Exam C, Fall 2005

FINAL ANSWER KEY

Question #	*Answer*
1	D
2	A
3	E
4	B
5	E
6	E
7	A
8	D
9	B
10	D and E
11	D
12	C
13	C
14	C
15	A
16	D
17	D
18	A

Question #	*Answer*
19	B
20	A
21	B
22	A
23	E
24	B and C
25	C
26	C
27	A
28	B
29	C
30	D
31	B
32	B
33	E
34	A
35	E

****BEGINNING OF EXAMINATION****

1. A portfolio of policies has produced the following claims:

100 100 100 200 300 300 300 400 500 600

Determine the empirical estimate of $H(300)$.

(A) Less than 0.50

(B) At least 0.50, but less than 0.75

(C) At least 0.75, but less than 1.00

(D) At least 1.00, but less than 1.25

(E) At least 1.25

2. You are given:

(i) The conditional distribution of the number of claims per policyholder is Poisson with mean λ.

(ii) The variable λ has a gamma distribution with parameters α and θ.

(iii) For policyholders with 1 claim in Year 1, the credibility estimate for the number of claims in Year 2 is 0.15.

(iv) For policyholders with an average of 2 claims per year in Year 1 and Year 2, the credibility estimate for the number of claims in Year 3 is 0.20.

Determine θ.

(A) Less than 0.02

(B) At least 0.02, but less than 0.03

(C) At least 0.03, but less than 0.04

(D) At least 0.04, but less than 0.05

(E) At least 0.05

3. A random sample of claims has been drawn from a Burr distribution with known parameter $\alpha = 1$ and unknown parameters θ and γ. You are given:

(i) 75% of the claim amounts in the sample exceed 100.

(ii) 25% of the claim amounts in the sample exceed 500.

Estimate θ by percentile matching.

(A) Less than 190

(B) At least 190, but less than 200

(C) At least 200, but less than 210

(D) At least 210, but less than 220

(E) At least 220

4. You are given:

(i) $f(x)$ is a cubic spline with knots (0, 0) and (2, 2).

(ii) $f'(0) = 1$ and $f''(2) = -24$

Determine $f(1)$.

(A) 1

(B) 4

(C) 6

(D) 8

(E) 10

5. For a portfolio of policies, you are given:

(i) There is no deductible and the policy limit varies by policy.

(ii) A sample of ten claims is:

350 350 500 500 500^{+} 1000 1000^{+} 1000^{+} 1200 1500

where the symbol + indicates that the loss exceeds the policy limit.

(iii) $\hat{S}_1(1250)$ is the product-limit estimate of $S(1250)$.

(iv) $\hat{S}_2(1250)$ is the maximum likelihood estimate of $S(1250)$ under the assumption that the losses follow an exponential distribution.

Determine the absolute difference between $\hat{S}_1(1250)$ and $\hat{S}_2(1250)$.

(A) 0.00

(B) 0.03

(C) 0.05

(D) 0.07

(E) 0.09

6. The random variable X has survival function:

$$S_X(x) = \frac{\theta^4}{\left(\theta^2 + x^2\right)^2}$$

Two values of X are observed to be 2 and 4. One other value exceeds 4.

Calculate the maximum likelihood estimate of θ.

(A) Less than 4.0

(B) At least 4.0, but less than 4.5

(C) At least 4.5, but less than 5.0

(D) At least 5.0, but less than 5.5

(E) At least 5.5

7. For a portfolio of policies, you are given:

(i) The annual claim amount on a policy has probability density function:

$$f(x|\theta) = \frac{2x}{\theta^2}, \quad 0 < x < \theta$$

(ii) The prior distribution of θ has density function:

$$\pi(\theta) = 4\theta^3, \quad 0 < \theta < 1$$

(iii) A randomly selected policy had claim amount 0.1 in Year 1.

Determine the Bühlmann credibility estimate of the claim amount for the selected policy in Year 2.

(A) 0.43

(B) 0.45

(C) 0.50

(D) 0.53

(E) 0.56

8. Total losses for a group of insured motorcyclists are simulated using the aggregate loss model and the inversion method.

The number of claims has a Poisson distribution with $\lambda = 4$. The amount of each claim has an exponential distribution with mean 1000.

The number of claims is simulated using $u = 0.13$. The claim amounts are simulated using $u_1 = 0.05$, $u_2 = 0.95$ and $u_3 = 0.10$ in that order, as needed.

Determine the total losses.

