Question #1  
Key: D

\( \hat{S}(300) = 3/10 \) (there are three observations greater than 300)
\( \hat{H}(300) = -\ln[\hat{S}(300)] = -\ln(0.3) = 1.204 \).

Question #2  
Key: A

\[ E(X \mid \lambda) = Var(X \mid \lambda) = \lambda \]
\[ \mu = \nu = E(\lambda) = \alpha \theta; a = Var(\lambda) = \alpha \theta^2; k = \nu/\alpha = 1/\theta \]
\[ Z = \frac{n}{n+1/\theta} = \frac{n\theta}{n\theta + 1} \]
\[ 0.15 = \frac{\theta}{\theta + 1}(1) + \frac{1}{\theta + 1} \mu = \frac{\theta + \mu}{\theta + 1} \]
\[ 0.20 = \frac{2\theta}{2\theta + 1}(2) + \frac{1}{2\theta + 1} \mu = \frac{4\theta + \mu}{2\theta + 1} \]
From the first equation,
\[ 0.15\theta + 0.15 = \theta + \mu \]
and so \( \mu = 0.15 - 0.85\theta \)
Then the second equation becomes
\[ 0.4\theta + 0.2 = 4\theta + 0.15 - 0.85\theta \]
\[ 0.05 = 2.75\theta; \theta = 0.01818 \]

Question #3  
Key: E

\[ 0.75 = \frac{1}{1 + (100/\theta)^{\gamma}} \]; \[ 0.25 = \frac{1}{1 + (500/\theta)^{\gamma}} \]
\( (100/\theta)^{\gamma} = 1/3; (500/\theta)^{\gamma} = 3 \)
Taking the ratio of these two equalities produces \( 5^{\gamma} = 9 \). From the second equality,
\[ 9 = [(500/\theta)^{2}]^\gamma = 5^{\gamma}; (500/\theta)^{2} = 5; \theta = 223.61 \]
Question #4
Key: B

\[ f(x) = a + bx + cx^2 + dx^3 \]
0 = \( f(0) = a \)
2 = \( f(2) = a + 2b + 4c + 8d \)
1 = \( f'(0) = b \)
\(-24 = f''(2) = 2c + 12d; c = -12 - 6d \)
Insert the values for \( a, b, \) and \( c \) into the second equation to obtain
\[ 2 = 2 + 4(-12 - 6d) + 8d; 48 = -16d; d = -3 \]
Then \( c = 6 \) and \( f(x) = x + 6x^2 - 3x^3; f(1) = 4 \)

Question #5
Key: E

Begin with

<table>
<thead>
<tr>
<th>( y )</th>
<th>350</th>
<th>500</th>
<th>1000</th>
<th>1200</th>
<th>1500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( r )</td>
<td>10</td>
<td>8</td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Then \( \hat{S}_1(1250) = \frac{8 \ 6 \ 4 \ 1}{10 \ 8 \ 5 \ 2} = 0.24 \)

The likelihood function is
\[ L(\theta) = \left[ \theta^{-1} e^{-350/\theta} \right]^2 \left[ \theta^{-1} e^{-500/\theta} \right]^2 \left[ e^{-1000/\theta} \theta^{-1} e^{-1200/\theta} \theta^{-1} e^{-1500/\theta} \right] \]
\[ = \theta^{-7} e^{-7900/\theta} \]
\[ l(\theta) = -7 \ln \theta - \frac{7900}{\theta}; l'(\theta) = -\frac{7}{\theta} + \frac{7900}{\theta^2} = 0; \hat{\theta} = 7900 / 7 \]
\[ \hat{S}_2(1250) = e^{-1250(7)/7900} = 0.33 \]
The absolute difference is 0.09.
Question #6
Key: E

\[ f(x) = -S'(x) = \frac{4x\theta^4}{(\theta^2 + x^2)^3} \]

\[ L(\theta) = f(2)f(4)S(4) = \frac{4(2)\theta^4}{(\theta^2 + 2^2)^3} \cdot \frac{4(4)\theta^4}{(\theta^2 + 4^2)^3} = \frac{\theta^4}{(\theta^2 + 4)(\theta^2 + 16)^3} = \frac{128\theta^2}{(\theta^2 + 4)(\theta^2 + 16)^5} \]

