****BEGINNING OF EXAMINATION****

1. Given:

(i) $\overset{\circ}{e}_0 = 25$

(ii) $l_x = \omega - x,\ 0 \le x \le \omega$

(iii) $T(x)$ is the future lifetime random variable.

Calculate $Var[T(10)]$.

(A) 65

(B) 93

(C) 133

(D) 178

(E) 333

2. Lucky Tom finds coins on his way to work at a Poisson rate of 0.5 coins/minute. The denominations are randomly distributed:

(i) 60% of the coins are worth 1;

(ii) 20% of the coins are worth 5; and

(iii) 20% of the coins are worth 10.

Calculate the conditional expected value of the coins Tom found during his one-hour walk today, given that among the coins he found exactly ten were worth 5 each.

(A) 108

(B) 115

(C) 128

(D) 165

(E) 180

3. For a fully discrete two-year term insurance of 400 on (x):

(i) $i = 0.1$

(ii) $400\, P^{1}_{x:\overline{2}|} = 74.33$

(iii) $400\ {}_{1}V^{1}_{x:\overline{2}|} = 16.58$

(iv) The contract premium equals the benefit premium.

Calculate the variance of the loss at issue.

(A) 21,615

(B) 23,125

(C) 27,450

(D) 31,175

(E) 34,150

4. You are given:

(i) The claim count N has a Poisson distribution with mean Λ.

(ii) Λ has a gamma distribution with mean 1 and variance 2.

Calculate the probability that $N = 1$.

(A) 0.19

(B) 0.24

(C) 0.31

(D) 0.34

(E) 0.37

5. An insurance company has agreed to make payments to a worker age x who was injured at work.

(i) The payments are 150,000 per year, paid annually, starting immediately and continuing for the remainder of the worker's life.

(ii) After the first 500,000 is paid by the insurance company, the remainder will be paid by a reinsurance company.

(iii) $${}_t p_x = \begin{cases} (0.7)^t, & 0 \le t \le 5.5 \\ 0, & 5.5 < t \end{cases}$$

(iv) $i = 0.05$

Calculate the actuarial present value of the payments to be made by the reinsurer.

(A) Less than 50,000

(B) At least 50,000, but less than 100,000

(C) At least 100,000, but less than 150,000

(D) At least 150,000, but less than 200,000

(E) At least 200,000

6. A special purpose insurance company is set up to insure one single life. The risk consists of a single possible claim.

(i) The claim amount distribution is:

Amount	Probability
100	0.60
200	0.40

(ii) The probability that the claim does not occur by time t is $\frac{1}{1+t}$.

(iii) The insurer's surplus at time t is $U(t) = 60 + 20t - S(t)$, where $S(t)$ is the aggregate claim amount paid by time t.

(iv) The claim is payable immediately.

Calculate the probability of ruin.

(A) $\frac{4}{7}$

(B) $\frac{3}{5}$

(C) $\frac{2}{3}$

(D) $\frac{3}{4}$

(E) $\frac{7}{8}$

7. In a triple decrement table, lives are subject to decrements of death (d), disability (i), and withdrawal (w).

You are given:

(i) The total decrement is uniformly distributed over each year of age.

(ii) $l_x^{(\tau)} = 25{,}000$

(iii) $l_{x+1}^{(\tau)} = 23{,}000$

(iv) $m_x^{(d)} = 0.02$

(v) $m_x^{(w)} = 0.05$

Calculate $q_x^{(i)}$, the probability of decrement by disability at age x.

(A) 0.0104

(B) 0.0112

(C) 0.0120

(D) 0.0128

(E) 0.0136

8. For a two-year term insurance on a randomly chosen member of a population:

(i) 1/3 of the population are smokers and 2/3 are nonsmokers.

(ii) The future lifetimes follow a Weibull distribution with:

- $t = 2$ and $q = 1.5$ for smokers
- $t = 2$ and $q = 2.0$ for nonsmokers

(iii) The death benefit is 100,000 payable at the end of the year of death.

(iv) $i = 0.05$

Calculate the actuarial present value of this insurance.

(A) 64,100

(B) 64,300

(C) 64,600

(D) 64,900

(E) 65,100

9. For a 10-year deferred whole life annuity of 1 on (35) payable continuously:

(i) Mortality follows De Moivre's law with $w = 85$.

(ii) $i = 0$

(iii) Level benefit premiums are payable continuously for 10 years.

Calculate the benefit reserve at the end of five years.

(A) 9.38

(B) 9.67

(C) 10.00

(D) 10.36

(E) 10.54

10. Taxicabs leave a hotel with a group of passengers at a Poisson rate $l = 10$ per hour. The number of people in each group taking a cab is independent and has the following probabilities:

Number of People	Probability
1	0.60
2	0.30
3	0.10

Using the normal approximation, calculate the probability that at least 1050 people leave the hotel in a cab during a 72-hour period.

(A) 0.60

(B) 0.65

(C) 0.70

(D) 0.75

(E) 0.80

11. A company provides insurance to a concert hall for losses due to power failure. You are given:

(i) The number of power failures in a year has a Poisson distribution with mean 1.

(ii) The distribution of ground up losses due to a single power failure is:

x	Probability of x
10	0.3
20	0.3
50	0.4

(iii) The number of power failures and the amounts of losses are independent.

(iv) There is an annual deductible of 30.

Calculate the expected amount of claims paid by the insurer in one year.

(A) 5

(B) 8

(C) 10

(D) 12

(E) 14

12. For a certain mortality table, you are given:

(i) $m(80.5) = 0.0202$

(ii) $m(81.5) = 0.0408$

(iii) $m(82.5) = 0.0619$

(iv) Deaths are uniformly distributed between integral ages.

Calculate the probability that a person age 80.5 will die within two years.

(A) 0.0782

(B) 0.0785

(C) 0.0790

(D) 0.0796

(E) 0.0800

13. An investment fund is established to provide benefits on 400 independent lives age x.

(i) On January 1, 2001, each life is issued a 10-year deferred whole life insurance of 1000, payable at the moment of death.

(ii) Each life is subject to a constant force of mortality of 0.05.

(iii) The force of interest is 0.07.

Calculate the amount needed in the investment fund on January 1, 2001, so that the probability, as determined by the normal approximation, is 0.95 that the fund will be sufficient to provide these benefits.

(A) 55,300

(B) 56,400

(C) 58,500

(D) 59,300

(E) 60,100

14. The Rejection Method is used to generate a random variable with density function $f(x)$ by using the density function $g(x)$ and constant c as the basis, where:

$$f(x) = 12\left(x - 2x^2 + x^3\right), \quad 0 < x < 1$$

$$g(x) = 1, \quad 0 < x < 1$$

The constant c has been chosen so as to minimize n, the expected number of iterations needed to generate a random variable from $f(x)$.

