Exam C Solutions
Spring 2005

Question #1
Key: D

The CDF is $F(x) = 1 - \frac{1}{(1+x)^4}$

Observation (x)	F(x)	compare to:	Maximum difference
0.2	0.518	0, 0.2	0.518
0.7	0.880	0.2, 0.4	0.680
0.9	0.923	0.4, 0.6	0.523
1.1	0.949	0.6, 0.8	0.349
1.3	0.964	0.8, 1.0	0.164

The K-S statistic is the maximum from the last column, 0.680.
Critical values are: 0.546, 0.608, 0.662, and 0.729 for the given levels of significance. The test statistic is between 0.662 (2.5%) and 0.729 (1.0%) and therefore the test is rejected at 0.025 and not at 0.01.

Question #2
Key: E

For claim severity,
$\mu_S = 1(0.4) + 10(0.4) + 100(0.2) = 24.4,$
$\sigma_S^2 = 1^2(0.4) + 10^2(0.4) + 100^2(0.2) - 24.4^2 = 1,445.04.$
For claim frequency,
$\mu_F = r\beta = 3r, \quad \sigma_F^2 = r\beta(1+\beta) = 12r.$
For aggregate losses,
$\mu = \mu_S\mu_F = 24.4(3r) = 73.2r,$
$\sigma^2 = \mu_S^2\sigma_F^2 + \sigma_S^2\mu_F = 24.4^2(12r) + 1,445.04(3r) = 11,479.44r.$
For the given probability and tolerance, $\lambda_0 = (1.96/0.1)^2 = 384.16.$
The number of observations needed is
$\lambda_0\sigma^2/\mu^2 = 384.16(11,479.44r)/(73.2r)^2 = 823.02/r.$
The average observation produces 3*r* claims and so the required number of claims is
$(823.02/r)(3r) = 2,469.$

Question #3
Key: A

$$\hat{H}(t_2) = \frac{1}{n} + \frac{1}{n-1} = \frac{2n-1}{n(n-1)} = \frac{39}{380} \Rightarrow 39n^2 - 799n + 380 = 0 \Rightarrow n = 20, n = 0.487\,.$$

Discard the non-integer solution to have $n = 20$.
The Kaplan-Meier Product-Limit Estimate is:

$$\hat{S}(t_9) = \frac{19}{20}\frac{18}{19}\cdots\frac{11}{12} = \frac{11}{20} = 0.55.$$

Question #4
Key: E

There are 27 possible bootstrap samples, which produce four different results. The results, their probabilities, and the values of g are:

Bootstrap Sample	Prob	g
1, 1, 1	8/27	0
1, 1, 4	12/27	2
1, 4, 4	6/27	-2
4, 4, 4	1/27	0

The third central moment of the original sample is 2. Then,

$$\text{MSE} = \left[\frac{8}{27}(0-2)^2 + \frac{12}{27}(2-2)^2 + \frac{6}{27}(-2-2)^2 + \frac{1}{27}(0-2)^2\right] = \frac{44}{9}.$$

Question #5
Key: A

Pick one of the points, say the fifth one. The vertical coordinate is $F(30)$ from the model and should be slightly less than 0.6. Inserting 30 into the five answers produces 0.573, 0.096, 0.293, 0.950, and something less than 0.5. Only the model in answer A is close.

Question #6
Key: C or E

This question was considered ambiguous, hence two solutions were accepted. Had the question begun with "Claim amounts are…" then the correct interpretation is that there are five observations (whose values are not stated), each from the Poisson distribution. In that case, the solution proceeds as follows:

The distribution of Θ is Pareto with parameters 1 and 2.6. Then,

$$v = EVPV = E(\Theta) = \frac{1}{2.6-1} = 0.625, \quad a = VHM = Var(\Theta) = \frac{2}{1.6(0.6)} - 0.625^2 = 1.6927,$$

$$k = v/a = 0.625/1.6927 = 0.3692, \quad Z = \frac{5}{5+0.3692} = 0.9312.$$

Alternatively, had the question begun with "The number of claims is …" then the correct interpretation is that there was a single observation whose value was 5. The only change is that now n = 1 rather than 5 and so $Z = \frac{1}{1+0.3692} = 0.7304.$

