Living to Age 100 in Canada in 2000
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Oldest-Old Mortality Rates and the Gompertz Law: A Theoretical and Empirical Study Based on Four Countries
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This is an interesting and important study of the oldest-old mortality in Canada, which includes construction of reliable life tables by the method of extinct generations, testing different statistical models to explain mortality age-trajectories, and analysis of temporal improvement in survival to advanced ages.

In my opinion, this study has the following important strengths:

1. The study is based on survival data generated by the method of extinct generations. This is the best possible method to study survival at extreme ages, because traditional methods based on census data and age claims by nonagenarians and centenarians are extremely unreliable. Now, when a profound change in the oldest-old mortality is observed in many developed countries, it is important to organize a continuous international monitoring of mortality trends at advanced ages (which is also important for regular correction of forecasts), and the method of extinct generations is the best possible way to address the problem of data quality.

2. The study uses several alternative models (mortality laws) that are applied to survival data and compared to each other. This is an important approach that needs to be developed further in the future studies. There are many different mortality models available in the scientific literature (Gavrilov, Gavrilova, 1991; 2001), which deserve to be compared with each other using the same data sets. This is how "the best mortality law" for extreme old ages could be eventually found.

3. The mortality models used in this study are not just arbitrary combinations of symbols and parameters, but rather the "theories" having fundamental justification in statistics and mathematical biology. For example, the Perks model used in the study directly follows from the mathematical theory of aging assuming the avalanche-like mechanism of age-related destruction of an organism (Gavrilov, Gavrilova, 1991). This mechanism of "domino effect" in aging can be illustrated by the following scheme:

**Avalanche-like Mechanism of Organism's Destruction with Age**

![Avalanche-like Mechanism Diagram](image)

In the the initial state (S0) organism has no defects. Then, as a result of random damage, it enters states S1, S2, ..., Sn, where n corresponds to the number of defects. Rate of new defects emergence has avalanche-like growth with the number of already accumulated defects (horizontal arrows). Hazard rate (vertical arrows directed down) also has avalanche-like growth with number of defects.

One suggestion for further developments of this study is to address the problem of secular mortality trends (period effects). This is because the life tables calculated by the method of extinct generations are cohort life tables, where deleterious effects of aging on mortality are confounded by simultaneous mortality improvement over time. It is this type of confounding that may result in the failure of tested models, which was observed in the study. The tested models were developed for situations where the age was the only predictor variable for mortality, while all other covariates were supposed to be fixed. This assumption is violated in the case of cohort life tables when the calendar time is also increasing with the age of surviving cohort. One way to overcome this problem is to use a set of generated cohort life tables in order to calculate the current (cross-sectional) life tables for the oldest-old. Then the models can be tested with these current life tables, where mortality rates are calculated for the same time periods and are, therefore, not confounded by secular mortality trends and fluctuations. Another interesting approach is to analyze the oldest-old mortality surface (at the Lexis diagram) by the 'age-period-cohort' models (APC models), where the changes in mortality rates are partitioned into the effects of age, period (calendar time), and cohort (birth year).

It is interesting to discuss possible limitations of this study, because this may be useful for future work. The method of extinct generations used in this study assumes that there is no migration of the population, and I agree with the author that this assumption seems to be reasonable for nonagenarians and centenarians. In fact some of them are even not able to leave the house where they live. However, it would be interesting to ask a question whether there are any real data proving negligible migration of the oldest-old in Canada. Canada is an interesting country with population mostly concentrated at the US border, and this border is not a Berlin Wall at all. There may be incentives for those who live in Canada and have aged parents in the United States to move aged parents in Canadian nursing homes, which may be cheaper and more convenient in care providing. On the other hand, those who live in the United States and have aged parents in Canada may also be interested to move aged parents closer to them, particularly if they believe that they can afford better care for their parents in the United States. It would be interesting to explore these issues in the future, and perhaps to conduct a survey in order to estimate migration among the Canadian oldest-old.