Calculate n.

(A) 1.52

(B) 1.78

(C) 1.86

(D) 2.05

(E) 2.11

15. In a double decrement table:

(i) $l_{30}^{(\tau)} = 1000$

(ii) $q_{30}'^{(1)} = 0.100$

(iii) $q_{30}'^{(2)} = 0.300$

(iv) ${}_{1|}q_{30}^{(1)} = 0.075$

(v) $l_{32}^{(\tau)} = 472$

Calculate $q_{31}^{(2)}$.

(A) 0.11

(B) 0.13

(C) 0.14

(D) 0.15

(E) 0.17

16. You are given:

	Mean	Standard Deviation
Number of Claims	8	3
Individual Losses	10,000	3,937

Using the normal approximation, determine the probability that the aggregate loss will exceed 150% of the expected loss.

(A) $\Phi(1.25)$

(B) $\Phi(1.5)$

(C) $1-\Phi(1.25)$

(D) $1-\Phi(1.5)$

(E) $1.5\,\Phi(1)$

17. The future lifetimes of a certain population can be modeled as follows:

(i) Each individual's future lifetime is exponentially distributed with constant hazard rate q.

(ii) Over the population, q is uniformly distributed over (1,11).

Calculate the probability of surviving to time 0.5, for an individual randomly selected at time 0.

(A) 0.05

(B) 0.06

(C) 0.09

(D) 0.11

(E) 0.12

18. The pricing actuary at Company XYZ sets the premium for a fully continuous whole life insurance of 1000 on (80) using the equivalence principle and the following assumptions:

(i) The force of mortality is 0.15.

(ii) $i = 0.06$

The pricing actuary's supervisor believes that the Illustrative Life Table with deaths uniformly distributed over each year of age is a better mortality assumption.

Calculate the insurer's expected loss at issue if the premium is not changed and the supervisor is right.

(A) -124

(B) -26

(C) 0

(D) 37

(E) 220

19. An insurance company sold 300 fire insurance policies as follows:

Number of Policies	Policy Maximum	Probability of Claim Per Policy
100	400	0.05
200	300	0.06

You are given:

(i) The claim amount for each policy is uniformly distributed between 0 and the policy maximum.

(ii) The probability of more than one claim per policy is 0.

(iii) Claim occurrences are independent.

Calculate the variance of the aggregate claims.

(A) 150,000

(B) 300,000

(C) 450,000

(D) 600,000

(E) 750,000

20. For a last-survivor insurance of 10,000 on independent lives (70) and (80), you are given:

(i) The benefit, payable at the end of the year of death, is paid only if the second death occurs during year 5.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.03$

Calculate the actuarial present value of this insurance.

(A) 235

(B) 245

(C) 255

(D) 265

(E) 275

21. A risky investment with a constant rate of default will pay:

(i) principal and accumulated interest at 16% compounded annually at the end of 20 years if it does not default; and

(ii) zero if it defaults.

A risk-free investment will pay principal and accumulated interest at 10% compounded annually at the end of 20 years.

The principal amounts of the two investments are equal.

The actuarial present values of the two investments are equal at time zero.

Calculate the median time until default or maturity of the risky investment.

(A) 9

(B) 10

(C) 11

(D) 12

(E) 13

22. For a special annual whole life annuity-due on independent lives (30) and (50):

(i) Y is the present-value random variable.

(ii) The benefit is 1000 while both are alive and 500 while only one is alive.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) You are doing a simulation of $K(30)$ and $K(50)$ to study the distribution of Y, using the Inverse Transform Method (where small random numbers correspond to early deaths).

(vi) In your first trial, your random numbers from the uniform distribution on [0,1] are 0.63 and 0.40 for generating $K(30)$ and $K(50)$ respectively.

(vii) F is the simulated value of Y in this first trial.

Calculate F.

(A) 15,150

(B) 15,300

(C) 15,450

(D) 15,600

(E) 15,750

23. For a birth and death process, you are given:

(i) There are four possible states $\{0,1,2,3\}$.

(ii) These limiting probabilities:

$$P_0 = 0.10$$
$$P_1 = 0.30$$

(iii) These instantaneous transition rates:

$$q_{21} = 0.40$$
$$q_{32} = 0.12$$

(iv) If the system is in state 2, the time until it leaves state 2 is exponentially distributed with mean 0.5.

Calculate the limiting probability P_2.

(A) 0.04

(B) 0.07

(C) 0.27

(D) 0.33

(E) 0.55

24. For a fully discrete whole life insurance with non-level benefits on (70):

(i) The level benefit premium for this insurance is equal to P_{50}.

(ii) $q_{70+k} = q_{50+k} + 0.01, \ k = 0, 1, \ldots, 19$

(iii) $q_{60} = 0.01368$

(iv) ${}_kV = {}_kV_{50}, \ k = 0, 1, \ldots, 19$

(v) ${}_{11}V_{50} = 0.16637$

Calculate b_{11}, the death benefit in year 11.

(A) 0.482

(B) 0.624

(C) 0.636

(D) 0.648

(E) 0.834

25. An insurance agent will receive a bonus if his loss ratio is less than 70%. You are given:

(i) His loss ratio is calculated as incurred losses divided by earned premium on his block of business.

(ii) The agent will receive a percentage of earned premium equal to 1/3 of the difference between 70% and his loss ratio.

(iii) The agent receives no bonus if his loss ratio is greater than 70%.

(iv) His earned premium is 500,000.

(v) His incurred losses are distributed according to the Pareto distribution:

$$F(x) = 1 - \left(\frac{600{,}000}{x + 600{,}000} \right)^3, \quad x > 0$$

Calculate the expected value of his bonus.

(A) 16,700

(B) 31,500

(C) 48,300

(D) 50,000

(E) 56,600

26. For a fully discrete 3-year endowment insurance of 1000 on (x):

(i) $q_x = q_{x+1} = 0.20$

(ii) $i = 0.06$

(iii) $1000\,P_{x:\overline{3}|} = 373.63$

Calculate $1000\left({}_2V_{x:\overline{3}|} - {}_1V_{x:\overline{3}|}\right)$.

(A) 320

(B) 325

(C) 330

(D) 335

(E) 340

27. For an insurer with initial surplus of 2:

(i) The annual aggregate claim amount distribution is:

Amount	Probability
0	0.6
3	0.3
8	0.1

(ii) Claims are paid at the end of the year.

(iii) A total premium of 2 is collected at the beginning of each year.

(iv) $i = 0.08$

Calculate the probability that the insurer is surviving at the end of year 3.

(A) 0.74

(B) 0.77

(C) 0.80

(D) 0.85

(E) 0.86

28. For a mortality study on college students:

(i) Students entered the study on their birthdays in 1963.