Question #7
Key: E

At 300, there are 400 policies available, of which 350 survive to 500. At 500 the risk set increases to 875, of which 750 survive to 1,000. Of the 750 at 1,000, 450 survive to 5,000. The probability of surviving to 5,000 is $\frac{350}{400}\frac{750}{875}\frac{450}{750} = 0.45$. The distribution function is 1 – 0.45 = 0.55.

Alternatively, the formulas in *Loss Models* could be applied as follows:

j	Interval $(c_j, c_{j+1}]$	d_j	u_j	x_j	P_j	r_j	q_j	$\hat{F}(c_j)$
0	(300, 500]	400	0	50	0	400	0.125	0
1	(500, 1,000]	525	0	125	350	875	0.142857	0.125
2	(1,000, 5,000]	0	120	300	750	750	0.4	0.25
3	(5,000, 10,000]	0	30	300	330	330	0.90909	0.55

where
d_j = number of observations with a lower truncation point of c_j,
u_j = number of observations censored from above at c_{j+1},
x_j = number of uncensored observations in the interval $(c_j, c_{j+1}]$.
Then, $P_{j+1} = P_j + d_j - u_j - x_j$, $r_j = P_j + d_j$ (using $\alpha = 1$ and $\beta = 0$), and $q_j = x_j / r_j$.

Finally, $\hat{F}(c_j) = 1 - \prod_{i=0}^{j-1}(1-q_i)$.

Question #8
Key: D

Let the function be $f(x) = a + bx + cx^2 + dx^3$. The four conditions imply the following:

$$
\begin{aligned}
f(0) &= 0 \quad \Rightarrow \quad a = 0, \\
f(2) &= 0 \quad \Rightarrow \quad 2b + 4c + 8d = 0, \\
f'(0) &= -2 \quad \Rightarrow \quad b = -2, \\
f'(2) &= 2 \quad \Rightarrow \quad b + 4c + 12d = 2.
\end{aligned}
$$

The solution is $c = 1$ and $d = 0$ and thus the function is $f(x) = x^2 - 2x$. The curvature measure is $\int_0^2 [f''(x)]^2 dx = \int_0^2 2^2 dx = 8.$

Question #9
Key: D

For an exponential distribution, the maximum likelihood estimate of θ is the sample mean, 6. Let $Y = X_1 + X_2$ where each X has an exponential distribution with mean 6. The sample mean is $Y/2$ and Y has a gamma distribution with parameters 2 and 6. Then

$$
\Pr(Y/2 > 10) = \Pr(Y > 20) = \int_{20}^{\infty} \frac{xe^{-x/6}}{36} dx
$$

$$
= -\frac{xe^{-x/6}}{6} - e^{-x/6} \Big|_{20}^{\infty} = \frac{20e^{-20/6}}{6} + e^{-20/6} = 0.1546.
$$

Question #10
Key: A

From Question 9, $F(10) = 1 - \frac{10e^{-10/\theta}}{\theta} - e^{-10/\theta} = 1 - e^{-10/\theta}(1 + 10\theta^{-1}) = g(\theta).$

$$
g'(\theta) = -\frac{20}{\theta^2} e^{-20/\theta}(1 + 20\theta^{-1}) + e^{-20/\theta} \frac{20}{\theta^2} = -\frac{400e^{-20/\theta}}{\theta^3}.
$$

At the maximum likelihood estimate of 6, $g'(6) = -0.066063.$

The maximum likelihood estimator is the sample mean. Its variance is the variance of one observation divided by the sample size. For the exponential distribution the variance is the square of the mean, so the estimated variance of the sample mean is 36/2 = 18. The answer is $(-0.066063)^2(18) = 0.079.$

