Preliminary
Exam C, Fall 2006

ANSWER KEY

Question #	*Answer*
1	E
2	D
3	B
4	C
5	A
6	D
7	B
8	C
9	E
10	D
11	E
12	B
13	C
14	A
15	B
16	E
17	D
18	D

Question #	*Answer*
19	B
20	D
21	A
22	A
23	E
24	E
25	D
26	A
27	C
28	C
29	C
30	B
31	C
32	A
33	B
34	A
35	A

BEGINNING OF EXAMINATION

1. You are given:

(i) Losses follow a Burr distribution with $\alpha = 2$.

(ii) A random sample of 15 losses is:

195 255 270 280 350 360 365 380 415 450 490 550 575 590 615

(iii) The parameters γ and θ are estimated by percentile matching using the smoothed empirical estimates of the 30th and 65th percentiles.

Calculate the estimate of γ.

(A) Less than 2.9

(B) At least 2.9, but less than 3.2

(C) At least 3.2, but less than 3.5

(D) At least 3.5, but less than 3.8

(E) At least 3.8

2. An insurance company sells three types of policies with the following characteristics:

Type of Policy	Proportion of Total Policies	Annual Claim Frequency
I	5%	Poisson with $\lambda = 0.25$
II	20%	Poisson with $\lambda = 0.50$
III	75%	Poisson with $\lambda = 1.00$

A randomly selected policyholder is observed to have a total of one claim for Year 1 through Year 4.

For the same policyholder, determine the Bayesian estimate of the expected number of claims in Year 5.

(A) Less than 0.4

(B) At least 0.4, but less than 0.5

(C) At least 0.5, but less than 0.6

(D) At least 0.6, but less than 0.7

(E) At least 0.7

3. You are given a random sample of 10 claims consisting of two claims of 400, seven claims of 800, and one claim of 1600.

Determine the empirical skewness coefficient.

(A) Less than 1.0

(B) At least 1.0, but less than 1.5

(C) At least 1.5, but less than 2.0

(D) At least 2.0, but less than 2.5

(E) At least 2.5

4. You are given:

(i) The cumulative distribution for the annual number of losses for a policyholder is:

n	$F_N(n)$
0	0.125
1	0.312
2	0.500
3	0.656
4	0.773
5	0.855
⋮	⋮

(ii) The loss amounts follow the Weibull distribution with $\theta = 200$ and $\tau = 2$.

(iii) There is a deductible of 150 for each claim subject to an annual maximum out-of-pocket of 500 per policy.

The inversion method is used to simulate the number of losses and loss amounts for a policyholder.

(a) For the number of losses use the random number 0.7654.

(b) For loss amounts use the random numbers:

0.2738 0.5152 0.7537 0.6481 0.3153

Use the random numbers in order and only as needed.

Based on the simulation, calculate the insurer's aggregate payments for this policyholder.

(A) 106.93

(B) 161.32

(C) 224.44

(D) 347.53

(E) 520.05

5. You have observed the following three loss amounts:

186 91 66

Seven other amounts are known to be less than or equal to 60. Losses follow an inverse exponential with distribution function

$$F(x) = e^{-\theta/x}, \quad x > 0$$

Calculate the maximum likelihood estimate of the population mode.

(A) Less than 11

(B) At least 11, but less than 16

(C) At least 16, but less than 21

(D) At least 21, but less than 26

(E) At least 26

6. For a group of policies, you are given:

(i) The annual loss on an individual policy follows a gamma distribution with parameters $\alpha = 4$ and θ.

(ii) The prior distribution of θ has mean 600.

(iii) A randomly selected policy had losses of 1400 in Year 1 and 1900 in Year 2.

(iv) Loss data for Year 3 was misfiled and unavailable.

(v) Based on the data in (iii), the Bühlmann credibility estimate of the loss on the selected policy in Year 4 is 1800.