\[ l(\theta) = \ln 128 + 12 \ln \theta - 3 \ln(\theta^2 + 4) - 5 \ln(\theta^2 + 16) \]

\[ l'(\theta) = \frac{12}{\theta} - \frac{6\theta}{\theta^2 + 4} - \frac{10\theta}{\theta^2 + 16} = 0; \quad 12\theta^4 - 20\theta^2 + 64 - 6\theta^4 - 16\theta^2 - 10\theta^4 + 4\theta^2 = 0 \]
\[ 0 = -4\theta^4 + 104\theta^2 + 768 = \theta^4 - 26\theta^2 - 192 \]
\[ \theta^2 = \frac{26 \pm \sqrt{26^2 + 4(192)}}{2} = 32; \quad \theta = 5.657 \]

Question #7
Key: A

\[ E(X | \theta) = \int_0^\theta x \frac{2x}{\theta^2} \, dx = \frac{2\theta}{3}; \quad Var(X | \theta) = \int_0^\theta x^2 \frac{2x}{\theta^2} \, dx - \frac{4\theta^2}{9} = \frac{\theta^2}{2} - \frac{4\theta^2}{9} = \frac{\theta^2}{18} \]
\[ \mu = (2/3)E(\theta) = (2/3)\int_0^1 4\theta^4 \, d\theta = 8/15 \]

\[ EVPV = v = (1/18)E(\theta^2) = (1/18)\int_0^1 4\theta^5 \, d\theta = 1/27 \]

\[ VHM = a = (2/3)^2 Var(\theta) = (4/9) \left[ \frac{4}{6} - (4/5)^2 \right] = 8/675 \]

\[ k = \frac{1/27}{8/675} = 25/8; \quad Z = \frac{1}{1 + 25/8} = 8/33 \]

Estimate is \((8/33)(0.1) + (25/33)(8/15) = 0.428\).

Question #8
Key: D

From the Poisson(4) distribution the probabilities at 0, 1, and 2 are 0.0183, 0.0733, and 0.1463. The cumulative probabilities are 0.0183, 0.0916, and 0.2381. Because 0.0916 < 0.13 < 0.2381 the simulated number of claims is 2. Claim amounts are simulated from solving \[ u = 1 - e^{-x/1000} \] for \[ x = -1000 \ln(1 - u) \]. The two simulated amounts are 51.29 and 2995.73 for a total of 3047.02.
Question #9
Key: B

It may be easiest to show this by graphing the density functions. For the first function the three components are each constant. One is of height 1/20 from 0 to 2 (representing the empirical probability of 1/10 at 1, one is height 1/20 from 1 to 3 and one is height 8/20 from 2 to 4. The following figure shows each of them and their sum, the kernel density estimator.

The triangular one is similar. For the triangle from 0 to 2, the area must be 1/10. With a base of 2, the height is 1/10. the same holds for the second triangle. The third has height 8/10. When added they look as follows;

The question asks about cumulative probabilities. From 0 to 1 the first is linear and the second is quadratic, but by $x = 1$ both have accumulated 0.05 of probability. Because the cumulative distribution functions are the same at 1 and the density functions are identical from 1 to 2, the distribution functions must be identical from 1 to 2.
Question #10  
Key: D and E  

For the Poisson distribution, the mean, $\lambda$, is estimated as $230/1000 = 0.23$.

<table>
<thead>
<tr>
<th># of Days</th>
<th>Poisson Probability</th>
<th>Expected # of Workers</th>
<th>Observed # of Workers</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.794533</td>
<td>794.53</td>
<td>818</td>
<td>0.69</td>
</tr>
<tr>
<td>1</td>
<td>0.182743</td>
<td>182.74</td>
<td>153</td>
<td>4.84</td>
</tr>
<tr>
<td>2</td>
<td>0.021015</td>
<td>21.02</td>
<td>25</td>
<td>0.75</td>
</tr>
<tr>
<td>3 or more</td>
<td>0.001709</td>
<td>1.71</td>
<td>4</td>
<td>3.07</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>1000</td>
<td>9.35</td>
</tr>
</tbody>
</table>

The $\chi^2$ distribution has 2 degrees of freedom because there are four categories and the Poisson parameter is estimated (d.f. = 4 – 1 – 1 = 2).