(ii) You have no information about mortality before birthdays in 1963.

(iii) Dick, who turned 20 in 1963, died between his 32^{nd} and 33^{rd} birthdays.

(iv) Jane, who turned 21 in 1963, was alive on her birthday in 1998, at which time she left the study.

(v) All lifetimes are independent.

(vi) Likelihoods are based upon the Illustrative Life Table.

Calculate the likelihood for these two students.

(A) 0.00138

(B) 0.00146

(C) 0.00149

(D) 0.00156

(E) 0.00169

29. For a whole life annuity-due of 1 on (x), payable annually:

(i) $q_x = 0.01$

(ii) $q_{x+1} = 0.05$

(iii) $i = 0.05$

(iv) $\ddot{a}_{x+1} = 6.951$

Calculate the change in the actuarial present value of this annuity-due if p_{x+1} is increased by 0.03.

(A) 0.16

(B) 0.17

(C) 0.18

(D) 0.19

(E) 0.20

30. X is a random variable for a loss.

Losses in the year 2000 have a distribution such that:

$$E[X \wedge d] = -0.025d^2 + 1.475d - 2.25, \quad d = 10, 11, 12, \ldots, 26$$

Losses are uniformly 10% higher in 2001.

An insurance policy reimburses 100% of losses subject to a deductible of 11 up to a maximum reimbursement of 11.

Calculate the ratio of expected reimbursements in 2001 over expected reimbursements in the year 2000.

(A) 110.0%

(B) 110.5%

(C) 111.0%

(D) 111.5%

(E) 112.0%

31. Company ABC issued a fully discrete three-year term insurance of 1000 on Pat whose stated age at issue was 30. You are given:

(i)

x	q_x
30	0.01
31	0.02
32	0.03
33	0.04

(ii) $i = 0.04$

(iii) Premiums are determined using the equivalence principle.

During year 3, Company ABC discovers that Pat was really age 31 when the insurance was issued. Using the equivalence principle, Company ABC adjusts the death benefit to the level death benefit it should have been at issue, given the premium charged.

Calculate the adjusted death benefit.

(A) 646

(B) 664

(C) 712

(D) 750

(E) 963

32. Insurance for a city's snow removal costs covers four winter months.

(i) There is a deductible of 10,000 per month.

(ii) The insurer assumes that the city's monthly costs are independent and normally distributed with mean 15,000 and standard deviation 2,000.

(iii) To simulate four months of claim costs, the insurer uses the Inverse Transform Method (where small random numbers correspond to low costs).

(iv) The four numbers drawn from the uniform distribution on [0,1] are:

0.5398	0.1151	0.0013	0.7881

Calculate the insurer's simulated claim cost.

(A) 13,400

(B) 14,400

(C) 17,800

(D) 20,000

(E) 26,600

33. In the state of Elbonia all adults are drivers. It is illegal to drive drunk. If you are caught, your driver's license is suspended for the following year. Driver's licenses are suspended only for drunk driving. If you are caught driving with a suspended license, your license is revoked and you are imprisoned for one year. Licenses are reinstated upon release from prison.

Every year, 5% of adults with an active license have their license suspended for drunk driving. Every year, 40% of drivers with suspended licenses are caught driving.

Assume that all changes in driving status take place on January 1, all drivers act independently, and the adult population does not change.

Calculate the limiting probability of an Elbonian adult having a suspended license.

(A) 0.019

(B) 0.020

(C) 0.028

(D) 0.036

(E) 0.047

34. For a last-survivor whole life insurance of 1 on (x) and (y):

(i) The death benefit is payable at the moment of the second death.

(ii) The independent random variables $T^*(x)$, $T^*(y)$, and Z are the components of a common shock model.

(iii) $T^*(x)$ has an exponential distribution with $m_x^{T^*(x)}(t) = 0.03, \quad t \geq 0.$

(iv) $T^*(y)$ has an exponential distribution with $m_y^{T^*(y)}(t) = 0.05, \quad t \geq 0.$

(v) Z, the common shock random variable, has an exponential distribution with $m^Z(t) = 0.02, \quad t \geq 0.$

(vi) $d = 0.06$

Calculate the actuarial present value of this insurance.

(A) 0.216

(B) 0.271

(C) 0.326

(D) 0.368

(E) 0.423

35. The distribution of Jack's future lifetime is a two-point mixture:

(i) With probability 0.60, Jack's future lifetime follows the Illustrative Life Table, with deaths uniformly distributed over each year of age.

(ii) With probability 0.40, Jack's future lifetime follows a constant force of mortality $m = 0.02$.

A fully continuous whole life insurance of 1000 is issued on Jack at age 62.

Calculate the benefit premium for this insurance at $i = 0.06$.

(A) 31

(B) 32

(C) 33

(D) 34

(E) 35

36. A new insurance salesperson has 10 friends, each of whom is considering buying a policy.

(i) Each policy is a whole life insurance of 1000, payable at the end of the year of death.

(ii) The friends are all age 22 and make their purchase decisions independently.

(iii) Each friend has a probability of 0.10 of buying a policy.

(iv) The 10 future lifetimes are independent.

(v) *S* is the random variable for the present value at issue of the total payments to those who purchase the insurance.

(vi) Mortality follows the Illustrative Life Table.

(vii) $i = 0.06$

Calculate the variance of *S*.

(A) 9,200

(B) 10,800

(C) 12,300

(D) 13,800

(E) 15,400

37. Given:

(i) p_k denotes the probability that the number of claims equals k for $k = 0,1,2, \ldots$

(ii) $\dfrac{p_n}{p_m} = \dfrac{m!}{n!}$, $m \geq 0,\ n \geq 0$

Using the corresponding zero-modified claim count distribution with $p_0^M = 0.1$, calculate p_1^M.

(A) 0.1

(B) 0.3

(C) 0.5

(D) 0.7

(E) 0.9

38. For Shoestring Swim Club, with three possible financial states at the end of each year:

(i) State 0 means cash of 1500. If in state 0, aggregate member charges for the next year are set equal to operating expenses.

(ii) State 1 means cash of 500. If in state 1, aggregate member charges for the next year are set equal to operating expenses plus 1000, hoping to return the club to state 0.

(iii) State 2 means cash less than 0. If in state 2, the club is bankrupt and remains in state 2.

(iv) The club is subject to four risks each year. These risks are independent. Each of the four risks occurs at most once per year, but may recur in a subsequent year.

(v) Three of the four risks each have a cost of 1000 and a probability of occurrence 0.25 per year.

(vi) The fourth risk has a cost of 2000 and a probability of occurrence 0.10 per year.

(vii) Aggregate member charges are received at the beginning of the year.