(A) 0

(B) 51

(C) 2996

(D) 3047

(E) 3152

9. You are given:

(i) The sample:

1 2 3 3 3 3 3 3 3 3

(ii) $\hat{F}_1(x)$ is the kernel density estimator of the distribution function using a uniform kernel with bandwidth 1.

(iii) $\hat{F}_2(x)$ is the kernel density estimator of the distribution function using a triangular kernel with bandwidth 1.

Determine which of the following intervals has $\hat{F}_1(x) = \hat{F}_2(x)$ for all x in the interval.

(A) $0 < x < 1$

(B) $1 < x < 2$

(C) $2 < x < 3$

(D) $3 < x < 4$

(E) None of (A), (B), (C) or (D)

10. 1000 workers insured under a workers compensation policy were observed for one year. The number of work days missed is given below:

Number of Days of Work Missed	Number of Workers
0	818
1	153
2	25
3 or more	4
Total	1000
Total Number of Days Missed	230

The chi-square goodness-of-fit test is used to test the hypothesis that the number of work days missed follows a Poisson distribution where:

(i) The Poisson parameter is estimated by the average number of work days missed.

(ii) Any interval in which the expected number is less than one is combined with the previous interval.

Determine the results of the test.

(A) The hypothesis is not rejected at the 0.10 significance level.

(B) The hypothesis is rejected at the 0.10 significance level, but is not rejected at the 0.05 significance level.

(C) The hypothesis is rejected at the 0.05 significance level, but is not rejected at the 0.025 significance level.

(D) The hypothesis is rejected at the 0.025 significance level, but is not rejected at the 0.01 significance level.

(E) The hypothesis is rejected at the 0.01 significance level.

11. You are given the following data:

	Year 1	Year 2
Total Losses	12,000	14,000
Number of Policyholders	25	30

The estimate of the variance of the hypothetical means is 254.

Determine the credibility factor for Year 3 using the nonparametric empirical Bayes method.

(A) Less than 0.73

(B) At least 0.73, but less than 0.78

(C) At least 0.78, but less than 0.83

(D) At least 0.83, but less than 0.88

(E) At least 0.88

12. A smoothing spline is to be fit to the points (0, 3), (1, 2), and (3, 6).

The candidate function is:

$$f(x)=\begin{cases} 2.6-(4/15)x+(4/15)x^3, & 0\le x\le 1 \\ 2.6+(8/15)(x-1)+0.8(x-1)^2-(2/15)(x-1)^3 & 1\le x\le 3 \end{cases}$$

Determine the value of S, the squared norm smoothness criterion.

(A) Less than 2.35

(B) At least 2.35, but less than 2.50

(C) At least 2.50, but less than 2.65

(D) At least 2.65, but less than 2.80

(E) At least 2.80

13. You are given the following about a Cox proportional hazards model for mortality:

(i) There are two covariates: $z_1 = 1$ for smoker and 0 for non-smoker, and $z_2 = 1$ for male and 0 for female.

(ii) The parameter estimates are $\hat{\beta}_1 = 0.05$ and $\hat{\beta}_2 = 0.15$.

(iii) The covariance matrix of the parameter estimates, $\hat{\beta}_1$ and $\hat{\beta}_2$, is:

$$\begin{pmatrix} 0.0002 & 0.0001 \\ 0.0001 & 0.0003 \end{pmatrix}$$

Determine the upper limit of the 95% confidence interval for the relative risk of a female non-smoker compared to a male smoker.

(A) Less than 0.6

(B) At least 0.6, but less than 0.8

(C) At least 0.8, but less than 1.0

(D) At least 1.0, but less than 1.2

(E) At least 1.2

14. You are given:

(i) Fifty claims have been observed from a lognormal distribution with unknown parameters μ and σ.

(ii) The maximum likelihood estimates are $\hat{\mu} = 6.84$ and $\hat{\sigma} = 1.49$.

(iii) The covariance matrix of $\hat{\mu}$ and $\hat{\sigma}$ is:

$$\begin{bmatrix} 0.0444 & 0 \\ 0 & 0.0222 \end{bmatrix}$$

(iv) The partial derivatives of the lognormal cumulative distribution function are:

$$\frac{\partial F}{\partial \mu} = \frac{-\phi(z)}{\sigma} \quad \text{and} \quad \frac{\partial F}{\partial \sigma} = \frac{-z \times \phi(z)}{\sigma}$$

(v) An approximate 95% confidence interval for the probability that the next claim will be less than or equal to 5000 is:

$$[P_L, P_H]$$

Determine P_L.

