# Exam C Construction and Evaluation of Actuarial Models

The examination for this material consists of four hours of multiple-choice questions and is identical to CAS Exam 4.

This material provides an introduction to modeling and covers important actuarial methods that are useful in modeling. A thorough knowledge of calculus, probability and mathematical statistics is assumed.

The candidate will be introduced to useful frequency and severity models beyond those covered in Exam M. The candidate will be required to understand the steps involved in the modeling process and how to carry out these steps in solving business problems. The candidate should be able to: 1) analyze data from an application in a business context; 2) determine a suitable model including parameter values; and 3) provide measures of confidence for decisions based upon the model. The candidate will be introduced to a variety of tools for the calibration and evaluation of the models.

A variety of tables will be provided to the candidate in the study note package and at the examination. These include values for the standard normal distribution, chi-square distribution, and abridged inventories of discrete and continuous probability distributions. These tables are also available on the SOA and CAS Web sites. Since they will be included with the examination, candidates will not be allowed to bring copies of the tables into the examination room.

#### LEARNING OUTCOMES

The candidate is expected to be familiar with survival, severity, frequency and aggregate models, and use statistical methods to estimate parameters of such models given sample data. The candidate is further expected to identify steps in the modeling process, understand the underlying assumptions implicit in each family of models, recognize which assumptions are applicable in a given business application, and appropriately adjust the models for impact of insurance coverage modifications.

Specifically, the candidate is expected to be able to perform the tasks listed below:

#### LEARNING OUTCOMES

#### A. Severity Models

- 1. Calculate the basic distributional quantities:
  - a) Moments,
  - b) Percentiles,
  - c) Generating functions.
- 2. Describe how changes in parameters affect the distribution.
- 3. Recognize classes of distributions and their relationships.
- 4. Apply the following techniques for creating new families of distributions:
  - a) Multiplication by a constant,
  - b) Raising to a power
  - c) Exponentiation,
  - d) Mixing.
- 5. Identify the applications in which each distribution is used and reasons why.
- 6. Apply the distribution to an application, given the parameters.
- Calculate various measures of tail weight and interpret the results to compare the tail weights.
- 8. Explain the properties of the lognormal distribution.
- 9. Explain the Black-Scholes formula as a limited expected value for a lognormal distribution.

- B. Frequency Models
  - a. For the Poisson, Mixed Poisson, Binomial, Negative Binomial, Geometric distribution and mixtures thereof (as well as compound distributions):
    - o Describe how changes in parameters affect the distribution,
    - o Calculate moments,
    - o Identify the applications for which each distribution is used and reasons why,
    - Apply the distribution to an application given the parameters.
- C. Aggregate Models
  - 1. Compute relevant parameters and statistics for collective risk models.
  - 2. Evaluate compound models for aggregate claims.
  - 3. Compute aggregate claims distributions.
- D. For severity, frequency and aggregate models,
  - 1. Evaluate the impacts of coverage modifications:
    - a) Deductibles,
    - b) Limits, and
    - c) Coinsurance.
  - 2. Calculate Loss Elimination Ratios.
  - 3. Evaluate effects of inflation on losses.
- E. Risk Measures
  - 1. Calculate risk measures VaR, CTE and explain their use and limitations
- F. Ruin Theory
  - 1. Calculate survival and ruin probabilities using discrete models.
  - 2. Describe the considerations included in a ruin model
- G. Construction of Empirical Models
  - 1. Estimate failure time and loss distributions using
    - a) Kaplan-Meier estimator, including approximations for large data sets
    - b) Nelson-Aalen estimator
    - c) Kernel density estimators
  - 2. Estimate the variance of estimators and confidence intervals for failure time and loss distributions.
  - 3. Estimate failure time and loss distributions with the Cox proportional hazards model and other basic models with covariates.
  - 4. Apply the following concepts in estimating failure time and loss distribution
    - a) Unbiasedness
    - b) Consistency
    - c) Mean squared error
- H. Construction and Selection of Parametric Models
  - 1. Estimate the parameters of failure time and loss distributions using
    - a) Maximum likelihood
    - b) Method of moments
    - c) Percentile matching
    - d) Bayesian procedures
  - 2. Estimate the parameters of failure time and loss distributions with censored and/or truncated data using maximum likelihood.
  - 3. Estimate the variance of estimators and the confidence intervals for the parameters and functions of parameters of failure time and loss distributions.
  - 4. Apply the following concepts in estimating failure time and loss distributions
    - a) Unbiasedness