(vi) After the estimate in (v) was calculated, the data for Year 3 was located. The loss on the selected policy in Year 3 was 2763.

Calculate the Bühlmann credibility estimate of the loss on the selected policy in Year 4 based on the data for Years 1, 2 and 3.

(A) Less than 1850

(B) At least 1850, but less than 1950

(C) At least 1950, but less than 2050

(D) At least 2050, but less than 2150

(E) At least 2150

7. The following is a sample of 10 payments:

$4 \quad 4 \quad 5^+ \quad 5^+ \quad 5^+ \quad 8 \quad 10^+ \quad 10^+ \quad 12 \quad 15$

where + indicates that a loss exceeded the policy limit.

Determine Greenwood's approximation to the variance of the product-limit estimate $\hat{S}(11)$.

(A) 0.016

(B) 0.031

(C) 0.048

(D) 0.064

(E) 0.075

8. Determine $f(3)$ using the second degree polynomial that interpolates the points:

(2, 25) (4, 20) (5, 30)

(A) Less than 15

(B) At least 15, but less than 18

(C) At least 18, but less than 21

(D) At least 21, but less than 23

(E) At least 23

9. You are given:

(i) For $Q = q$, $X_1, X_2, \ldots, X_m$ are independent, identically distributed Bernoulli random variables with parameter q.

(ii) $S_m = X_1 + X_2 + \cdots + X_m$

(iii) The prior distribution of Q is beta with $a = 1$, $b = 99$, and $\theta = 1$.

Determine the smallest value of m such that the mean of the marginal distribution of S_m is greater than or equal to 50.

(A) 1082

(B) 2164

(C) 3246

(D) 4950

(E) 5000

10. You are given:

(i) A portfolio consists of 100 identically and independently distributed risks.

(ii) The number of claims for each risk follows a Poisson distribution with mean λ.

(iii) The prior distribution of λ is:

$$\pi(\lambda) = \frac{(50\lambda)^4 e^{-50\lambda}}{6\lambda}, \quad \lambda > 0$$

During Year 1, the following loss experience is observed:

Number of Claims	Number of Risks
0	90
1	7
2	2
3	1
Total	100

Determine the Bayesian expected number of claims for the portfolio in Year 2.

(A) 8

(B) 10

(C) 11

(D) 12

(E) 14

11. You are planning a simulation to estimate the mean of a non-negative random variable. It is known that the population standard deviation is 20% larger than the population mean.

Use the central limit theorem to estimate the smallest number of trials needed so that you will be at least 95% confident that the simulated mean is within 5% of the population mean.

(A) 944

(B) 1299

(C) 1559

(D) 1844

(E) 2213

12. You are given:

(i) The distribution of the number of claims per policy during a one-year period for 10,000 insurance policies is:

Number of Claims per Policy	Number of Policies
0	5000
1	5000
2 or more	0

(ii) You fit a binomial model with parameters m and q using the method of maximum likelihood.

Determine the maximum value of the loglikelihood function when $m = 2$.

(A) −10,397

(B) −7,781

(C) −7,750

(D) −6,931

(E) −6,730

13. You are given:

(i) Over a three-year period, the following claim experience was observed for two insureds who own delivery vans:

		Year		
Insured		1	2	3
A	Number of Vehicles	2	2	1
	Number of Claims	1	1	0
B	Number of Vehicles	N/A	3	2
	Number of Claims	N/A	2	3

(ii) The number of claims for each insured each year follows a Poisson distribution.

Determine the semiparametric empirical Bayes estimate of the claim frequency per vehicle for Insured A in Year 4.

(A) Less than 0.55

(B) At least 0.55, but less than 0.60

(C) At least 0.60, but less than 0.65

(D) At least 0.65, but less than 0.70

(E) At least 0.70

14. For the data set

200 300 100 400 X

you are given:

(i) $k = 4$

(ii) $s_2 = 1$

(iii) $r_4 = 1$

(iv) The Nelson-Åalen Estimate $\hat{H}(410) > 2.15$

Determine X.