(A) 0.73

(B) 0.76

(C) 0.79

(D) 0.82

(E) 0.85

15. For a particular policy, the conditional probability of the annual number of claims given $\Theta = \theta$, and the probability distribution of Θ are as follows:

Number of Claims	0	1	2
Probability	2θ	θ	$1-3\theta$

θ	0.10	0.30
Probability	0.80	0.20

One claim was observed in Year 1.

Calculate the Bayesian estimate of the expected number of claims for Year 2.

(A) Less than 1.1

(B) At least 1.1, but less than 1.2

(C) At least 1.2, but less than 1.3

(D) At least 1.3, but less than 1.4

(E) At least 1.4

16. You simulate observations from a specific distribution $F(x)$, such that the number of simulations N is sufficiently large to be at least 95 percent confident of estimating $F(1500)$ correctly within 1 percent.

Let P represent the number of simulated values less than 1500.

Determine which of the following could be values of N and P.

(A) N = 2000 P = 1890

(B) N = 3000 P = 2500

(C) N = 3500 P = 3100

(D) N = 4000 P = 3630

(E) N = 4500 P = 4020

17. For a survival study, you are given:

(i) Deaths occurred at times $y_1 < y_2 < \ldots < y_9$.

(ii) The Nelson-Aalen estimates of the cumulative hazard function at y_3 and y_4 are:

$$\hat{H}(y_3) = 0.4128 \text{ and } \hat{H}(y_4) = 0.5691$$

(iii) The estimated variances of the estimates in (ii) are:

$$\widehat{\text{Var}}[\hat{H}(y_3)] = 0.009565 \quad \text{and} \quad \widehat{\text{Var}}[\hat{H}(y_4)] = 0.014448$$

Determine the number of deaths at y_4.

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

18. A random sample of size n is drawn from a distribution with probability density function:

$$f(x) = \frac{\theta}{(\theta + x)^2} \quad , \qquad 0 < x < \infty, \quad \theta > 0$$

Determine the asymptotic variance of the maximum likelihood estimator of θ.

(A) $\dfrac{3\theta^2}{n}$

(B) $\dfrac{1}{3n\theta^2}$

(C) $\dfrac{3}{n\theta^2}$

(D) $\dfrac{n}{3\theta^2}$

(E) $\dfrac{1}{3\theta^2}$

19. For a portfolio of independent risks, the number of claims for each risk in a year follows a Poisson distribution with means given in the following table:

Class	Mean Number of Claims per Risk	Number of Risks
1	1	900
2	10	90
3	20	10

You observe x claims in Year 1 for a randomly selected risk.

The Bühlmann credibility estimate of the number of claims for the same risk in Year 2 is 11.983.

Determine x.

(A) 13

(B) 14

(C) 15

(D) 16

(E) 17

20. A survival study gave (0.283, 1.267) as the symmetric linear 95% confidence interval for $H(5)$.

Using the delta method, determine the symmetric linear 95% confidence interval for $S(5)$.

(A) (0.23, 0.69)

(B) (0.26, 0.72)

(C) (0.28, 0.75)

(D) (0.31, 0.73)

(E) (0.32, 0.80)

21. You are given:

(i) Losses on a certain warranty product in Year *i* follow a lognormal distribution with parameters μ_i and σ_i.

(ii) $\sigma_i = \sigma$, for $i = 1, 2, 3, \ldots$

(iii) The parameters μ_i vary in such a way that there is an annual inflation rate of 10% for losses.

(iv) The following is a sample of seven losses:

Year 1:	20	40	50	
Year 2:	30	40	90	120

Using trended losses, determine the method of moments estimate of μ_3.

(A) 3.87

(B) 4.00

(C) 30.00

(D) 55.71

(E) 63.01

22. You are given:

(i) A region is comprised of three territories. Claims experience for Year 1 is as follows:

Territory	Number of Insureds	Number of Claims
A	10	4
B	20	5
C	30	3

(ii) The number of claims for each insured each year has a Poisson distribution.

(iii) Each insured in a territory has the same expected claim frequency.

(iv) The number of insureds is constant over time for each territory.

Determine the Bühlmann-Straub empirical Bayes estimate of the credibility factor Z for Territory A.

(A) Less than 0.4

(B) At least 0.4, but less than 0.5

(C) At least 0.5, but less than 0.6

(D) At least 0.6, but less than 0.7

(E) At least 0.7

23. Determine which of the following is a natural cubic spline passing through the three points (0, y_1), (1, y_2), and (3, 6).