(viii) $i = 0$

Calculate the probability that the club is in state 2 at the end of three years, given that it is in state 0 at time 0.

(A) 0.24

(B) 0.27

(C) 0.30

(D) 0.37

(E) 0.56

39. For a continuous whole life annuity of 1 on (x):

(i) $T(x)$, the future lifetime of (x), follows a constant force of mortality 0.06.

(ii) The force of interest is 0.04.

Calculate $\Pr\left(\bar{a}_{\overline{T(x)}|} > \bar{a}_x\right)$.

(A) 0.40

(B) 0.44

(C) 0.46

(D) 0.48

(E) 0.50

40. Rain is modeled as a Markov process with two states:

(i) If it rains today, the probability that it rains tomorrow is 0.50.

(ii) If it does not rain today, the probability that it rains tomorrow is 0.30.

Calculate the limiting probability that it rains on two consecutive days.

(A) 0.12

(B) 0.14

(C) 0.16

(D) 0.19

(E) 0.22

END OF EXAMINATION

Course 3
Answer Key

May 2000

1	C		21	E
2	C		22	B
3	E		23	A
4	A		24	D
5	B		25	E
6	D		26	B
7	D		27	A
8	C		28	C
9	A		29	C
10	D		30	D
11	E		31	B
12	A		32	B
13	A		33	E
14	B		34	D
15	B		35	A
16	C		36	E
17	E		37	C
18	A		38	E
19	D		39	C
20	A		40	D

Solutions to Course 3 Exam – May 2000

Question # 1
Key: C

$$\overset{\circ}{e}_0 = \int_0^w \left(1 - \frac{t}{w}\right) dt = w - \frac{w^2}{2w} = \frac{w}{2} = 25 \Rightarrow w = 50$$

$$\overset{\circ}{e}_{10} = \int_0^{40} \left(1 - \frac{t}{40}\right) dt = 40 - \frac{40^2}{(2)(40)} = 20$$

$$\begin{aligned} \text{Var}\left[T(x)\right] &= 2\int_0^{40} t\left(1 - \frac{t}{40}\right) dt - (20)^2 \\ &= 2\left[\frac{t^2}{2} - \frac{t^3}{3 \times 40}\right]_0^{40} - (20)^2 \\ &= 133 \end{aligned}$$

Question # 2
Key: C

A priori, expect 30 coins: 18 worth one, 6 worth 5, 6 worth 10.

Given 10 worth 5, expect: 18 @ 1; 10 @ 5; 6 @ 10.

Total = 18 + 50 + 60 = 128

Question # 3
Key: E

Need to determine q_x and q_{x+1}.

Formula 7.23: $16.58 = {}_1V = \frac{400 q_{x+1}}{1.1} - 74.33 \Rightarrow q_{x+1} = 0.25$

Formula 8.39: ${}_0V = 0 = 400 v q_x - p + v\, {}_1V p_x$

$$q_x = \frac{p \times 1.1 - {}_1V}{400 - {}_1V} = 0.17$$

Loss (Example 8.51):

0 $400\left(\frac{1}{1.1}\right) - 74.33 = 289.30$

1 $400\left(\frac{1}{1.1}\right)^2 - 74.33\left(1 + \frac{1}{1.1}\right) = 188.68$

2 $-74.33\left(1 + \frac{1}{1.1}\right) = -141.90$

Since $E[L] = 0$,
$Var = (289.30)^2(0.17) + (188.68)^2(0.25)(1-0.17) + (141.90)^2(1-0.17)(1-0.25) = 34150$

Question # 4
Key: A

The distribution is negative binomial

$$E[N]=1$$

$$\begin{aligned}\operatorname{Var}[N] &= E_\Lambda\left[\operatorname{Var}(N|\Lambda)\right]+\operatorname{Var}_\Lambda\left[E(N|\Lambda)\right]\\ &= E_\Lambda[\Lambda]+\operatorname{Var}_\Lambda[\Lambda]\\ &=1+2=3\end{aligned}$$

From the KPW appendix, we have:

$$rb=1$$

$$rb(1+b)=3$$

$$(1+b)=\frac{3}{1}\Rightarrow b=2$$

$$rb=1\Rightarrow r=\frac{1}{2}$$

$$\Pr[N=1]=\frac{rb}{(1+b)^{r+1}}=\frac{1}{3^{3/2}}=0.19245$$

Question # 5
Key: B

If the worker survives for three years, the reinsurance will pay 100,000 at t=3, and everything after that. So the actuarial present value of the reinsurer's portion of the claim

$$=100{,}000\times\left(\frac{.7}{1.05}\right)^3+150{,}000\times\left(\left(\frac{.7}{1.05}\right)^4+\left(\frac{.7}{1.05}\right)^5\right)$$

$$=79{,}012$$

Question # 6
Key: D

- Surplus = $U(t) = 60 + 20t - S(t)$.

- $P(\text{Ruin}) = P\left(t < \dfrac{S(t) - 60}{20}\right)$

- Hence claims of 100, 200 causing ruin can only occur on the intervals $\left[0, \dfrac{100-60}{20} = 2\right], \left[0, \dfrac{200-60}{20} = 7\right]$ respectively.

- $P(\text{Claim by } t) = 1 - P(\text{no claim}) = \dfrac{t}{1+t}$

- Therefore,

$$P(\text{Ruin}) = \sum_s \text{p(s)} P(\text{claim of size s causes ruin})$$

$$= 60\% \frac{2}{1+2} + 40\% \frac{7}{1+7} = 75\%$$

Question # 7
Key: D

$$L_x^{(t)} = \frac{l_x^{(t)} + l_{x+1}^{(t)}}{2} = 24{,}000$$

$$q_x^{(i)} = \frac{d_x^{(i)}}{25{,}000}$$

$$m_x^{(d)} = \frac{d_x^{(d)}}{L_x^{(t)}} = \frac{d_x^{(d)}}{24{,}000} = 0.02 \Rightarrow d_x^{(d)} = 480$$

$$m_x^{(w)} = \frac{d_x^{(w)}}{L_x^{(t)}} = \frac{d_x^{(w)}}{24{,}000} = 0.05 \Rightarrow d_x^{(w)} = 1200$$

$$\Rightarrow d_x^{(t)} = 2000 = d_x^{(i)} + d_x^{(d)} + d_x^{(w)} \Rightarrow d_x^{(i)} = 320$$

$$\Rightarrow q_x^{(i)} = \frac{320}{25{,}000} = 0.0128$$

Question # 8
Key: C

$$Z = \begin{cases} 10^5 v^{k+1} & k = 0,1 \\ 0 & k = 2,3,\ldots \end{cases} \qquad E[Z] = E[Z|S]1/3 + E[Z|N]2/3$$

$$= 10^5\left\{\left(vq^s_{[x]} + v^2{}_1|q^s_{[x]}\right)1/3 + \left(vq^N_{x[x]} + v^2{}_1|q^N_{[x]}\right)2/3\right\}$$

$$q_{[x]} = 1 - p_{[x]} \qquad p_{[x]} = e^{-\left(\frac{1}{q}\right)^t}$$

$$_1|q_{[x]} = p_{[x]} - {}_2p_{[x]} \qquad {}_2p_{[x]} = e^{-\left(\frac{2}{q}\right)^t}$$

