Exam M Fall 2005

FINAL ANSWER KEY

Question #	*Answer*		*Question #*	*Answer*
1	C		21	E
2	C		22	B
3	C		23	E
4	D		24	E
5	C		25	C
6	B		26	E
7	A		27	E
8	D		28	D
9	B		29	A
10	A		30	D
11	A		31	A
12	A		32	A
13	D		33	B
14	C		34	C
15	A		35	A
16	D		36	A
17	D		37	C
18	D		38	C
19	B		39	E
20	B		40	B

****BEGINNING OF EXAMINATION****

1. For a special whole life insurance on (x), you are given:

(i) Z is the present value random variable for this insurance.

(ii) Death benefits are paid at the moment of death.

(iii) $\mu_x(t) = 0.02, \quad t \geq 0$

(iv) $\delta = 0.08$

(v) $b_t = e^{0.03t}, \quad t \geq 0$

Calculate $\text{Var}(Z)$.

(A) 0.075

(B) 0.080

(C) 0.085

(D) 0.090

(E) 0.095

2. For a whole life insurance of 1 on (x), you are given:

(i) Benefits are payable at the moment of death.

(ii) Level premiums are payable at the beginning of each year.

(iii) Deaths are uniformly distributed over each year of age.

(iv) $i = 0.10$

(v) $\ddot{a}_x = 8$

(vi) $\ddot{a}_{x+10} = 6$

Calculate the 10th year terminal benefit reserve for this insurance.

(A) 0.18

(B) 0.25

(C) 0.26

(D) 0.27

(E) 0.30

3. A special whole life insurance of 100,000 payable at the moment of death of (x) includes a double indemnity provision. This provision pays during the first ten years an additional benefit of 100,000 at the moment of death for death by accidental means.

You are given:

(i) $\mu_x^{(\tau)}(t) = 0.001, \quad t \geq 0$

(ii) $\mu_x^{(1)}(t) = 0.0002, \quad t \geq 0$, where $\mu_x^{(1)}$ is the force of decrement due to death by accidental means.

(iii) $\delta = 0.06$

Calculate the single benefit premium for this insurance.

(A) 1640

(B) 1710

(C) 1790

(D) 1870

(E) 1970

4. Kevin and Kira are modeling the future lifetime of (60).

(i) Kevin uses a double decrement model:

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
60	1000	120	80
61	800	160	80
62	560	–	–

(ii) Kira uses a non-homogeneous Markov model:
(a) The states are 0 (alive), 1 (death due to cause 1), 2 (death due to cause 2).
(b) Q_{60} is the transition matrix from age 60 to 61; Q_{61} is the transition matrix from age 61 to 62.

(iii) The two models produce equal probabilities of decrement.

Calculate Q_{61}.

(A) $\begin{pmatrix} 1.00 & 0.12 & 0.08 \\ 0 & 1.00 & 0 \\ 0 & 0 & 1.00 \end{pmatrix}$

(B) $\begin{pmatrix} 0.80 & 0.12 & 0.08 \\ 0.56 & 0.16 & 0.08 \\ 0 & 0 & 1.00 \end{pmatrix}$

(C) $\begin{pmatrix} 0.76 & 0.16 & 0.08 \\ 0 & 1.00 & 0 \\ 0 & 0 & 1.00 \end{pmatrix}$

(D) $\begin{pmatrix} 0.70 & 0.20 & 0.10 \\ 0 & 1.00 & 0 \\ 0 & 0 & 1.00 \end{pmatrix}$

(E) $\begin{pmatrix} 0.60 & 0.28 & 0.12 \\ 0 & 1.00 & 0 \\ 0 & 0 & 1.00 \end{pmatrix}$

5. A certain species of flower has three states: sustainable, endangered and extinct. Transitions between states are modeled as a non-homogeneous Markov chain with transition matrices Q_i as follows:

$$Q_1 = \begin{array}{l} \\ \text{Sustainable} \\ \text{Endangered} \\ \text{Extinct} \end{array} \begin{pmatrix} \text{Sustainable} & \text{Endangered} & \text{Extinct} \\ 0.85 & 0.15 & 0 \\ 0 & 0.7 & 0.3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 0.9 & 0.1 & 0 \\ 0.1 & 0.7 & 0.2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_3 = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.2 & 0.7 & 0.1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_i = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad i = 4, 5, \ldots$$

Calculate the probability that a species endangered at the start of year 1 will ever become extinct.