The critical values for a chi-square test with two degrees of freedom are shown in the following table.

<table>
<thead>
<tr>
<th>Significance Level</th>
<th>Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>4.61</td>
</tr>
<tr>
<td>5%</td>
<td>5.99</td>
</tr>
<tr>
<td>2.5%</td>
<td>7.38</td>
</tr>
<tr>
<td>1%</td>
<td>9.21</td>
</tr>
</tbody>
</table>

9.35 is greater than 9.21 so the null hypothesis is rejected at the 1% significance level.

Question #11  
Key: D  

$$EVPV = \hat{\nu} = \frac{25(480 - 472.73)^2 + 30(466.67 - 472.73)^2}{2-1} = 2423.03$$ where $480 = 12,000/25$, $466.67 = 14,000/30$, and $472.73 = 26,000/55$.

$k = 2423.03/254 = 9.54$; $Z = \frac{55}{55+9.54} = 0.852$

Question #12  
Key: C  

$$f^\ast(x) = \begin{cases} 1.6x, & 0 < x < 1 \\ 1.6 - 0.8(x-1) = 2.4 - 0.8x, & 1 < x < 3 \end{cases}$$

$$S = \int_0^1 (1.6x)^2 dx + \int_1^3 (2.4 - 0.8x)^2 dx = 2.56$$
**Question #13**

**Key: C**

Relative risk \( = e^{-\hat{\beta}_1 - \hat{\beta}_2} \)

which has partial derivatives \(-e^{-0.2}\) at \(\hat{\beta}_1 = 0.05\) and \(\hat{\beta}_2 = 0.15\)

Using the delta method, the variance of the relative risk is

\[
\frac{1}{10,000} \begin{pmatrix} -e^{-0.2} & -e^{-0.2} \\ 2 & 1 \\ 1 & 3 \\ -e^{-0.2} \end{pmatrix} = \frac{7e^{-0.4}}{10,000} = 0.000469
\]

Std dev = 0.0217

upper limit = \(e^{-0.2} + 1.96(0.0217)\)

\[= 0.8613\]

Alternatively, consider the quantity \(\beta_1 + \beta_2\). The variance is

\[
\frac{1}{10,000} \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 3 \\ 1 \end{pmatrix} = \frac{7}{10,000} = 0.0007.
\]

The lower limit for this quantity is

\[0.2 - 1.96\sqrt{0.0007} = 0.1481\]

and the upper limit for the relative risk is \(e^{-0.1481} = 0.8623\).

**Question #14**

**Key: C**

The quantity of interest is \(P = \Pr(X \leq 5000) = \Phi\left(\frac{\ln 5000 - \mu}{\sigma}\right)\). The point estimate is

\[\Phi\left(\frac{\ln 5000 - 6.84}{1.49}\right) = \Phi(1.125) = 0.87.\]

For the delta method:

\[
\frac{\partial P}{\partial \mu} = -\frac{\phi(1.125)}{1.49} = -0.1422; \quad \frac{\partial P}{\partial \sigma} = -\frac{1.125\phi(1.125)}{1.49} = -0.1600\]

where \(\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\).