	Smokers	Nonsmokers
$p_{[x]}$	0.64118	0.77880
$_2p_{[x]}$	0.16901	0.36788

Now "plug in"

$$= 10^5\left\{\left[\frac{1}{1.05}(0.35882) + \left(\frac{1}{1.05}\right)^2(0.47217)\right]1/3 + \left[\frac{1}{1.05}(0.2212) + \left(\frac{1}{1.05}\right)^2(0.41092)\right]2/3\right\}$$

$$= (58{,}338)2/3 + (77{,}000)1/3 = 64{,}559$$

Question # 9
Key: A

$${}_{5}^{10}\bar{V}\left({}_{10|}\bar{a}_{35}\right) = \bar{P}\left({}_{10|}\bar{a}_{35}\right)\bar{s}_{35:\overline{5}|}$$

$$\Rightarrow \bar{P}\left({}_{10|}\bar{a}_{35}\right) = \frac{\bar{a}_{45}}{\bar{s}_{35:\overline{10}|}}$$

$$\bar{a}_{45} = \int_0^{40} v^t\,{}_tp_{45}\,dt = \int_0^{40} {}_tp_{45}\,dt \ \ (i=0)$$

$$= \mathring{e}_{45} = \frac{1}{40}\left(40 - \frac{t^2}{2}\right)_0^{40} = 20$$

$$\bar{s}_{35:\overline{10}|} = \bar{a}_{35:\overline{10}|} / {}_{10}E_{35} = \bar{a}_{35:\overline{10}|} \ / \ v^{10}\ {}_{10}p_{35}$$

$$= \frac{1}{50}\int_0^{10}(50-t)\,dt\,\frac{l_{35}}{l_{45}}$$

$$= \frac{450}{40}$$

$$\Rightarrow \bar{P}\left({}_{10|}\bar{a}_{35}\right) = \frac{20}{450}\times 40 = 1.778$$

$${}_{5}^{10}\bar{V} = \bar{P}\left({}_{10|}\bar{a}_{35}\right)\bar{s}_{35:\overline{5}|} = \frac{(1.778)}{50}\int_0^5 (50-t)\,dt\,\frac{l_{35}}{l_{40}}$$

$$= \frac{1}{50}(1.778)\left(\frac{50}{45}\right)\left(50t - \frac{t^2}{2}\right)_0^5 = 9.38$$

Question # 10
Key: D

Let X(t) be the number leaving by cab in a t hour interval and let Y_i denote the number of people in the ith group. Then:

$$E[Y_i] = (1\times 0.6) + (2\times 0.3) + (3\times 0.1) = 1.5$$
$$E[Y_i^2] = (1^2 \times 0.6) + (2^2 \times 0.3) + (3^2 \times 0.1) = 2.7$$

$$E[X(72)] = 10\times 72\times 1.5 = 1{,}080$$
$$Var[X(72)] = 10\times 72\times 2.7 = 1{,}944$$

$$\begin{aligned} P[X(72)\geq 1{,}050] &= P[X(72)\geq 1049.5] \\ &= P\left[\frac{X(72)-1080}{\sqrt{1944}} \geq \frac{1049.5-1080}{\sqrt{1944}}\right] \\ &= 1-\Phi(-0.691754) \\ &= \Phi(0.691754) \\ &= 0.7553 \end{aligned}$$

The answer is D with or without using the 0.5 continuity correction.

Question # 11
Key: E

Calculate convolutions of $f_x(x)$:

x	f	f^{*2}
10	0.3	0.00
20	0.3	0.09

n	0	1	2	3	4
$\Pr(N=n)$	0.37	0.37	0.18	0.06	0.02

$f_s(0) = 0.37$ $\quad$ $F_s(0) = 0.37$

$f_s(10) = 0.37 \times 0.3 = 0.11$ $\quad$ $F_s(10) = 0.37 + 0.11 = 0.48$

$f_s(20) = 0.18 \times 0.09 + 0.3 \times 0.37 = 0.13$ $\quad$ $F_s(20) = 0.48 + 0.13 = 0.61$

$$E[S] = 1[(0.3)(10) + (0.3)(20) + (0.4)(50)] = 29$$

$$E[(S-30)_+] = E[S] - 10(1 - F_S(0)) - 10(1 - F_S(10)) - 10(1 - F_S(20))$$
$$= 29 - 10(3 - 0.37 - 0.48 - 0.61) = 29 - 15.4 = 13.6$$

Question # 12
Key: A

$$0.0408 = m(81.5) = \frac{q_{81}}{(1 - 1/2 q_{81})} \Rightarrow q_{81} = 0.0400$$

Similarly, $q_{80} = 0.0200$ *and* $q_{82} = 0.0600$

$${}_2q_{80.5} = {}_{1/2}q_{80.5} + {}_{1/2}p_{80.5}[q_{81} + p_{81}\ {}_{1/2}q_{82}]$$
$$= \frac{0.01}{0.99} + \frac{0.98}{0.99}[0.04 + 0.96(0.03)] = 0.0782$$

Question # 13
Key: A

$$E(Z) = 1000\frac{m}{m+d}e^{-10(m+d)}$$
$$= 1000\left(\frac{5}{12}\right)e^{-1.2}$$
$$= 125.5$$

$$\text{Var}(Z) = 1000^2\left(\frac{m}{m+2d}e^{-10(m+2d)} - \left(\frac{5}{12}\right)^2 e^{-2.4}\right)$$
$$= 23{,}610.16$$

$$E(S) = 400E(Z) = 50{,}200$$

$$\text{Var}(S) = 400\,\text{Var}(Z) = 9{,}444{,}064$$

$$0.95 = \Pr\left(\frac{S - E(S)}{\sqrt{\text{Var}(S)}} \le \frac{k - 50{,}200}{\sqrt{9{,}444{,}064}}\right) \Rightarrow k = 1.645 \times \sqrt{9{,}444{,}064} + 50{,}200 = 55{,}255$$