(A) 100

(B) 200

(C) 300

(D) 400

(E) 500

15. You are given:

(i) A hospital liability policy has experienced the following numbers of claims over a 10-year period:

10 2 4 0 6 2 4 5 4 2

(ii) Numbers of claims are independent from year to year.

(iii) You use the method of maximum likelihood to fit a Poisson model.

Determine the estimated coefficient of variation of the estimator of the Poisson parameter.

(A) 0.10

(B) 0.16

(C) 0.22

(D) 0.26

(E) 1.00

16. You are given:

(i) Claim sizes follow an exponential distribution with mean θ.

(ii) For 80% of the policies, $\theta = 8$.

(iii) For 20% of the policies, $\theta = 2$.

A randomly selected policy had one claim in Year 1 of size 5.

Calculate the Bayesian expected claim size for this policy in Year 2.

(A) Less than 5.8

(B) At least 5.8, but less than 6.2

(C) At least 6.2, but less than 6.6

(D) At least 6.6, but less than 7.0

(E) At least 7.0

17. For a double-decrement study, you are given:

(i) The following survival data for individuals affected by both decrements (1) and (2):

j	c_j	$q_j^{(T)}$
0	0	0.100
1	20	0.182
2	40	0.600
3	60	1.000

(ii) $q_j'^{(2)} = 0.05$ for all j

(iii) Group A consists of 1000 individuals observed at age 0.

(iv) Group A is affected by only decrement (1).

Determine the Kaplan-Meier multiple-decrement estimate of the number of individuals in Group A that survive to be at least 40 years old.

(A) 343

(B) 664

(C) 736

(D) 816

(E) 861

18. You are given:

(i) At time 4 hours, there are 5 working light bulbs.

(ii) The 5 bulbs are observed for p more hours.

(iii) Three light bulbs burn out at times 5, 9, and 13 hours, while the remaining light bulbs are still working at time $4 + p$ hours.

(iv) The distribution of failure times is uniform on $(0,\omega)$.

(v) The maximum likelihood estimate of ω is 29.

Determine p.

(A) Less than 10

(B) At least 10, but less than 12

(C) At least 12, but less than 14

(D) At least 14, but less than 16

(E) At least 16

19. You are given:

(i) The number of claims incurred in a month by any insured follows a Poisson distribution with mean λ.

(ii) The claim frequencies of different insureds are independent.

(iii) The prior distribution of λ is Weibull with $\theta = 0.1$ and $\tau = 2$.

(iv) Some values of the gamma function are

$$\Gamma(0.5) = 1.77245, \quad \Gamma(1) = 1, \quad \Gamma(1.5) = 0.88623, \quad \Gamma(2) = 1$$

(v)

Month	Number of Insureds	Number of Claims
1	100	10
2	150	11
3	250	14

Determine the Bühlmann-Straub credibility estimate of the number of claims in the next 12 months for 300 insureds.

(A) Less than 255

(B) At least 255, but less than 275

(C) At least 275, but less than 295

(D) At least 295, but less than 315

(E) At least 315

20. You are given:

(i) The following data set:

2500 2500 2500 3617 3662 4517 5000 5000 6010 6932 7500 7500

(ii) $\hat{H}_1(7000)$ is the Nelson-Åalen estimate of the cumulative hazard rate function calculated under the assumption that all of the observations in (i) are uncensored.

(iii) $\hat{H}_2(7000)$ is the Nelson-Åalen estimate of the cumulative hazard rate function calculated under the assumption that all occurrences of the values 2500, 5000 and 7500 in (i) reflect right-censored observations and that the remaining observed values are uncensored.

Calculate $|\hat{H}_1(7000) - \hat{H}_2(7000)|$.

(A) Less than 0.1

(B) At least 0.1, but less than 0.3

(C) At least 0.3, but less than 0.5

(D) At least 0.5, but less than 0.7

(E) At least 0.7

21. For a warranty product you are given:

(i) Paid losses follow the lognormal distribution with $\mu = 13.294$ and $\sigma = 0.494$.