Question #11
Key B

$$\mu(\lambda,\theta) = E(S \mid \lambda,\theta) = \lambda\theta,$$
$$v(\lambda,\theta) = Var(S \mid \lambda,\theta) = \lambda 2\theta^2,$$
$$v = EVPV = E(\lambda 2\theta^2) = 1(2)(1+1) = 4,$$
$$a = VHM = Var(\lambda\theta) = E(\lambda^2)E(\theta^2) - [E(\lambda)E(\theta)]^2 = 2(2) - 1 = 3,$$
$$k = v / a = 4/3.$$

Question #12
Key: B

The distribution is binomial with m = 100 and q = 0.03. The first three probabilities are:
$$p_0 = 0.97^{100} = 0.04755,\ p_1 = 100(0.97)^{99}(0.03) = 0.14707,$$
$$p_2 = \frac{100(99)}{2}(0.97)^{98}(0.03)^2 = 0.22515.$$
Values between 0 and 0.04755 simulate a 0, between 0.04755 and 0.19462 simulate a 1, and between 0.19462 and 0.41977 simulate a 2. The three simulated values are 2, 0, and 1. The mean is 1.

Question #13
Key: A

A mixture of two Poissons or negative binomials will always have a variance greater than its mean. A mixture of two binomials could have a variance equal to its mean, because a single binomial has a variance less than its mean.

Question #14
Key: D

The posterior distribution can be found from

$$\pi(\lambda\,|\,10) \propto \frac{e^{-\lambda}\lambda^{10}}{10!}\left(\frac{0.4}{6}e^{-\lambda/6} + \frac{0.6}{12}e^{-\lambda/12}\right) \propto \lambda^{10}\left(0.8e^{-7\lambda/6} + 0.6e^{-13\lambda/12}\right).$$

The required constant is found from

$$\int_0^\infty \lambda^{10}\left(0.8e^{-7\lambda/6} + 0.6e^{-13\lambda/12}\right)d\lambda = 0.8(10!)(6/7)^{11} + 0.6(10!)(12/13)^{11} = 0.395536(10!).$$

The posterior mean is

$$E(\lambda\,|\,10) = \frac{1}{0.395536(10!)}\int_0^\infty \lambda^{11}\left(0.8e^{-7\lambda/6} + 0.6e^{-13\lambda/12}\right)d\lambda$$

$$= \frac{0.8(11!)(6/7)^{12} + 0.6(11!)(12/13)^{12}}{0.395536(10!)} = 9.88.$$

Question #15
Key: A

$\hat{H}(4.5) = \frac{2}{12} + \frac{1}{10} + \frac{2}{9} + \frac{2}{7} = 0.77460.$ The variance estimate is $\frac{2}{12^2} + \frac{1}{10^2} + \frac{2}{9^2} + \frac{2}{7^2} = 0.089397.$ The confidence interval is $0.77460 \pm 1.96\sqrt{0.089397} = 0.77460 \pm 0.58603.$ The interval is from 0.1886 to 1.3606.

Question #16
Key: D

$$bias = E(\hat{\theta}) - \theta = \frac{k}{k+1}\theta - \theta = -\frac{\theta}{k+1},$$

$$Var(\hat{\theta}) = Var\left(\frac{k\theta}{k+1}\right) = \frac{k^2\theta^2}{25(k+1)^2},$$

$$MSE = Var(\hat{\theta}) + bias^2 = \frac{k^2\theta^2}{25(k+1)^2} + \frac{\theta^2}{(k+1)^2},$$

$$MSE = 2bias^2 = \frac{2\theta^2}{(k+1)^2}.$$

Setting the last two equal and canceling the common terms gives

$\frac{k^2}{25} + 1 = 2$ for k = 5.