(A) 0.45

(B) 0.47

(C) 0.49

(D) 0.51

(E) 0.53

6. For a special 3-year term insurance:

(i) Insureds may be in one of three states at the beginning of each year: active, disabled, or dead. All insureds are initially active. The annual transition probabilities are as follows:

	Active	Disabled	Dead
Active	0.8	0.1	0.1
Disabled	0.1	0.7	0.2
Dead	0.0	0.0	1.0

(ii) A 100,000 benefit is payable at the end of the year of death whether the insured was active or disabled.

(iii) Premiums are paid at the beginning of each year when active. Insureds do not pay any annual premiums when they are disabled.

(iv) $d = 0.10$

Calculate the level annual benefit premium for this insurance.

(A) 9,000

(B) 10,700

(C) 11,800

(D) 13,200

(E) 20,800

7. Customers arrive at a bank according to a Poisson process at the rate of 100 per hour. 20% of them make only a deposit, 30% make only a withdrawal and the remaining 50% are there only to complain. Deposit amounts are distributed with mean 8000 and standard deviation 1000. Withdrawal amounts have mean 5000 and standard deviation 2000.

The number of customers and their activities are mutually independent.

Using the normal approximation, calculate the probability that for an 8-hour day the total withdrawals of the bank will exceed the total deposits.

(A) 0.27

(B) 0.30

(C) 0.33

(D) 0.36

(E) 0.39

8. A Mars probe has two batteries. Once a battery is activated, its future lifetime is exponential with mean 1 year.

The first battery is activated when the probe lands on Mars. The second battery is activated when the first fails.

Battery lifetimes after activation are independent.

The probe transmits data until both batteries have failed.

Calculate the probability that the probe is transmitting data three years after landing.

(A) 0.05

(B) 0.10

(C) 0.15

(D) 0.20

(E) 0.25

9. For a special fully discrete 30-payment whole life insurance on (45), you are given:

(i) The death benefit of 1000 is payable at the end of the year of death.

(ii) The benefit premium for this insurance is equal to $1000P_{45}$ for the first 15 years followed by an increased level annual premium of π for the remaining 15 years.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

Calculate π.

(A) 16.8

(B) 17.3

(C) 17.8

(D) 18.3

(E) 18.8

10. For a special fully discrete 2-year endowment insurance on (x):

(i) The pure endowment is 2000.

(ii) The death benefit for year k is $(1000k)$ plus the benefit reserve at the end of year k, $k = 1, 2$.

(iii) π is the level annual benefit premium.

(iv) $i = 0.08$

(v) $p_{x+k-1} = 0.9, \quad k = 1, 2$

Calculate π.

(A) 1027

(B) 1047

(C) 1067

(D) 1087

(E) 1107

11. For a group of 250 individuals age x, you are given:

(i) The future lifetimes are independent.

(ii) Each individual is paid 500 at the beginning of each year, if living.

(iii) $A_x = 0.369131$

(iv) ${}^2A_x = 0.1774113$

(v) $i = 0.06$

Using the normal approximation, calculate the size of the fund needed at inception in order to be 90% certain of having enough money to pay the life annuities.

(A) 1.43 million

(B) 1.53 million

(C) 1.63 million

(D) 1.73 million

(E) 1.83 million

12. For a double decrement table, you are given:

Age	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
40	1000	60	55
41	–	–	70
42	750	–	–

Each decrement is uniformly distributed over each year of age in the double decrement table.

Calculate $q_{41}^{\prime(1)}$.

(A) 0.077

(B) 0.078

(C) 0.079

(D) 0.080

(E) 0.081

13. The actuarial department for the SharpPoint Corporation models the lifetime of pencil sharpeners from purchase using a generalized DeMoivre model with $s(x) = (1 - x/\omega)^{\alpha}$, for $\alpha > 0$ and $0 \le x \le \omega$.

A senior actuary examining mortality tables for pencil sharpeners has determined that the original value of α must change. You are given:

(i) The new complete expectation of life at purchase is half what it was previously.

(ii) The new force of mortality for pencil sharpeners is 2.25 times the previous force of mortality for all durations.

(iii) ω remains the same.

Calculate the original value of α.

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

14. You are given:

(i) T is the future lifetime random variable.

(ii) $\mu(t) = \mu, \qquad t \geq 0$

(iii) $\mathrm{Var}[T] = 100$.

