

- Member Who Did Not Retire:
  $\text{Loss} = AL \text{ as active} - \widetilde{AL} \text{ as active}$
  $= 39{,}946 - [37{,}407 + 1292] \times 1.07$
  $= -1462$

Total Loss on Retirement = 5610 – 1462 = 4148

Investment:

$\text{Investment Return} = 100{,}000(i) - (t)$

$\text{Loss} = 100{,}000(0.07 - 0.02) = 5{,}000 \text{ Loss}$

Solution #4

(a)

$$\underbrace{AL@\,1/1/2001}_{\text{(As active)}} = B(y)\frac{D_y}{D_x}\left(\ddot{a}^{(12)}_{\overline{5}|} + \frac{N^{(12)}_{70}}{D_{65}}\right) \times \frac{N_w - N_x}{N_w - N_y}$$

$$= 30 \times 12 \times 40 \times \left[\ddot{a}^{(12)}_{\overline{5}|} + {}_5p_{65}\, v^5 \ddot{a}^{(12)}_{70}\right] v^{65-60} \frac{\ddot{a}_{\overline{x-w=35}|}}{\ddot{a}_{\overline{y-w=40}|}}$$

$$= 14{,}400 \times \left[4.2541 + \frac{0.9039 \times 8.4642}{(1.07)^5}\right] \frac{1}{1.07^5} \times \frac{13.854}{14.2649}$$

$$= 96{,}811$$

$$\underbrace{AL@\,1/1/2001}_{\text{(As retired)}} = B(x)\frac{D_y}{D_x}\left(\ddot{a}^{(12)}_{\overline{5}|} + \frac{N^{(12)}_{70}}{D_{65}}\right)$$

$$= 30 \times 12 \times 35 \times v^5 \times \left[\ddot{a}^{(12)}_{\overline{5}|} + {}_5p_{65}\, v^5\, \ddot{a}^{(12)}_{70}\right]$$

$$= \frac{30 \times 12 \times 35}{1.07^5}\left[4.2541 + \frac{0.9039 \times 8.4642}{1.07^5}\right]$$

$$= \$87{,}222$$

$$\text{Gain} / (\text{Loss}) = AL(\text{as active}) - AL(\text{as retired})$$

$$= \$96{,}811 - \$87{,}222$$

$$= \$9{,}589$$

Solution #4 – Continued

(b)

$$\begin{aligned}\text{Factor 75\% J\&S} &= \ddot{a}_x^{(12)} + 75\%\left(\ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)}\right)\\ &= \ddot{a}_{60}^{(12)} + 75\%\left(\ddot{a}_{57}^{(12)} - \ddot{a}_{60:57}^{(12)}\right)\\ &= 10.8387 + 75\%(12.5296 - 9.7460)\\ &= 12.9264\end{aligned}$$

$$\begin{aligned}\text{Optional pension} &= \frac{AL(\text{as retired})}{\text{Factor 75\% J\&S}}\\ &= \frac{\$87{,}222}{12.9264}\\ &= \$6{,}747.59\,/\,\text{year} \Rightarrow \$562.30\,/\,\text{month}\end{aligned}$$

Solution #5

(a) $$FIL\ NC_t = \frac{\left[\sum PVFB_t - AL\right]}{\sum PVFS_t} \times \sum S_t$$

where AL_0 is determined using the EAN method.

$$EAN\ NC_x = \frac{PVFBW_x}{PVFSW_x} \times S_x$$

and $EAN\ AL_x = PVFB_x - PVFNC_x$

<u>Member J:</u>

$$PVFBW = 0.85 \times \left[6000 + 750 S_{\overline{15}|4\%}\right] \times \ddot{a}_{60}^{(12)} \times v^{(60-30)}$$
$$= 0.85 \times [6000 + 750 \times 27.671] \times 11.4 \times v^{30}$$
$$= 0.85 \times 26{,}753 \times 11.4 \times v^{30}$$
$$= 39{,}193$$

$$PVFSW = \frac{50{,}000}{(1.04)^{11}} \times \ddot{a}_{\overline{30}|j\%} \text{ where } j = \frac{1.065}{1.04} - 1$$
$$= \frac{50{,}000}{(1.04)^{11}} \times 21.711$$
$$= 705{,}153$$

$$EAN\ NC_t = \frac{39{,}193}{705{,}153} \times 50{,}000 = 2779$$

$$PVFNC_t = 2779 \times \ddot{a}_{\overline{19}|j\%}$$
$$= 2779 \times 15.473 = 42{,}999$$

$$EAN\ AL_t = 78{,}353 - 42{,}999 = 35{,}354$$

$$PVFS_t = 50{,}000 \times \ddot{a}_{\overline{19}|j\%}$$
$$= 50{,}000 \times 15.473 = 773{,}650$$

Solution #5 – Continued

Member K:

$$PVFBW = 0.85 \times \left[12{,}000 + 900_{s_{\overline{7}|j}}\right] \times \ddot{a}_{60}^{(12)} \times v^{(60-35)}$$
$$= 0.85 \times [12{,}000 + 900 \times 7.898] \times 11.4 \times v^{25}$$
$$= 0.85 \times 19{,}108 \times 11.4 \times v^{25} = 38{,}353$$

$$PVFSW = \frac{60{,}000}{(1.04)^{18}} \times \ddot{a}_{\overline{25}|j\%}$$
$$= \frac{60{,}000}{(1.04)^{18}} \times 19.076 = 564{,}987$$

$$EAN\ NC_t = \frac{38{,}353}{564{,}987} \times 60{,}000 = 4073$$

$$PVFB_t = 38{,}353 \times 1.065^{18} = 119{,}150$$

$$PVFNC_t = 4073 \times \ddot{a}_{\overline{7}|j\%}$$
$$= 4073 \times 6.526 = 26{,}580$$

$$EAN\ AL_t = 119{,}150 - 26{,}580 = 92{,}570$$

$$PVFS_t = 60{,}000 \times \ddot{a}_{\overline{7}|j\%}$$
$$= 60{,}000 \times 6.526 = 391{,}560$$

Aggregate

$$AL_t = 35{,}354 + 92{,}570 = 127{,}924$$

$$\sum PVFB_t = 78{,}353 + 119{,}150 = 197{,}503$$

$$\sum PVFS_t = 773{,}650 + 391{,}560 = 1{,}165{,}210$$

$$\sum S_t = 50{,}000 + 60{,}000 = 110{,}000$$

$$FIL\ NC_t = \frac{[197{,}503 - 127{,}924]}{1{,}165{,}210} \times 110{,}000$$
$$= 5.97\% \times 110{,}000 = 6569$$

Solution #5 – Continued

(b) $IA\ NC_x = \frac{(PVFB - F)}{PVFS} \times S$

$F = B(x) \times ER \text{ red'n} \times \ddot{a}_{60}^{(12)} \times v^{(60-x)}$

Member J:

$F = 6000 \times 0.85 \times \ddot{a}_{60}^{(12)} \times v^{19} = 17{,}572$

$PVFB = 78{,}353$

$PVFS = 773{,}650$

$IA\ NC_x = \frac{(78{,}353 - 17{,}572)}{773{,}650} \times 50{,}000 = 3928$

Member K:

$F = 12{,}000 \times 0.85 \times \ddot{a}_{60}^{(12)} \times v^{7} = 74{,}827$

$PVFB = 119{,}150$

$PVFS = 391{,}560$

$IA\ NC_x = \frac{(119{,}150 - 74{,}827)}{391{,}560} \times 60{,}000 = 6792$

Total $NC = 3928 + 6792 = 10{,}720$