(ii) The ratio of estimated unpaid losses to paid losses, y, is modeled by

$$y = 0.801x^{0.851}e^{-0.747x}$$

where

x = 2006 − contract purchase year

The inversion method is used to simulate four paid losses with the following four uniform (0,1) random numbers:

0.2877 0.1210 0.8238 0.6179

Using the simulated values, calculate the empirical estimate of the average unpaid losses for purchase year 2005.

(A) Less than 300,000

(B) At least 300,000, but less than 400,000

(C) At least 400,000, but less than 500,000

(D) At least 500,000, but less than 600,000

(E) At least 600,000

22. Five models are fitted to a sample of $n = 260$ observations with the following results:

Model	Number of Parameters	Loglikelihood
I	1	−414
II	2	−412
III	3	−411
IV	4	−409
V	6	−409

Determine the model favored by the Schwarz Bayesian criterion.

(A) I

(B) II

(C) III

(D) IV

(E) V

23. You are given:

(i) The annual number of claims for an individual risk follows a Poisson distribution with mean λ.

(ii) For 75% of the risks, $\lambda = 1$.

(iii) For 25% of the risks, $\lambda = 3$.

A randomly selected risk had r claims in Year 1. The Bayesian estimate of this risk's expected number of claims in Year 2 is 2.98.

Determine the Bühlmann credibility estimate of the expected number of claims for this risk in Year 2.

(A) Less than 1.9

(B) At least 1.9, but less than 2.3

(C) At least 2.3, but less than 2.7

(D) At least 2.7, but less than 3.1

(E) At least 3.1

24. You are given the following ages at time of death for 10 individuals:

25	30	35	35	37	39	45	47	49	55

Using a uniform kernel with bandwidth $b = 10$, determine the kernel density estimate of the probability of survival to age 40.

(A) 0.377

(B) 0.400

(C) 0.417

(D) 0.439

(E) 0.485

25. The following is a natural cubic spline passing through the points (0, 3), (1, 2), (3, 6):

$$f(x) = \begin{cases} 3 - \left(\tfrac{3}{2}\right)x + \left(\tfrac{1}{2}\right)x^3, & 0 \le x \le 1 \\ 2 + \left(\tfrac{3}{2}\right)(x-1)^2 - \left(\tfrac{1}{4}\right)(x-1)^3, & 1 \le x \le 3 \end{cases}$$

Using the method of extrapolation as given in the Loss Models text, determine $f(4)$.

(A) 7.0

(B) 8.0

(C) 8.8

(D) 9.0

(E) 10.0

26. The random variables $X_1, X_2, \ldots, X_n$ are independent and identically distributed with probability density function

$$f(x) = \frac{e^{-x/\theta}}{\theta}, \quad x \geq 0$$

Determine $E\left[\bar{X}^2\right]$.

(A) $\left(\frac{n+1}{n}\right)\theta^2$

(B) $\left(\frac{n+1}{n^2}\right)\theta^2$

(C) $\frac{\theta^2}{n}$

(D) $\frac{\theta^2}{\sqrt{n}}$

(E) θ^2

27. Three individual policyholders have the following claim amounts over four years:

Policyholder	Year 1	Year 2	Year 3	Year 4
X	2	3	3	4
Y	5	5	4	6
Z	5	5	3	3

Using the nonparametric empirical Bayes procedure, calculate the estimated variance of the hypothetical means.

(A) Less than 0.40

(B) At least 0.40, but less than 0.60

(C) At least 0.60, but less than 0.80

(D) At least 0.80, but less than 1.00

(E) At least 1.00

28. You are given:

(i) A Cox proportional hazards model was used to compare the fuel economies of traditional and hybrid cars.

(ii) A single covariate z was used with $z = 0$ for a traditional car and $z = 1$ for a hybrid car.