Question #17
Key: D

For the geometric distribution $\mu(\beta)=\beta$ and $v(\beta)=\beta(1+\beta)$. The prior density is Pareto with parameters α and 1. Then,

$$\mu = E(\beta) = \frac{1}{\alpha-1},$$

$$v = EVPV = E[\beta(1+\beta)] = \frac{1}{\alpha-1} + \frac{2}{(\alpha-1)(\alpha-2)} = \frac{\alpha}{(\alpha-1)(\alpha-2)},$$

$$a = VHM = Var(\beta) = \frac{2}{(\alpha-1)(\alpha-2)} - \frac{1}{(\alpha-1)^2} = \frac{\alpha}{(\alpha-1)^2(\alpha-2)},$$

$$k = v/a = \alpha - 1, \quad Z = \frac{1}{1+k} = \frac{1}{\alpha}.$$

The estimate is

$$\frac{1}{\alpha}x + \left(1-\frac{1}{\alpha}\right)\frac{1}{\alpha-1} = \frac{x+1}{\alpha}.$$

Question #18
Key: C

The hazard rate function is $h(x) = h = \frac{1}{\theta}e^{\beta_1 z_1 + \beta_2 z_2}$, a constant. For the four observations, those constants are

$$\frac{1}{18}e^{.1(0)+.01(20)} = 0.06786, \frac{1}{18}e^{.1(0)+.01(30)} = 0.07449,$$
$$\frac{1}{18}e^{.1(1)+.01(30)} = 0.08288, \frac{1}{18}e^{.1(1)+.01(40)} = 0.09160.$$

Each observation is from an exponential distribution with density function he^{-hx} and survival function e^{-hx}. The contributions to the loglikelihood function are $\ln(h)-hx$ and $-hx$, respectively. The answer is,

$$\ln(0.06786) - 0.06786(3) - 0.07499(6) + \ln(0.08288) - 0.08288(7) - 0.09160(8) = -7.147.$$

Question #19
Key: E

A is false. Using sample data gives a better than expected fit and therefore a test statistic that favors the null hypothesis, thus increasing the Type II error probability. The K-S test works only on individual data and so B is false. The A-D test emphasizes the tails, thus C is false. D is false because the critical value depends on the degrees of freedom which in turn depends on the number of cells, not the sample size.

Question #20
Key: B

$$E(\theta) = 0.05(0.8) + 0.3(0.2) = 0.1,$$
$$E(\theta^2) = 0.05^2(0.8) + 0.3^2(0.2) = 0.02,$$
$$\mu(\theta) = 0(2\theta) + 1(\theta) + 2(1-3\theta) = 2 - 5\theta,$$
$$v(\theta) = 0^2(2\theta) + 1^2(\theta) + 2^2(1-3\theta) - (2-5\theta)^2 = 9\theta - 25\theta^2,$$
$$\mu = E(2-5\theta) = 2 - 5(0.1) = 1.5,$$
$$v = EVPV = E(9\theta - 25\theta^2) = 9(0.1) - 25(0.02) = 0.4,$$
$$a = VHM = Var(2-5\theta) = 25Var(\theta) = 25(0.02 - 0.1^2) = 0.25,$$
$$k = v/a = 0.4/0.25 = 1.6,\ Z = \frac{1}{1+1.6} = \frac{5}{13},$$
$$P = \frac{5}{13}2 + \frac{8}{13}1.5 = 1.6923.$$

Question #21
Key: B

$f(\lambda\,|\,5,3) \propto \frac{e^{-\lambda}\lambda}{5!}\frac{e^{-\lambda}\lambda^3}{3!}\frac{2^5\lambda^5 e^{-2\lambda}}{24\lambda} \propto \lambda^{12}e^{-4\lambda}$. This is a gamma distribution with parameters 13 and 0.25. The expected value is 13(0.25) = 3.25.

Alternatively, if the Poisson-gamma relationships are known, begin with the prior parameters $\alpha = 5$ and $\beta = 2$ where $\beta = 1/\theta$ if the parameterization from *Loss Models* is considered. Then the posterior parameters are $\alpha' = 5+5+3 = 13$ where the second 5 and the 3 are the observations and $\beta' = 2+2 = 4$ where the second 2 is the number of observations. The posterior mean is then 13/4 = 3.25.