- b) Asymptotic unbiasedness
- c) Consistency
- d) Mean squared error
- e) Uniform minimum variance
- 5. Determine the acceptability of a fitted model using
  - a) Graphical procedures
  - b) Kolmogorov-Smirnov test
  - c) Anderson-Darling test
  - d) Chi-square goodness-of-fit test
  - e) Likelihood ratio test
- I. Credibility
  - 1. Apply limited fluctuation (classical) credibility including criteria for both full and partial credibility.
  - 2. Perform Bayesian analysis using both discrete and continuous models.
  - 3. Apply Bühlmann and Bühlmann-Straub models and understand the relationship of these to the Bayesian model.
  - 4. Apply conjugate priors in Bayesian analysis and in particular the Poisson-gamma model.
  - 5. Apply empirical Bayesian methods in the nonparametric and semiparametric cases.
- J. Simulation
  - 1. Simulate both discrete and continuous random variables using the inversion method.
  - 2. Estimate the number of simulations needed to obtain an estimate with a given error and a given degree of confidence.
  - 3. Use simulation to determine the p-value for a hypothesis test.
  - 4. Use the bootstrap method to estimate the mean squared error of an estimator.
  - 5. Apply simulation methods within the context of actuarial models.
  - 6. Simulate lognormal stock prices.
  - 7. Incorporate jumps in stock prices by mixing Poisson and lognormal random variables.
  - 8. Use variance reduction techniques to accelerate convergence.
  - 9. Use the Cholesky decomposition method for simulating correlated random variables.

#### Texts

- Loss Models: From Data to Decisions, (Second Edition), 2004, by Klugman, S.A., Panjer, H.H. and Willmot, G.E., Chapter 3, Chapter 4, Sections 4.1-4.6.6 only, Chapter 5, Chapter 6, Sections 6.1-6.7, 6.11.1 only, Chapter 7, Sections 7.1, 7.2.3, 7.3.1, 7.3.2 only, Chapters 9–11, Chapter 12 (excluding 12.5.4, 12.5.5 and 12.6), Chapter 13, and Chapter 17.
- *# Derivatives Markets* (Second Edition), 2006, by McDonald, R.L., Chapters 18-19, excluding appendices.

# **Reading Options for Credibility**

The candidate may use any of the alternatives shown below.

# **Option A**

Loss Models: From Data to Decisions, (Second Edition), 2004, by Klugman, S.A., Panjer, H.H., and Willmot, G.E., Chapter 16, Sections 16.3, 16.4 (excluding16.4.7), 16.5 (excluding 16.5.3, 16.1 (background only), 16.2 (background only).

# **Option B**

 Foundations of Casualty Actuarial Science (Fourth Edition), 2001, Casualty Actuarial Society, Chapter 8, "Credibility", by Mahler, H.C., and Dean C.G., Section 1 (background only) Sections 2–5 (Available as SN C-21-01). • Topics in Credibility Theory (Study Note C-24-05) by Dean, C.G.

#### **Option C**

• Introduction to Credibility Theory (Third Edition), 1999, Herzog, T.N., Chapter 1-3 (background only), 4–8, and 9 (background only).

# **Study Notes**

SNs for the Preliminary Education examinations are available on the SOA Web site under Exams and Jobs/Candidate and Exam Information/Spring Exam Session/Spring 2007 Basic Education Catalog – Study Notes Information. Hard copies may be purchased by using the Study Note and Published Reference order form in the back of the printed catalog or by downloading the form from the Spring Exam Session Web page.

Title
Exam C Introductory Study Note
Sample Problem Mapping for Exam C
Past Examinations and Solutions
Exam C Sample Questions and Solutions
May 2005 Exam C Questions and Solutions
November 2005 Exam C Questions and Solutions
Credibility (to be used with Option B only)
Topics in Credibility Theory (to be used with Option B only)
An Introduction to Risk Measures for Actuarial Applications