(A) $$f(x)=\begin{cases} 3-x-(7/6)x^3, & 0\le x<1 \\ 2+(1/6)(x-1)+(11/6)(x-1)^2-(11/24)(x-1)^3, & 1\le x\le 3 \end{cases}$$

(B) $$f(x)=\begin{cases} 3-x-x^2+x^3, & 0\le x<1 \\ 2+2(x-1)^2-(1/2)(x-1)^3, & 1\le x\le 3 \end{cases}$$

(C) $$f(x)=\begin{cases} 3-x-(1/2)x^2+(1/2)x^3, & 0\le x<1 \\ 2-(1/2)(x-1)+(x-1)^2-(1/8)(x-1)^3, & 1\le x\le 3 \end{cases}$$

(D) $$f(x)=\begin{cases} 3-(5/4)x-(1/2)x^2+(3/4)x^3, & 0\le x<1 \\ 2+(7/4)(x-1)^2-(3/8)(x-1)^3, & 1\le x\le 3 \end{cases}$$

(E) $$f(x)=\begin{cases} 3-(3/2)x+(1/2)x^3, & 0\le x<1 \\ 2+(3/2)(x-1)^2-(1/4)(x-1)^3, & 1\le x\le 3 \end{cases}$$

24. You are given:

(i) A Cox proportional hazards model was used to study the survival times of patients with a certain disease from the time of onset to death.

(ii) A single covariate z was used with $z = 0$ for a male patient and $z = 1$ for a female patient.

(iii) A sample of five patients gave the following survival times (in months):

Males:	10	18	25
Females:	15	21	

(iv) The parameter estimate is $\hat{\beta} = 0.27$.

Using the Nelson-Aalen estimate of the baseline cumulative hazard function, estimate the probability that a future female patient will survive more than 20 months from the time of the onset of the disease.

(A) 0.33

(B) 0.36

(C) 0.40

(D) 0.43

(E) 0.50

25. You are given:

(i) A random sample of losses from a Weibull distribution is:

595 700 789 799 1109

(ii) At the maximum likelihood estimates of θ and τ, $\sum \ln\left(f\left(x_i\right)\right) = -33.05$.

(iii) When $\tau = 2$, the maximum likelihood estimate of θ is 816.7.

(iv) You use the likelihood ratio test to test the hypothesis

$$H_0 : \tau = 2$$
$$H_1 : \tau \neq 2$$

Determine the result of the test.

(A) Do not reject H_0 at the 0.10 level of significance.

(B) Reject H_0 at the 0.10 level of significance, but not at the 0.05 level of significance.

(C) Reject H_0 at the 0.05 level of significance, but not at the 0.025 level of significance.

(D) Reject H_0 at the 0.025 level of significance, but not at the 0.01 level of significance.

(E) Reject H_0 at the 0.01 level of significance.

26. For each policyholder, losses $X_1, \ldots, X_n$, conditional on Θ, are independently and identically distributed with mean,

$$\mu(\theta) = \mathrm{E}\left(X_j \middle| \Theta = \theta\right), \quad j = 1, 2, \ldots, n$$

and variance,

$$v(\theta) = \mathrm{Var}\left(X_j \middle| \Theta = \theta\right), \quad j = 1, 2, \ldots, n.$$

You are given:

(i) The Bühlmann credibility assigned for estimating X_5 based on $X_1, \ldots, X_4$ is $Z = 0.4$.

(ii) The expected value of the process variance is known to be 8.

Calculate $\mathrm{Cov}\left(X_i, X_j\right)$, $i \neq j$.

(A) Less than -0.5

(B) At least -0.5, but less than 0.5

(C) At least 0.5, but less than 1.5

(D) At least 1.5, but less than 2.5

(E) At least 2.5

27. Losses for a warranty product follow the lognormal distribution with underlying normal mean and standard deviation of 5.6 and 0.75 respectively.

You use simulation to estimate claim payments for a number of contracts with different deductibles.

The following are four uniform (0,1) random numbers:

0.6217 0.9941 0.8686 0.0485

Using these numbers and the inversion method, calculate the average payment per loss for a contract with a deductible of 100.

(A) Less than 630

(B) At least 630, but less than 680

(C) At least 680, but less than 730

(D) At least 730, but less than 780

(E) At least 780

28. The random variable X has the exponential distribution with mean θ.

Calculate the mean-squared error of X^2 as an estimator of θ^2.