Question #22
Key: D

Because the kernel extends one unit each direction there is no overlap. The result will be three replications of the kernel. The middle one will be twice has high due to having two observations of 3 while only one observation at 1 and 5. Only graphs A and D fit this description. The kernel function is smooth, which rules out graph A.

Question #23
Key: D

A is false, the collocation polynomial is too wiggly. Both collocation polynomials and cubic splines pass through all points, so B is false. C is false because the curvature adjusted spline sets the values, but not necessarily to zero. The inequality in E should be the other direction. Answer D is true.

Question #24
Key: C

The two moment equations are

$508=\dfrac{\theta}{\alpha-1},\quad 701,401.6=\dfrac{2\theta^2}{(\alpha-1)(\alpha-2)}$. Dividing the square of the first equation into the second equation gives $\dfrac{701,401.6}{508^2}=2.7179366=\dfrac{2(\alpha-1)}{\alpha-2}$. The solution is $\alpha = 4.785761$. From the first equation, $\theta = 1,923.167$. The requested LEV is

$$E(X\wedge 500)=\frac{1,923.167}{3.785761}\left[1-\left(\frac{1,923.167}{1,923.167+500}\right)^{3.785761}\right]=296.21.$$

Question #25
Key: B

$$\hat{v}=\widehat{EVPV}=\frac{50(200-227.27)^2+60(250-227.27)^2+100(160-178.95)^2+90(200-178.95)^2}{1+1}$$

$$=71,985.647,$$

$$\hat{k}=71,985.647/651.03=110.57,$$

$$\hat{Z}=\frac{110}{110+110.57}=0.499.$$

Question #26
Key: B

$$F_{100}(1,000)=0.16,\ F_{100}(3,000)=0.38,\ F_{100}(5,000)=0.63,\ F_{100}(10,000)=0.81,$$

$$F_{100}(2,000)=0.5(0.16)+0.5(0.38)=0.27,$$

$$F_{100}(6,000)=0.8(0.63)+0.2(0.81)=0.666.$$

$$\Pr(2,000<X<6,000)=0.666-0.27=0.396.$$

Question #27
Key: C

$$L=\left[\frac{f(750)}{1-F(200)}\right]^3 f(200)^3 f(300)^4[1-F(10,000)]^6\left[\frac{f(400)}{1-F(300)}\right]^4$$

$$=\left[\frac{\alpha 10,200^{\alpha}}{10,750^{\alpha+1}}\right]^3\left[\frac{\alpha 10,000^{\alpha}}{10,200^{\alpha+1}}\right]^3\left[\frac{\alpha 10,000^{\alpha}}{10,300^{\alpha+1}}\right]^4\left[\frac{10,000^{\alpha}}{20,000^{\alpha}}\right]^6\left[\frac{\alpha 10,300^{\alpha}}{10,400^{\alpha+1}}\right]^4$$

$$=\alpha^{14}10,200^{-3}10,000^{13\alpha}10,300^{-4}10,750^{-3\alpha-3}20,000^{-6\alpha}10,400^{-4\alpha-4}$$

$$\propto \alpha^{14}10,000^{13\alpha}10,750^{-3\alpha}20,000^{-6\alpha}10,400^{-4\alpha},$$

$$\ln L=14\ln\alpha+13\alpha\ln(10,000)-3\alpha\ln(10,750)-6\alpha\ln(20,000)-4\alpha\ln(10,400)$$

$$=14\ln\alpha-4.5327\alpha.$$

The derivative is $14/\alpha-4.5327$ and setting it equal to zero gives $\hat{\alpha}=3.089$.