Then the variance of \(\hat{P}\) is estimated as \((-0.1422)^2 0.0444 + (-0.16)^2 0.0222 = 0.001466\) and the lower limit is \(P_L = 0.87 - 1.96\sqrt{0.001466} = 0.79496\).
Question #15
Key: A

\[ \Pr(\theta = 0.1 \mid X_1 = 1) = \frac{\Pr(X_1 = 1 \mid \theta = 0.1) \Pr(\theta = 0.1)}{\Pr(X_1 = 1 \mid \theta = 0.1) \Pr(\theta = 0.1) + \Pr(X_1 = 1 \mid \theta = 0.3) \Pr(\theta = 0.3)} \]

\[= \frac{0.1(0.8)}{0.1(0.8) + 0.3(0.2)} = \frac{4}{7} \]

Then,
\[E(X_2 \mid \theta = 0.1) = 0(0.2) + 1(0.1) + 2(0.7) = 1.5 \]
\[E(X_2 \mid \theta = 0.3) = 0(0.6) + 1(0.3) + 2(0.1) = 0.5 \]
\[E(X_2 \mid X_1 = 1) = (1.5)\frac{4}{7} + (0.5)\frac{3}{7} = 1.071 \]

Question #16
Key: D

The requirement is that
\[0.01\hat{F}(1500) \geq 1.96\sqrt{\frac{\hat{F}(1500)\hat{S}(1500)}{N}} \]
\[0.0001\frac{P^2}{N^2} \geq 3.8416\frac{P(N - P)}{N^3} \]
\[\frac{NP}{N - P} \geq 38,416. \]
For the five answer choices, the left hand side is 34,364, 15,000, 27,125, 39,243, and 37,688. Only answer D meets the condition.

Question #17
Key: D

\[\frac{s_4}{r_4} = \hat{H}(y_4) - \hat{H}(y_1) = 0.5691 - 0.4128 = 0.1563. \]
\[\frac{s_4}{r_4^2} = \hat{V}[\hat{H}(y_4)] - \hat{V}[\hat{H}(y_3)] = 0.014448 - 0.009565 = 0.004883. \]
Therefore,\[s_4 = \left(\frac{s_4}{r_4}\right)^2 = \frac{0.1563^2}{0.004833} = 5. \]
Question #18
Key: A

\[
\ln f(x) = \ln \theta - 2 \ln(\theta + x)
\]

\[
\frac{\partial \ln f(x)}{\partial \theta} = \frac{1}{\theta} - \frac{2}{\theta + x}
\]

\[
\frac{\partial^2 \ln f(x)}{\partial \theta^2} = -\frac{1}{\theta^2} + \frac{2}{(\theta + x)^2}
\]

\[
E \left[ \frac{\partial^2 \ln f(x)}{\partial \theta^2} \right] = -\frac{1}{\theta^2} + \int_0^\infty \frac{2\theta}{(\theta + x)^3} \, dx = -\frac{1}{\theta^2} + \left[ -\frac{2\theta}{3(\theta + x)^2} \right]_0^\infty = -\frac{1}{\theta^2} + \frac{2}{3\theta^2} = -\frac{1}{3\theta^2}
\]

\[
I(\theta) = \frac{n}{3\theta^2}; \quad \text{Var} = \frac{3\theta^2}{n}
\]

Question #19
Key: B

\[
\mu = E[E(X \mid \lambda)] = E(\lambda) = 1(0.9) + 10(0.09) + 20(0.01) = 2
\]

\[
EVPV = \nu = E[Var(X \mid \lambda)] = E(\lambda) = 2
\]

\[
VHM = a = Var[E(X \mid \lambda)] = Var(\lambda) = 1(0.9) + 100(0.09) + 400(0.01) - 2^2 = 9.9
\]

\[
Z = \frac{1}{1 + 2/9.9} = 0.83193; \quad 11.983 = 0.83193x + 0.16807(2); \quad x = 14
\]

Question #20
Key: A

The given interval for \( H \) can be written as \( 0.775 \pm 1.96\sqrt{0.063} \) and therefore the estimated variance of \( \hat{H} \) is 0.063. To apply the delta method,

\[
S = e^{-H}; \quad \frac{dS}{dH} = -e^{-H}; \quad \text{Var}(\hat{S}) = (-e^{-\hat{H}})^2 \text{Var}(\hat{H}) = (-e^{-0.775})^2(0.063) = 0.134.
\]

The point estimate of \( S \) is \( e^{-0.775} = 0.4607 \) and the confidence interval is \( 0.4607 \pm 1.96\sqrt{0.0134} = 0.2269 \) or (0.23, 0.69).
Question #21  
Key: B  