Question # 14
Key: B

According to the theorem on p. 67, $\Rightarrow$ the expected number of interations is $c \approx 1.78$, as follows:

$$\frac{f(x)}{g(x)} = 12\left(x - 2x^2 + x^3\right)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = 12\left(1 - 4x + 3x^2\right) = 0$$

$$\Rightarrow 12(3x-1)(x-1) = 0$$

$$\Rightarrow x = \frac{1}{3}$$

$$\left.\frac{d^2}{dx^2}\left(\frac{f(x)}{g(x)}\right)\right|_{1/3} = 12(-4+6x)\Big|_{\frac{1}{3}} = 12(-4+2) = -24 < 0$$

$$\therefore \frac{f(x)}{g(x)} \text{ is maximum when } x = \frac{1}{3}$$

$$\Rightarrow c = 12\left(\frac{1}{3} - 2\times\frac{1}{9} + \frac{1}{27}\right)$$

$$= 12\left[\frac{9-6+1}{27}\right] = 12\times\frac{4}{27} = \frac{48}{27} \approx 1.78$$

Question # 15
Key: B

$$p_{30}^{(t)} = p_{30}'^{(1)} p_{30}'^{(2)} = (0.9)(0.7) = 0.63$$

$$l_{31}^{(t)} = p_{30}^{(t)} l_{30}^{(t)} = 630$$

$$d_{31}^{(1)} = {}_{1|}q_{30}^{(1)} \, l_{30}^{(t)} = 75$$

$$d_{31}^{(t)} = l_{31}^{(t)} - l_{32}^{(t)} = 630 - 472 = 158$$

$$d_{31}^{(2)} = d_{31}^{(t)} - d_{31}^{(1)} = 158 - 75 = 83$$

$$q_{31}^{(2)} = \frac{d_{31}^{(2)}}{l_{31}^{(t)}} = \frac{83}{630} = 0.1317 \approx 0.13$$

Question # 16
Key: C

Let S = the aggregate loss.

$$E(S) = 8 \times 10{,}000$$

$$\text{Var}(S) = 8 \times 3937^2 + 3^2 \times (10{,}000)^2 = (32{,}000)^2$$

$$\begin{aligned} P(S > 1.5E(S)) &= P\left(\frac{S - 80{,}000}{32{,}000} > \frac{40{,}000}{32{,}000}\right) \\ &= P(Z > 1.25) \\ &= \Phi(-1.25) \\ &= 1 - \Phi(1.25) \end{aligned}$$

Question # 17
Key: E

$$s(x) = \Pr(X > x) = E\left[\Pr(X > x|q)\right] = \int_1^{11} 0.1 e^{-qx} dq$$

$$= 0.1\frac{\left(e^{-x} - e^{-11x}\right)}{x}$$

So: $s(0.5) = 0.1205$

Question # 18
Key: A

$$p = \frac{1000\int_0^\infty me^{-(d+m)t}dt}{\int_0^\infty e^{-(d+m)t}dt} = \frac{1000\frac{m}{d+m}}{\frac{1}{d+m}} = 1000m = 150$$

Expected loss $= 1000\overline{A}_{80} - p\,\overline{a}_{80}$.

$$1000\overline{A}_{80} = 1000\frac{i}{d}A_{80} = (1.029708672)(665.75) = 685.53$$

$$\overline{a}_{80} = \frac{1 - 0.68553}{d} = 5.3969$$

Expected loss = $685.53 - 150(5.3969) = -124$

Question # 19
Key: D

- $E(S) = E(N)\text{Var}(X) + E(X)^2\,\text{Var}(N)$
- N is binomial $E(N) = nq$, $\text{Var}(N) = nq(1-q)$, where q is the probability of a claim.
- X is uniform. $E(X) = \dfrac{\text{Max}}{2}$, $\text{Var}(X) = \dfrac{\text{Max}^2}{12}$

Hence,

$$= \sum_{\text{policies}} \#\text{policies}\left\{q\frac{\text{Max}^2}{12} + q(1-q)\left(\frac{\text{Max}}{2}\right)^2\right\}$$

$$\text{Total variance } = 100\left[0.05\frac{400^2}{12} + (0.05)(0.95)\left(\frac{400}{2}\right)^2\right] + 200\left[0.06\frac{300^2}{12} + (0.06)(0.94)\left(\frac{300}{2}\right)^2\right]$$

$$= 600{,}466.67$$

Question # 20
Key: A

$$APV = v^5\left({}_4p_{\overline{70:80}} - {}_5p_{\overline{70:80}}\right)$$

$$_4p_{70} = \frac{l_{74}}{l_{70}} = \frac{56{,}640.51}{66{,}161.55} = 0.856094$$

$$_4p_{80} = \frac{l_{84}}{l_{80}} = \frac{26{,}607.34}{39{,}143.65} = 0.679736$$

$$_4p_{\overline{70:80}} = {}_4p_{70} + {}_4p_{80} - {}_4p_{70} \times {}_4p_{80} = 0.953912$$

$$_5p_{70} = \frac{l_{75}}{l_{70}} = \frac{53{,}960.81}{66{,}161.55} = 0.815592$$

$$_5p_{80} = \frac{l_{85}}{l_{80}} = \frac{23{,}582.46}{39{,}143.65} = 0.602459$$

$$_5p_{\overline{70:80}} = {}_5p_{70} + {}_5p_{80} - {}_5p_{70} \times {}_5p_{80} = 0.926690$$

$$APV = (0.953912 - 0.926690)v^5 = \frac{0.027222}{1.03^5} = 0.02348$$

Question # 21
Key: E

The expected values at the end of 20 years are $(1.1)^{20} = (1.16)^{20} e^{-20m}$.
Actuarial present values at time zero are $(1.1)^{20} v^{20}$ and $(1.16)^{20} e^{-20m} v^{20}$

So $1.1 = 1.16 e^{-m}$

So $m = \log\left(\frac{1.16}{1.1}\right) = 0.0531$

$$S(x_{0.5}) = e^{-mx_{0.5}} = 0.5 \Rightarrow x_{0.5} = \frac{-\log 0.5}{m} = \frac{0.6931}{0.0531} = 13.05$$

Question # 22
Key: B

Note: These values of l are 1/100 of the ones in the Exam booklet, which does not affect the answer.

$l_{30} = 95{,}014$
$(1-0.63) l_{30} = 35{,}155$
$l_{81} < 35{,}155 < l_{82}$
So: $K(30) = 51$

$l_{50} = 89{,}509$
$(1-0.4) l_{50} = 53{,}705$
$l_{75} < 53{,}705 < l_{76}$
So: $K(50) = 25$

$$\begin{aligned} \text{Simulated Y} \quad &= 1000 \ddot{a}_{\overline{26}|} + 500\left(\ddot{a}_{\overline{52}|} - \ddot{a}_{\overline{26}|}\right) \\ &= 1000(13.783) + 500(16.813 - 13.783) \\ &= 15{,}298 \end{aligned}$$

Question # 23
Key: A

Mean time until leaving state 2 is $\dfrac{1}{2} \Rightarrow q_{20} + q_{21} + q_{23} = 2$.