Question #28
Key: C

$$\hat{v}=EVPV=\bar{x}=\frac{30+30+12+4}{100}=0.76.$$

$$\hat{a}=VHM=\frac{50(0-0.76)^2+30(1-0.76)^2+15(2-0.76)^2+4(3-0.76)^2+1(4-0.76)^2}{99}-0.76$$

$$=0.090909,$$

$$\hat{k}=\frac{0.76}{0.090909}=8.36,\quad \hat{Z}=\frac{1}{1+8.36}=0.10684,$$

$$P=0.10684(1)+0.89316(0.76)=0.78564.$$

The above analysis was based on the distribution of total claims for two years. Thus 0.78564 is the expected number of claims for the next two years. For the next one year the expected number is 0.78564/2 = 0.39282.

Question #29
Key: A

For members of Class A, $h(x)=2x/\theta, S_A(x)=e^{-x^2/\theta}, f_A(x)=(2x/\theta)e^{-x^2/\theta}$ and for members of Class B, $h(x)=2xe^{\beta}/\theta=2x\gamma/\theta, S_B(x)=e^{-x^2\gamma/\theta}, f_B(x)=(2x\gamma/\theta)e^{-x^2\gamma/\theta}$ where $\gamma=e^{\beta}$. The likelihood function is

$$\begin{aligned}L(\gamma,\theta)&=f_A(1)f_A(3)f_B(2)f_B(4)\\&=2\theta^{-1}e^{-1/\theta}6\theta^{-1}e^{-9/\theta}4\gamma\theta^{-1}e^{-4\gamma/\theta}8\gamma\theta^{-1}e^{-16\gamma/\theta}\\&\propto\theta^{-4}\gamma^2e^{-(10+20\gamma)/\theta}.\end{aligned}$$

The logarithm and its derivatives are

$$l(\gamma,\theta)=-4\ln\theta+2\ln\gamma-(10+20\gamma)\theta^{-1}$$

$$\partial l/\partial\gamma=2\gamma^{-1}+20\theta^{-1}$$

$$\partial l/\partial\theta=-4\theta^{-1}+(10+20\gamma)\theta^{-2}.$$

Setting each partial derivative equal to zero and solving the first equation for θ gives $\theta=10\gamma$. Substituting in the second equation yields

$$-\frac{4}{10\gamma}+\frac{10+20\gamma}{100\gamma^2}=0,\quad 400\gamma^2=100\gamma+200\gamma^2,\quad \gamma=0.5.\ \text{ Then, }\ \beta=\ln(\gamma)=\ln(0.5)=-0.69.$$

Question #30
Key: C

Let the two spline functions be

$$f_0(x)=a+b(x+2)+c(x+2)^2+d(x+2)^3,$$

$$f_1(x)=e+fx+gx^2+hx^3.$$

The eight spline conditions produce the following eight equations.

$$f_0(-2)=-32\Rightarrow a=-32,\quad f_1(0)=0\Rightarrow e=0,$$

$$f_0''(-2)=0\Rightarrow c=0,\quad f_0(0)=0\Rightarrow -32+2b+4c+8d=0,$$

$$f_1(2)=32\Rightarrow 2f+4g+8h=32,\quad f_0'(0)=f_1'(0)\Rightarrow b+4c+12d=f,$$

$$f_0''(0)=f_1''(0)\Rightarrow 2c+12d=2g,\quad f_1''(2)=0\Rightarrow 2g+12h=0.$$

Eliminating one variable at a time leads to b = 16, d = 0, f = 16, g = 0, and h = 0. Then, $f_0(x)=-32+16(x+2)=16x$ and the second derivative is zero.

The above work could be avoided by noting that the three knots are (-2, -32), (0,0), and (2, 32). These points lie on a straight line and so the spline will be that straight line.