The first step is to trend the year 1 data by 1.21 and the year 2 data by 1.1. The observations are now 24.2, 48.4, 60.5, 33, 44, 99, and 132. The first two sample moments are 63.014 and 5262.64. The equations to solve are

\[ 63.014 = e^{\mu + 0.5\sigma^2}; \quad 4.14336 = \mu + 0.5\sigma^2 \]

\[ 5262.64 = e^{2\mu + 2\sigma^2}; \quad 8.56839 = 2\mu + 2\sigma^2. \]

Taking four times the first equation and subtracting the second gives \(2\mu\) and therefore

\[ \mu = \frac{4(4.14336) - 8.56839}{2} = 4.00. \]

Question #22  
Key: A  

\[ \hat{\mu} = \bar{x} = 12/60 = 0.2, \quad EVPV = \hat{\nu} = \bar{x} = 0.2 \]

\[ VHM = \hat{a} = \frac{10(0.4 - 0.2)^2 + 20(0.25 - 0.2)^2 + 30(0.1 - 0.2)^2 - (3-1)(0.2)}{60 - 10^2 + 20^2 + 30^2} = 0.009545 \]

\[ \hat{k} = 20.9524; \quad Z = \frac{10}{10 + 20.9524} = 0.323 \]

Question #23  
Key: E  

By elimination, (A) is incorrect because \(f''(3) = -1.833 \neq 0\), (B) is incorrect because \(f''(0) = -2 \neq 0\), (C) is incorrect because \(f''(0) = -1 \neq 0\), and (D) is incorrect because \(f''(0) = -1 \neq 0\). Therefore (E) must be correct. Also, this function does meet all the requirements:

\(f''(0) = 0; \quad f_0 (l) = f_1 (l) = 2; \quad f_0 (l) = f_1 (l) = 0; \quad f_{\infty} (l) = f_1^\prime (l) = 3; \quad f(3) = 6; \quad f''(3) = 0 \)

Question #24  
Key: B and C  

For males, \(c_j = 1\) and for females, \(c_j = e^{0.27} = 1.31\). Then,

\[ \hat{H}(20) = \frac{1}{3 + 2(1.31)} + \frac{1}{2 + 2(1.31)} + \frac{1}{2 + 1.31} = 0.6965 \quad \text{and} \quad \hat{S}_{female}(20) = (e^{-0.6965})^{1.31} = 0.402. \]
Question #25
Key: C

\[ l(\tau, \theta) = \sum_{j=1}^{5} \ln f(x_j) = \sum_{j=1}^{5} \ln \tau + (\tau - 1) \ln x_j - \tau \ln \theta - (x_j / \theta) \]. Under the null hypothesis it is

\[ l(2, \theta) = \sum_{j=1}^{5} \ln 2 + \ln x_j - 2 \ln \theta - (x_j / \theta)^2 \]. Inserting the maximizing value of 816.7 for \( \theta \) gives 

\(-35.28\). The likelihood ratio test statistic is \( 2(-33.05 + 35.28) = 4.46 \). There is one degree of freedom. At a 5% significance level the critical value is 3.84 and at a 2.5% significance level it is 5.02.

Question #26
Key: C

It is given that \( n = 4, \nu = 8, \) and \( Z = 0.4. \) Then, \( 0.4 = \frac{4}{4 + \frac{8}{a}} \) which solves for \( a = 4/3. \) For the covariance,

\[ \text{Cov}(X_i, X_j) = E(X_i X_j) - E(X_i)E(X_j) \]

\[ = E[E(X_i X_j | \theta)] - E[E(X_i | \theta)]E[E(X_j | \theta)] \]

\[ = E[\mu(\theta)^2] - E[\mu(\theta)]^2 = Var[\mu(\theta)] = a = 4/3. \]

Question #27
Key: A

The value of \( z \) is obtained by inversion from the standard normal table. That is, \( u = \Pr(Z \leq z). \)

The value of \( x \) is obtained from \( x = 0.75z + 5.6. \) The lognormal value is obtained by exponentiating \( x \) and the final column applies the deductible.
**Question #28**