$(q_{20} = 0$ for birth and death process$)$

$0 + 0.4 + q_{23} = 2$

$q_{23} = 1.6$

$\sum P_i = 1 \Rightarrow P_3 = 0.6 - P_2$

Rate entering 3 = rate leaving 3 $\Rightarrow P_2 q_{23} = P_3 q_{32}$

$$1.6 P_2 = (0.6 - P_2)(0.12)$$
$$1.72 P_2 = 0.072$$
$$P_2 = 0.042$$

Question # 24
Key: D

$$ {}_{11}V = \left({}_{10}V + P_{50} \right)\left(1+i \right) - \left(b_{11} - {}_{11}V \right) q_{80} \qquad (1) $$

But by traditional formula:

${}_{11}V_{50} = \left({}_{10}V_{50} + P_{50} \right)\left(1+i \right) - \left(1 - {}_{11}V_{50} \right) q_{60}$ and since ${}_{10}V = {}_{10}V_{50}$ and ${}_{11}V = {}_{11}V_{50}$

So:

$$ {}_{11}V = \left({}_{10}V + P_{50} \right)\left(1+i \right) - \left(1 - {}_{11}V \right) q_{60} \qquad (2) $$

$(1)-(2) \Rightarrow \left(b_{11} - {}_{11}V \right) q_{80} = \left(1 - {}_{11}V \right) q_{60}$

So:

$$ \begin{aligned} b_{11} &= \frac{\left(1 - {}_{11}V \right) q_{60}}{q_{80}} + {}_{11}V \\ &= \frac{\left(1 - 0.16637 \right)\left(0.01368 \right)}{0.02368} + 0.16637 \\ &= 0.64796 \approx 0.648 \end{aligned} $$

Question # 25
Key: E

Let L be the loss, then the bonus $= \frac{1}{3} \quad 500{,}000 \begin{cases} .7 - \dfrac{L}{500{,}000} & \text{If } L \le 350{,}000 \\ 0 & \text{If } L > 350{,}000 \end{cases}$

$$
\begin{aligned}
E(\text{Bonus}) &= \frac{350{,}000}{3} - \frac{1}{3} E[L \wedge 350{,}000] \\
&= \frac{350{,}000}{3} - \left(\frac{1}{3}\right)\frac{600{,}000}{2}\left(1 - \left(\frac{600{,}000}{950{,}000}\right)^2\right) \\
&= 56{,}556
\end{aligned}
$$

Question # 26
Key: B

$$
\begin{aligned}
1000\,{}_1V_{x:\overline{3}|} &= \left[\frac{\left(1000P_{x:\overline{3}|}\right)(1+i) - (1000q_x)}{p_x}\right] \\
&= \left[\frac{(373.63)(1.06) - (1000)0.2}{0.8}\right] \\
&= \left[\frac{396.0478 - 200}{0.8}\right] \\
&= 245.06
\end{aligned}
$$

$$
\begin{aligned}
1000\,{}_2V_{x:\overline{3}|} &= \left[\frac{\left(1000P_{x:\overline{3}|} + {}_1V_{x:\overline{3}|}\right)(1+i) - (1000q_{x+1})}{p_{x+1}}\right] \\
&= \left[\frac{(373.63 + 245.06)(1.06) - (1000)0.2}{0.8}\right] \\
&= \left[\frac{655.8114 - 200}{0.8}\right] \\
&= 569.76
\end{aligned}
$$

$$\therefore 1000\left({}_2V_{x:\overline{3}|} - {}_1V_{x:\overline{3}|}\right) = 569.76 - 245.06 = 324.70$$

Question # 27
Key: A

	Probability	Cash Before Receiving Prem.	Cash After Premium
T=0	1.000	2	4
T=1	0.6	4.32	6.32
	0.3	1.32	3.32
	0.1	Ruin	--
T=2	0.36	6.83	8.83
	0.18	3.83	5.83
	0.06	Ruin	--
	0.18	3.59	5.59
	0.09	0.59	2.59
	0.03	Ruin	--
	0.10	Ruin (from above)	--
T=3	0.216	9.53	
	0.108	6.53	
	0.036	1.53	
	0.108	6.29	
	0.054	3.29	
	0.018	Ruin	
	0.108	6.03	
	0.054	3.03	
	0.018	Ruin	
	0.054	2.79	
	0.027	Ruin	
	0.009	Ruin	
	0.190	Ruin (From above)	

Question # 28
Key: C

For Dick: $$_{12|}q_{20} = \frac{l_{32}q_{32}}{l_{20}} = \frac{(9471591)(0.00170)}{9617802} = 0.001674$$

For Jane: $$_{35}p_{21} = \frac{l_{56}}{l_{21}} = \frac{8563435}{9607896} = 0.891291$$

For both: $$(0.001674)(0.891291) = 0.00149$$

Question # 29
Key: C

$$\ddot{a}_x = 1 + vp_x + v^2 p_x p_{x+1} \ddot{a}_{x+2}$$

Let y denote the change in p_{x+1}.

$$\ddot{a}(\text{with increase}) = 1 + vp_x + v^2 p_x (p_{x+1} + y) \ddot{a}_{x+2}$$
$$= \ddot{a}(\text{without}) + yv^2 p_x \ddot{a}_{x+2}$$

$yv^2 p_x \ddot{a}_{x+2}$ = change in actuarial present value.

$$\ddot{a}_{x+1} = 6.951 = 1 + vp_{x+1}\ddot{a}_{x+2} = 1 + \frac{1}{1.05}(1 - 0.05)\ddot{a}_{x+2} \Rightarrow 6.577 = \ddot{a}_{x+2}$$

$$0.03\left(\frac{1}{1.05}\right)^2 0.99 \times 6.577 = 0.177$$

Question # 30
Key: D

Expected claim payments in 2000

$$= E[X \wedge 22] - E[X \wedge 11]$$
$$= \left(-0.025 \times 22^2 + 1.475 \times 22 - 2.25\right) - \left(-0.025 \times 11^2 + 1.475 \times 11 - 2.25\right)$$
$$= (18.10 - 10.95)$$
$$= 7.15$$

From theorem 2.5:

Expected claim payments in 2001

$$= 1.1\left[E(X \wedge 20) - E(X \wedge 10)\right]$$
$$= 1.1\left[\left(-0.025 \times 20^2 + 1.475 \times 20 - 2.25\right) - \left(0.025 \times 10^2 + 1.475 \times 10 - 2.25\right)\right]$$
$$= 1.1(17.25 - 10)$$
$$= 7.975$$

$$\frac{7.975}{7.15} = 1.115$$

Question # 31
Key: B

$$A^{1}_{30:\overline{3|}} = \frac{0.01}{1.04} + \frac{0.99(0.02)}{1.04^2} + \frac{(0.99)(0.98)(0.03)}{1.04^3} = 0.053796725$$

$$\ddot{a}_{30:\overline{3|}} = 1 + \frac{0.99}{1.04} + \frac{(0.99)(0.98)}{1.04^2} = 2.848927515$$

$$1000P^{1}_{30:\overline{3|}} = 1000\frac{0.053796725}{2.848927515} = 18.88315$$

$$A^{1}_{31:\overline{3|}} = \frac{0.02}{1.04} + \frac{(0.98)(0.03)}{1.04^2} + \frac{(0.98)(0.97)(0.04)}{1.04^3} = 0.080215919$$

$$\ddot{a}_{31:\overline{3|}} = 1 + \frac{0.98}{1.04} + \frac{(0.98)(0.97)}{1.04^2} = 2.82119$$

$$1000P^{1}_{30:\overline{3|}} \times \ddot{a}_{31:\overline{3|}} = BA^{1}_{31:\overline{3|}}$$

$$53.272954 = B(0.080215919)$$
$$B = 664$$

Question # 32
Key: B

Results from the standard normal *cdf* are converted to $N(15{,}000; 2000)$ using $X = 15{,}000 + 2000Z$

Random Number	Standard Normal	Cost	Claim Paid
0.5398	0.1	15200	5200
0.1151	-1.2	12600	2600
0.0013	-3.0	9000	0
0.7881	0.8	16600	6600

Total claims = 5200 + 2600 + 0 + 6600 = 14,400

Question # 33
Key: E

Given:

$$(0.95)p_0 + (0.60)p_1 + p_2 = p_0$$
$$(0.05)p_0 = p_1$$
$$(0.40)p_1 = p_2$$
$$p_0 + p_1 + p_2 = 1$$

Solve for $p_1 = \dfrac{5}{107} \sim 0.047$.

Question # 34
Key: D

$$\mu_x^{T(x)} = \mu_x^{T^*(x)} + \mu^Z = 0.03 + 0.02 = 0.05$$

$$\mu_y^{T(y)} = \mu_y^{T^*(y)} + \mu^Z = 0.05 + 0.02 = 0.07$$

$$\mu_{xy} = \mu_x^{T^*(x)} + \mu_y^{T^*(y)} + \mu^Z = 0.03 + 0.05 + 0.02 = 0.10$$

$$\bar{A}_x = \frac{\mu_x^{T(x)}}{\mu_x^{T(x)} + \delta} = \frac{0.05}{0.11} = 0.4545$$

$$\bar{A}_y = \frac{\mu_y^{T(y)}}{\mu_y^{T(y)} + \delta} = \frac{0.07}{0.13} = 0.5385$$

$$\bar{A}_{xy} = \frac{\mu_{xy}}{\mu_{xy} + \delta} = \frac{0.10}{0.16} = 0.6250$$

$$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = 0.4545 + 0.5385 - 0.6250 = 0.3680$$

Question # 35
Key: A

Distribution of moment of death is 60% ILT and 40% constant force.

Therefore:

$$\bar{A}_{\text{Jack}} = (0.6)\bar{A}_{62} \text{ using ILT } + (0.4)\bar{A}_{62} \text{ constant force.}$$

$$\bar{A}_{62} \text{ ILT} = \frac{0.06}{\ln(1.06)}(0.39670) = 0.40849$$

$$\bar{A}_{62} \text{ constant force} = \frac{0.02}{0.02 + \ln(1.06)} = 0.25553$$

$$\bar{A}_{\text{Jack}} = (0.6)(0.40849) + (0.4)(0.25553) = 0.3473$$

$$\bar{a}_{\text{Jack}} = \frac{1 - \bar{A}_{\text{Jack}}}{\ln(1.06)} = \frac{1 - 0.3473}{0.05827} = 11.2013$$

$$1000\bar{P}_{\text{Jack}} = \frac{(1000)(0.3473)}{11.2013} = 31.01$$

Question # 36
Key: E

$$S = \sum_{j=1}^{N} Z_j$$

$$\begin{aligned}\text{Var}(S) &= E(N)\text{Var}(Z_1) + E(Z_1)^2 \text{ Var}(N) \\ &= (10)(0.1)(10{,}779.18) + (71.35)^2 (10)(0.1)(0.9) \\ &= 15{,}361\end{aligned}$$

Where $Z_j = 1000 v^{T_j(22)}$

$$\text{Var}(Z_1) = 10^6 \left({}^2A_{22} - A_{22}^2\right) = 10{,}779.18$$

$$E(Z_1) \quad = A_{22} \times 10^3 = 71.35$$

Question # 37
Key: C

The given relationship can be written $p_k = p_{k-1}\left(0 + \dfrac{1}{k}\right)$

so p_k is an (a,b,1) class with $a = 0$ and $b = 1$,

so p_k is Poisson with $l = 1$,

so $p_0 = e^{-l} = 0.368 = p_1$.

By 3.14 of Loss Models, $p_1^M = \dfrac{1 - p_0^M}{1 - p_0} p_1 = \dfrac{0.9}{0.632}\, 0.368 = 0.52$

Question # 38
Key: E

$P_{02} = P_{12}$, and therefore it is not necessary to track states 0 and 1 separately.

$$P_{12} = \Pr\left[1 \text{ big or (no big and 2 or more smalls)}\right]$$
$$= 0.1 + (0.9)\left[3(0.25)^2(0.75) + (0.25)^3\right]$$
$$= 0.1 + 0.14 = 0.24$$

If you enter 2, you stay there.
Probability of not entering in 3 years $= (1-0.24)^3 = 0.44$.
Probability of being in 2 after 3 years $= 1 - 0.44 = 0.56$.

Question # 39
Key: C

$$\Pr\left(\bar{a}_{\overline{T}|} > \bar{a}_x\right) = \Pr\left(\frac{1-v^T}{d} > \bar{a}_x\right)$$
$$= \Pr\left[T > \frac{-1}{d}\log\left(\frac{m}{d+m}\right)\right]$$
$$= {}_{t_0}p_x \text{ where } t_0 = \frac{-1}{d}\log\left(\frac{m}{d+m}\right)$$
$$= e^{-mt_0}$$
$$= \left(\frac{m}{m+d}\right)^{m/d}$$
$$= \left(\frac{6}{10}\right)^{3/2} = 0.4648$$

Question # 40
Key: D

p = limiting probability of rain.

$$p = 0.5p + 0.3(1 - p)$$

$$p = \frac{0.3}{0.8} = 0.375$$

Prob (rain on two consecutive days) = Prob (rain on first) x Prob (rain on second, given rain on first)
= (0.375)(0.5)
= 0.1875