Question #31
Key: E

The density function is $f(x)=\dfrac{0.2x^{-0.8}}{\theta^{0.2}}e^{-(x/\theta)^{0.2}}$. The likelihood function is

$$
\begin{aligned}
L(\theta) &= f(130)f(240)f(300)f(540)[1-F(1000)]^2 \\
&= \frac{0.2(130)^{-0.8}}{\theta^{0.2}}e^{-(130/\theta)^{0.2}}\frac{0.2(240)^{-0.8}}{\theta^{0.2}}e^{-(240/\theta)^{0.2}}\frac{0.2(300)^{-0.8}}{\theta^{0.2}}e^{-(300/\theta)^{0.2}}\frac{0.2(540)^{-0.8}}{\theta^{0.2}}e^{-(540/\theta)^{0.2}}e^{-(1000/\theta)^{0.2}}e^{-(1000/\theta)^{0.2}} \\
&\propto \theta^{-0.8}e^{-\theta^{-0.2}(130^{0.2}+240^{0.2}+300^{0.2}+540^{0.2}+1000^{0.2}+1000^{0.2})}, \\
l(\theta) &= -0.8\ln(\theta)-\theta^{-0.2}(130^{0.2}+240^{0.2}+300^{0.2}+540^{0.2}+1000^{0.2}+1000^{0.2}) \\
&= -0.8\ln(\theta)-20.2505\theta^{-0.2}, \\
l'(\theta) &= -0.8\theta^{-1}+0.2(20.2505)\theta^{-1.2}=0, \\
\theta^{-0.2} &= 0.197526, \quad \hat{\theta}=3{,}325.67.
\end{aligned}
$$

Question #32
Key: A

Buhlmann estimates are on a straight line, which eliminates E. Bayes estimates are never outside the range of the prior distribution. Because graphs B and D include values outside the range 1 to 4, they cannot be correct. The Buhlmann estimates are the linear least squares approximation to the Bayes estimates. In graph C the Bayes estimates are consistently higher and so the Buhlmann estimates are not the best approximation. This leaves A as the only feasible choice.

Question #33
Key: C

The expected counts are 300(0.035) = 10.5, 300(0.095) = 28.5, 300(0.5) = 150, 300(0.2) = 60, and 300(0.17) = 51 for the five groups. The test statistic is

$$\frac{(5-10.5)^2}{10.5}+\frac{(42-28.5)^2}{28.5}+\frac{(137-150)^2}{150}+\frac{(66-60)^2}{60}+\frac{(50-51)^2}{51}=11.02.$$

There are 5 – 1 = 4 degrees of freedom. From the table, the critical value for a 5% test is 9.488 and for a 2.5% test is 11.143. The hypothesis is rejected at 5%, but not at 2.5%.

Question #34
Key: A

To simulate a lognormal variate, first convert the uniform random number to a standard normal. This can be done be using the table of normal distribution values. For the six values given, the corresponding standard normal values are 0.3, -0.1, 1.6, -1.4, 0.8, and -0.2. Next, multiply each number by the standard deviation of 0.75 and add the mean of 5.6. This produces random observations from the normal 5.6, 0.75^2 distribution. These values are 5.825, 5,525, 6.8, 4.55, 6.2, and 5.45. To create lognormal observations, exponentiate these values. The results are 339, 251, 898, 95, 493, and 233. After imposing the policy limit of 400, the mean is (339 + 251 + 400 + 95 + 400 + 233)/6 = 286.

Question #35
Key: C

For the geometric distribution, $\Pr(X_1 = 2 \mid \beta) = \dfrac{\beta^2}{(1+\beta)^3}$ and the expected value is β.

$$\Pr(\beta = 2 \mid X_1 = 2) = \frac{\Pr(X_1 = 2 \mid \beta = 2)\Pr(\beta = 2)}{\Pr(X_1 = 2 \mid \beta = 2)\Pr(\beta = 2) + \Pr(X_1 = 2 \mid \beta = 5)\Pr(\beta = 5)}$$

$$= \frac{\frac{4}{27}\frac{1}{3}}{\frac{4}{27}\frac{1}{3} + \frac{25}{216}\frac{2}{3}} = 0.39024.$$

The expected value is then 0.39024(2) + 0.60976(5) = 3.83.