Key: B

\[ \text{MSE} = E[(X^2 - \theta^2)^2] = E(X^4 - 2X^2\theta^2 + \theta^4) = 24\theta^4 - 2\theta^2 + \theta^4 = 21\theta^4 \]

**Question #29**

Key: C

The sample mean of \( \frac{157(0) + 66(1) + 19(2) + 4(3) + 2(4)}{248} = 0.5 \) is the maximum likelihood estimate of the geometric parameter \( \beta \) as well as the method of moments estimate of the Poisson parameter \( \lambda \). Then, \( P = (1 + 0.5)^{-1} = 0.6667 \) and \( Q = e^{-0.5} = 0.6065 \). The absolute difference is 0.0602.

**Question #30**

Key: D

\[
\bar{x} = \frac{5000(0) + 2100(1) + 750(2) + 100(3) + 50(4)}{8000} = 0.5125 \quad \text{and} \quad s^2 = \frac{5000(0.5125)^2 + 2100(0.4875)^2 + 750(1.4875)^2 + 100(2.4875)^2 + 50(3.4875)^2}{7999} = 0.5874.
\]

Then, \( \hat{\mu} = \hat{\nu} = \bar{x} = 0.5125 \) and \( \hat{\sigma} = s^2 - \bar{x} = 0.0749 \). The credibility factor is \( Z = \frac{1}{1 + 0.5125 / 0.0749} = 0.1275 \) and the estimate is \( 0.1275(1) + 0.8725(0.5125) = 0.5747 \).

**Question #31**

Key: B

\( s = F_n(3000) = 4 / 8 = 0.5 \) because for the p-p plot the denominator is \( n+1 \).

\( t = F(3000) = 1 - e^{-3000/3300} = 0.59711 \). For the difference plot, \( D \) uses a denominator of \( n \) and so

\( D = 4 / 7 - 0.59711 = -0.02568 \) and the answer is \( 0.5 - 0.59711 + 0.02568 = -0.071 \).
Question #32
Key: B

\[\pi(q \mid 2, 2) \propto f(2 \mid q)f(2 \mid q)\pi(q) = q(q^2 / 0.039) \propto q^4.\] Because \(\int_{0.2}^{0.5} q^4 \, dq = 0.006186,\)
\[\pi(q \mid 2, 2) = q^4 / 0.006186.\] Given \(q,\) the expected number of claims is
\[E(N \mid q) = 0(0.1) + 1(0.9 - q) + 2q = 0.9 + q.\] The Bayesian estimate is
\[E(N \mid 2, 2) = \int_{0.2}^{0.5} (0.9 + q) \frac{q^4}{0.006186} \, dq = 1.319.\]

Question #33
Key: E

\[0.689 = F_{500}(1500) = 0.5 F_{500}(1000) + 0.5 F_{500}(2000) = 0.5 \left(\frac{200 + 110}{500} + \frac{310 + x}{500}\right) \implies x = 69\]
\[0.839 = F_{500}(3500) = 0.5 F_{500}(2000) + 0.5 F_{500}(5000) = 0.5 \left(\frac{310 + 69}{500} + \frac{379 + y}{500}\right) \implies y = 81\]

Question #34
Key: A

A is false because the test works best when the expected number of observations is about the same from interval to interval. B is true (Loss Models, 427-8), C is true (Loss Models, 428), and D is true (Loss Models, 430).

Question #35
Key: E

\[n\lambda \geq \lambda_0 \left[1 + \left(\frac{\sigma_y}{\theta_y}\right)^2\right]; \quad \theta_y = \alpha\theta = 10,000\alpha; \quad \sigma_y^2 = \alpha\theta^2 = 10^8\alpha\]
\[n\lambda \geq \left(\frac{1.96}{0.1}\right)^2 \left[1 + \frac{10^8\alpha}{10^8\alpha^2}\right] = 384.16(1 + \alpha^{-1})\]

Because \(\alpha\) is needed, but not given, the answer cannot be determined from the information given.