(A) $20\theta^4$

(B) $21\theta^4$

(C) $22\theta^4$

(D) $23\theta^4$

(E) $24\theta^4$

29. You are given the following data for the number of claims during a one-year period:

Number of Claims	Number of Policies
0	157
1	66
2	19
3	4
4	2
5+	0
Total	248

A geometric distribution is fitted to the data using maximum likelihood estimation.
Let P = probability of zero claims using the fitted geometric model.

A Poisson distribution is fitted to the data using the method of moments.
Let Q = probability of zero claims using the fitted Poisson model.

Calculate $|P-Q|$.

(A) 0.00

(B) 0.03

(C) 0.06

(D) 0.09

(E) 0.12

30. For a group of auto policyholders, you are given:

(i) The number of claims for each policyholder has a conditional Poisson distribution.

(ii) During Year 1, the following data are observed for 8000 policyholders:

Number of Claims	Number of Policyholders
0	5000
1	2100
2	750
3	100
4	50
5+	0

A randomly selected policyholder had one claim in Year 1.

Determine the semiparametric empirical Bayes estimate of the number of claims in Year 2 for the same policyholder.

(A) Less than 0.15

(B) At least 0.15, but less than 0.30

(C) At least 0.30, but less than 0.45

(D) At least 0.45, but less than 0.60

(E) At least 0.60

31. You are given:

(i) The following are observed claim amounts:

400 1000 1600 3000 5000 5400 6200

(ii) An exponential distribution with $\theta = 3300$ is hypothesized for the data.

(iii) The goodness of fit is to be assessed by a *p*-*p* plot and a *D*(*x*) plot.

Let (s, t) be the coordinates of the *p*-*p* plot for a claim amount of 3000.

Determine $(s-t)-D(3000)$.

(A) -0.12

(B) -0.07

(C) 0.00

(D) 0.07

(E) 0.12

32. You are given:

(i) In a portfolio of risks, each policyholder can have at most two claims per year.

(ii) For each year, the distribution of the number of claims is:

Number of Claims	Probability
0	0.10
1	$0.90-q$
2	q

(iii) The prior density is:

$$\pi(q)=\frac{q^2}{0.039}, \qquad 0.2<q<0.5$$

A randomly selected policyholder had two claims in Year 1 and two claims in Year 2.

For this insured, determine the Bayesian estimate of the expected number of claims in Year 3.

(A) Less than 1.30

(B) At least 1.30, but less than 1.40

(C) At least 1.40, but less than 1.50

(D) At least 1.50, but less than 1.60

(E) At least 1.60

33. For 500 claims, you are given the following distribution:

Claim Size	Number of Claims
[0, 500)	200
[500, 1,000)	110
[1,000, 2,000)	x
[2,000, 5,000)	y
[5,000, 10,000)	?
[10,000, 25,000)	?
[25,000, ∞)	?

You are also given the following values taken from the ogive:

$F_{500}(1500) = 0.689$

$F_{500}(3500) = 0.839$

Determine y.

(A) Less than 65

(B) At least 65, but less than 70

(C) At least 70, but less than 75

(D) At least 75, but less than 80

(E) At least 80

34. Which of statements (A), (B), (C), and (D) is false?

(A) The chi-square goodness-of-fit test works best when the expected number of observations varies widely from interval to interval.

(B) For the Kolmogorov-Smirnov test, when the parameters of the distribution in the null hypothesis are estimated from the data, the probability of rejecting the null hypothesis decreases.

(C) For the Kolmogorov-Smirnov test, the critical value for right censored data should be smaller than the critical value for uncensored data.

(D) The Anderson-Darling test does not work for grouped data.

(E) None of (A), (B), (C) or (D) is false.

35. You are given:

(i) The number of claims follows a Poisson distribution.

(ii) Claim sizes follow a gamma distribution with parameters α (unknown) and $\theta = 10{,}000$.

(iii) The number of claims and claim sizes are independent.

(iv) The full credibility standard has been selected so that actual aggregate losses will be within 10% of expected aggregate losses 95% of the time.

Using limited fluctuation (classical) credibility, determine the expected number of claims required for full credibility.

(A) Less than 400

(B) At least 400, but less than 450

(C) At least 450, but less than 500

(D) At least 500

(E) The expected number of claims required for full credibility cannot be determined from the information given.

****END OF EXAMINATION****