Calculate $\mathrm{E}[T \wedge 10]$.

(A) 2.6

(B) 5.4

(C) 6.3

(D) 9.5

(E) 10.0

15. For a fully discrete 15-payment whole life insurance of 100,000 on (x), you are given:

(i) The expense-loaded level annual premium using the equivalence principle is 4669.95.

(ii) $100,000A_x = 51,481.97$

(iii) $\ddot{a}_{x:\overline{15}|} = 11.35$

(iv) $d = 0.02913$

(v) Expenses are incurred at the beginning of the year.

(vi) Percent of premium expenses are 10% in the first year and 2% thereafter.

(vii) Per policy expenses are K in the first year and 5 in each year thereafter until death.

Calculate K.

(A) 10.0

(B) 16.5

(C) 23.0

(D) 29.5

(E) 36.5

16. For the future lifetimes of (x) and (y):

(i) With probability 0.4, $T(x) = T(y)$ (i.e., deaths occur simultaneously).

(ii) With probability 0.6, the joint density function is

$$f_{T(x),\, T(y)}(t,s) = 0.0005, \qquad 0 < t < 40, \qquad 0 < s < 50$$

Calculate $\text{Prob}\left[T(x) < T(y)\right]$.

(A) 0.30

(B) 0.32

(C) 0.34

(D) 0.36

(E) 0.38

17. The length of time, in years, that a person will remember an actuarial statistic is modeled by an exponential distribution with mean $1/Y$. In a certain population, Y has a gamma distribution with $\alpha = \theta = 2$.

Calculate the probability that a person drawn at random from this population will remember an actuarial statistic less than $1/2$ year.

(A) 0.125

(B) 0.250

(C) 0.500

(D) 0.750

(E) 0.875

18. In a CCRC, residents start each month in one of the following three states: Independent Living (State #1), Temporarily in a Health Center (State #2) or Permanently in a Health Center (State #3). Transitions between states occur at the end of the month.

If a resident receives physical therapy, the number of sessions that the resident receives in a month has a geometric distribution with a mean which depends on the state in which the resident begins the month. The numbers of sessions received are independent. The number in each state at the beginning of a given month, the probability of needing physical therapy in the month, and the mean number of sessions received for residents receiving therapy are displayed in the following table:

State #	Number in state	Probability of needing therapy	Mean number of visits
1	400	0.2	2
2	300	0.5	15
3	200	0.3	9

Using the normal approximation for the aggregate distribution, calculate the probability that more than 3000 physical therapy sessions will be required for the given month.

(A) 0.21

(B) 0.27

(C) 0.34

(D) 0.42

(E) 0.50

19. In a given week, the number of projects that require you to work overtime has a geometric distribution with $\beta = 2$. For each project, the distribution of the number of overtime hours in the week is the following:

x	$f(x)$
5	0.2
10	0.3
20	0.5

The number of projects and number of overtime hours are independent. You will get paid for overtime hours in excess of 15 hours in the week.

Calculate the expected number of overtime hours for which you will get paid in the week.

(A) 18.5

(B) 18.8

(C) 22.1

(D) 26.2

(E) 28.0

20. For a group of lives age *x*, you are given:

(i) Each member of the group has a constant force of mortality that is drawn from the uniform distribution on $[0.01, 0.02]$.

(ii) $\delta = 0.01$

For a member selected at random from this group, calculate the actuarial present value of a continuous lifetime annuity of 1 per year.

(A) 40.0

(B) 40.5

(C) 41.1

(D) 41.7

(E) 42.3

21. For a population whose mortality follows DeMoivre's law, you are given:

(i) $\mathring{e}_{\overline{40:40}} = 3\mathring{e}_{\overline{60:60}}$

(ii) $\mathring{e}_{\overline{20:20}} = k\,\mathring{e}_{\overline{60:60}}$

Calculate k.

(A) 3.0

(B) 3.5

(C) 4.0

(D) 4.5

(E) 5.0

22. For an insurance on (x) and (y):

(i) Upon the first death, the survivor receives the single benefit premium for a whole life insurance of 10,000 payable at the moment of death of the survivor.

(ii) $\mu_x(t) = \mu_y(t) = 0.06$ while both are alive.

(iii) $\mu_{xy}(t) = 0.12$

(iv) After the first death, $\mu(t) = 0.10$ for the survivor.

(v) $\delta = 0.04$

Calculate the actuarial present value of this insurance on (x) and (y).