(iii) The following are sample values of miles per gallon for the two types of car:

Traditional:	22	25	28	33	39
Hybrid:	27	31	35	42	45

(iv) The partial maximum likelihood estimate of the coefficient β is –1.

Calculate the estimate of the baseline cumulative hazard function $H_0(32)$ using an analog of the Nelson-Åalen estimator which is appropriate for proportional hazard models.

(A) Less than 0.7

(B) At least 0.7, but less than 0.9

(C) At least 0.9, but less than 1.1

(D) At least 1.1, but less than 1.3

(E) At least 1.3, but less than 1.5

29. You are given:

(i) The number of claims made by an individual in any given year has a binomial distribution with parameters $m = 4$ and q.

(ii) The prior distribution of q has probability density function

$$\pi(q) = 6q(1-q), \quad 0 < q < 1.$$

(iii) Two claims are made in a given year.

Determine the mode of the posterior distribution of q.

(A) 0.17

(B) 0.33

(C) 0.50

(D) 0.67

(E) 0.83

30. A company has determined that the limited fluctuation full credibility standard is 2000 claims if:

(i) The total number of claims is to be within 3% of the true value with probability p.

(ii) The number of claims follows a Poisson distribution.

The standard is changed so that the total cost of claims is to be within 5% of the true value with probability p, where claim severity has probability density function:

$$f(x) = \frac{1}{10,000}, \qquad 0 \le x \le 10,000$$

Using limited fluctuation credibility, determine the expected number of claims necessary to obtain full credibility under the new standard.

(A) 720

(B) 960

(C) 2160

(D) 2667

(E) 2880

31. For a mortality study with right censored data, you are given the following:

Time	Number of Deaths	Number at Risk
3	1	50
5	3	49
6	5	k
10	7	21

You are also told that the Nelson-Åalen estimate of the survival function at time 10 is 0.575.

Determine k.

(A) 28

(B) 31

(C) 36

(D) 44

(E) 46

32. A dental benefit is designed so that a deductible of 100 is applied to annual dental charges. The reimbursement to the insured is 80% of the remaining dental charges subject to an annual maximum reimbursement of 1000.

You are given:

(i) The annual dental charges for each insured are exponentially distributed with mean 1000.

(ii) Use the following uniform (0, 1) random numbers and the inversion method to generate four values of annual dental charges:

0.30 0.92 0.70 0.08

Calculate the average annual reimbursement for this simulation.

(A) 522

(B) 696

(C) 757

(D) 947

(E) 1042

33. For a group of policies, you are given:

(i) Losses follow the distribution function

$$F(x) = 1 - \theta / x, \qquad \theta < x < \infty .$$

(ii) A sample of 20 losses resulted in the following:

Interval	Number of Losses
$x \le 10$	9
$10 < x \le 25$	6
$x > 25$	5

Calculate the maximum likelihood estimate of θ.

(A) 5.00

(B) 5.50

(C) 5.75

(D) 6.00

(E) 6.25

34. You are given:

(i) Loss payments for a group health policy follow an exponential distribution with unknown mean.

(ii) A sample of losses is:

100	200	400	800	1400	3100

Use the delta method to approximate the variance of the maximum likelihood estimator of $S(1500)$.

(A) 0.019

(B) 0.025

(C) 0.032

(D) 0.039

(E) 0.045

35. You are given:

(i) A random sample of payments from a portfolio of policies resulted in the following:

Interval	Number of Policies
(0, 50]	36
(50, 150]	x
(150, 250]	y
(250, 500]	84
(500, 1000]	80
(1000, ∞)	0
Total	n

(ii) Two values of the ogive constructed from the data in (i) are:

$$F_n(90) = 0.21, \quad \text{and} \quad F_n(210) = 0.51$$

Calculate x.

(A) 120

(B) 145

(C) 170

(D) 195

(E) 220

****END OF EXAMINATION****