(A) 4500

(B) 5400

(C) 6000

(D) 7100

(E) 7500

23. Kevin and Kira are in a history competition:

(i) In each round, every child still in the contest faces one question. A child is out as soon as he or she misses one question. The contest will last at least 5 rounds.

(ii) For each question, Kevin's probability and Kira's probability of answering that question correctly are each 0.8; their answers are independent.

Calculate the conditional probability that both Kevin and Kira are out by the start of round five, given that at least one of them participates in round 3.

(A) 0.13

(B) 0.16

(C) 0.19

(D) 0.22

(E) 0.25

24. For a special increasing whole life annuity-due on (40), you are given:

(i) Y is the present-value random variable.

(ii) Payments are made once every 30 years, beginning immediately.

(iii) The payment in year 1 is 10, and payments increase by 10 every 30 years.

(iv) Mortality follows DeMoivre's law, with $\omega = 110$.

(v) $i = 0.04$

Calculate $\mathrm{Var}(Y)$.

(A) 10.5

(B) 11.0

(C) 11.5

(D) 12.0

(E) 12.5

25. For a special 3-year term insurance on (x), you are given:

(i) Z is the present-value random variable for this insurance.

(ii) $q_{x+k} = 0.02(k+1)$, $k = 0, 1, 2$

(iii) The following benefits are payable at the end of the year of death:

k	b_{k+1}
0	300
1	350
2	400

(iv) $i = 0.06$

Calculate $\mathrm{Var}(Z)$.

(A) 9,600

(B) 10,000

(C) 10,400

(D) 10,800

(E) 11,200

26. For an insurance:

(i) Losses have density function

$$f_X(x) = \begin{cases} 0.02x & 0 < x < 10 \\ 0 & \text{elsewhere} \end{cases}$$

(ii) The insurance has an ordinary deductible of 4 per loss.

(iii) Y^P is the claim payment per payment random variable.

Calculate $\mathrm{E}\left[Y^P\right]$.

(A) 2.9

(B) 3.0

(C) 3.2

(D) 3.3

(E) 3.4

27. An actuary has created a compound claims frequency model with the following properties:

(i) The primary distribution is the negative binomial with probability generating function

$$P(z) = \left[1 - 3(z-1)\right]^{-2}.$$

(ii) The secondary distribution is the Poisson with probability generating function

$$P(z) = e^{\lambda(z-1)}.$$

(iii) The probability of no claims equals 0.067.

Calculate λ.

(A) 0.1

(B) 0.4

(C) 1.6

(D) 2.7

(E) 3.1

28. In 2005 a risk has a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 3000$. In 2006 losses inflate by 20%.

An insurance on the risk has a deductible of 600 in each year. P_i, the premium in year *i*, equals 1.2 times the expected claims.

The risk is reinsured with a deductible that stays the same in each year. R_i, the reinsurance premium in year *i*, equals 1.1 times the expected reinsured claims.

$$R_{2005} / P_{2005} = 0.55$$

Calculate R_{2006} / P_{2006}.

(A) 0.46

(B) 0.52

(C) 0.55

(D) 0.58

(E) 0.66

29. For a fully discrete whole life insurance of 1000 on (60), you are given:

(i) The expenses, payable at the beginning of the year, are:

Expense Type	First Year	Renewal Years
% of Premium	20%	6%
Per Policy	8	2

(ii) The level expense-loaded premium is 41.20.

(iii) $i = 0.05$

Calculate the value of the expense augmented loss variable, ${}_0L_e$, if the insured dies in the third policy year.

(A) 770

(B) 790

(C) 810

(D) 830

(E) 850

30. For a fully discrete whole life insurance of 1000 on (45), you are given:

t	$1000\,{}_tV_{45}$	q_{45+t}
22	235	0.015
23	255	0.020
24	272	0.025

Calculate $1000\,{}_{25}V_{45}$.

(A) 279

(B) 282

(C) 284

(D) 286

(E) 288

31. The graph of a piecewise linear survival function, $s(x)$, consists of 3 line segments with endpoints (0, 1), (25, 0.50), (75, 0.40), (100, 0).

Calculate $\dfrac{{}_{20|55}q_{15}}{{}_{55}q_{35}}$.

(A) 0.69

(B) 0.71

(C) 0.73

(D) 0.75

(E) 0.77

32. For a group of lives aged 30, containing an equal number of smokers and non-smokers, you are given:

(i) For non-smokers, $\mu^n(x) = 0.08$, $x \geq 30$

(ii) For smokers, $\mu^s(x) = 0.16$, $x \geq 30$

Calculate q_{80} for a life randomly selected from those surviving to age 80.

(A) 0.078

(B) 0.086

(C) 0.095

(D) 0.104

(E) 0.112

33. For a 3-year fully discrete term insurance of 1000 on (40), subject to a double decrement model:

(i)

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
40	2000	20	60
41	–	30	50
42	–	40	–

(ii) Decrement 1 is death. Decrement 2 is withdrawal.

(iii) There are no withdrawal benefits.

(iv) $i = 0.05$

Calculate the level annual benefit premium for this insurance.

(A) 14.3

(B) 14.7

(C) 15.1

(D) 15.5

(E) 15.7

34. Each life within a group medical expense policy has loss amounts which follow a compound Poisson process with $\lambda = 0.16$. Given a loss, the probability that it is for Disease 1 is $\frac{1}{16}$.

Loss amount distributions have the following parameters:

	Mean per loss	Standard Deviation per loss
Disease 1	5	50
Other diseases	10	20

Premiums for a group of 100 independent lives are set at a level such that the probability (using the normal approximation to the distribution for aggregate losses) that aggregate losses for the group will exceed aggregate premiums for the group is 0.24.

A vaccine which will eliminate Disease 1 and costs 0.15 per person has been discovered.

Define:
A = the aggregate premium assuming that no one obtains the vaccine, and
B = the aggregate premium assuming that everyone obtains the vaccine and the cost of the vaccine is a covered loss.

Calculate A/B.

(A) 0.94

(B) 0.97

(C) 1.00

(D) 1.03

(E) 1.06

35. An actuary for a medical device manufacturer initially models the failure time for a particular device with an exponential distribution with mean 4 years.

This distribution is replaced with a spliced model whose density function:

(i) is uniform over [0, 3]

(ii) is proportional to the initial modeled density function after 3 years

(iii) is continuous

Calculate the probability of failure in the first 3 years under the revised distribution.

(A) 0.43

(B) 0.45

(C) 0.47

(D) 0.49

(E) 0.51

36. For a fully continuous whole life insurance of 1 on (30), you are given:

(i) The force of mortality is 0.05 in the first 10 years and 0.08 thereafter.

(ii) $\delta = 0.08$

Calculate the benefit reserve at time 10 for this insurance.

(A) 0.144

(B) 0.155

(C) 0.166

(D) 0.177

(E) 0.188

37. For a 10-payment, 20-year term insurance of 100,000 on Pat:

(i) Death benefits are payable at the moment of death.

(ii) Contract premiums of 1600 are payable annually at the beginning of each year for 10 years.

(iii) $i = 0.05$

(iv) L is the loss random variable at the time of issue.

Calculate the minimum value of L as a function of the time of death of Pat.

(A) $-21,000$

(B) $-17,000$

(C) $-13,000$

(D) $-12,400$

(E) $-12,000$

38. For an insurance:

(i) The number of losses per year has a Poisson distribution with $\lambda = 10$.

(ii) Loss amounts are uniformly distributed on (0, 10).

(iii) Loss amounts and the number of losses are mutually independent.

(iv) There is an ordinary deductible of 4 per loss.

Calculate the variance of aggregate payments in a year.

(A) 36

(B) 48

(C) 72

(D) 96

(E) 120

39. For an insurance portfolio:

(i) The number of claims has the probability distribution

n	p_n
0	0.1
1	0.4
2	0.3
3	0.2

(ii) Each claim amount has a Poisson distribution with mean 3; and

(iii) The number of claims and claim amounts are mutually independent.

Calculate the variance of aggregate claims.

(A) 4.8

(B) 6.4

(C) 8.0

(D) 10.2

(E) 12.4

40. Lucky Tom deposits the coins he finds on the way to work according to a Poisson process with a mean of 22 deposits per month.

5% of the time, Tom deposits coins worth a total of 10.
15% of the time, Tom deposits coins worth a total of 5.
80% of the time, Tom deposits coins worth a total of 1.

The amounts deposited are independent, and are independent of the number of deposits.

Calculate the variance in the total of the monthly deposits.

(A) 180

(B) 210

(C) 240

(D) 270

(E) 300

****END OF EXAMINATION****