Another interesting question is how reliable are the data on ages at death, recorded in Canadian death certificates for the oldest-old, which are used by the method of extinct generations. There are significant differences between different countries in the way on how this information (age at death) is reported. For example, in Russia everybody is supposed to have an internal passport (with exact birth date recorded there) and this passport has to be surrendered by the relatives in order to obtain death certificate for deceased person. In other words, age at death in a death certificate comes from the document (internal passport), rather than claims from the relatives or physicians. The situation is different in the United States, where ages at death in death certificates are sometimes taken from recollections of relatives or physician claims, and therefore may be less accurate for the oldest-old. It would be interesting to know what is the situation in Canada with age at death reporting and how accurate is this information in the case of the oldest-old death certificates.

Finally, there is a technical question related to application of the method of extinct generations to the Canadian data, where the annual numbers of deaths above age 100 are aggregated (denoted 100+). Since the annual numbers of deaths above age 100 may be different from the number of deaths (above age 100) in cohort life tables generated by the method of extinct generations, it would be interesting to know, how this problem (caused by data aggregation) has been addressed.

Overall, this is an interesting, important and thought-provoking study, which raises a number of new methodological issues for the future research.

References

This is an interesting study, which suggests and applies new rigorous statistical methods to test the validity of the Gompertz law at extreme old ages. The author also suggested an explanation why the Gompertz law fails at older ages, based on the idea of population heterogeneity.

It is important to discuss the scientific background related to this study because the significance of this work could be more fully appreciated in the context of previous findings.

The current situation with applicability of the Gompertz law to extreme old ages is paradoxical. On the one hand, it is well known for a long time that the Gompertz law is not applicable to mortality rates at advanced ages -- the observed mortality rates are always lower than predicted by the Gompertz model, and, not surprisingly, the actual numbers of survivors to extreme ages is always higher than predicted by the Gompertz law. The picture below illustrates the mortality deceleration observed at advanced ages contrary to the predictions of the Gompertz law:

**Mortality at Advanced Ages**

![Mortality graph](image)


The scientific literature on the deviation of mortality rates from the Gompertz law at advanced ages is abundant, starting with the study of Benjamin Gompertz himself (Gompertz, 1825; Makeham, 1867; Brownlee, 1919; Perks, 1932; Greenwood & Irwin, 1939; Mildvan & Strehler, 1960; Strehler, 1960; Economos, 1979, 1980, 1983, 1985; Gavrilov & Gavrilova, 1991). The most recent and detailed review of the scientific literature on this issue was made by Olshansky (1998).

Paradoxically, the Gompertz law and the Gompertz-Makeham law are nevertheless often applied to estimate the oldest-old mortality rates by extrapolation in order to "close" the life tables. When confronted with the question why these "wrong" formulas are used, the demographers/actuaries usually reply that this is not an important issue, because life expectancy at birth is not very sensitive to the way how exactly the life tables are closed. The same "wrong" formulas and related assumptions are often used for graduation (smoothing procedures) of the mortality trajectories at advanced ages. It is extremely important, therefore, to know exactly how a particular life table was closed and/or graduated, before using it for testing of any statistical models. If the Gompertz or the Gompertz-Makeham laws were already introduced into the data by the method of life table construction/graduation, these data would not be useful for statistical hypothesis testing.
To summarize the coverage of scientific background, we may conclude that the Gompertz law is known to be not applicable to the oldest-old mortality, unless the data are spoiled by artificial introduction of this law during extrapolation/graduation procedures. The discussed study by Jack Yue confirms that the Gompertz law is indeed not applicable to the oldest-old mortality, and this conclusion validates the new statistical methods suggested by the author for testing the Gompertz law.

The next important question is: why is the Gompertz law not applicable to the oldest-old mortality? The author of the discussed paper suggests that this may be the result of population heterogeneity at advanced ages. The same hypothesis was also suggested by other authors (Carnes, Olshansky, 2001). There is, however, one problem with testing this hypothesis. It may be not difficult to generate the mortality trajectories that will be close to the observed trajectories by assuming that population is a mixture of subgroups of people with different Gompertz parameters. The real problem here is whether there is any sense in such computational exercise. Is there any direct evidence indicating increased population heterogeneity at advanced ages?

The answer to this question is provided at the picture below. The graph illustrates the dependence of daughters’ lifespan (expressed as additional years of life gained/lost compared to the reference lifespan level in the same birth cohort) as a function of maternal lifespan. For more on methodological details of this study see (Gavrilova, Gavrilov, 2001). Interestingly the dependence of daughters’ lifespan on maternal lifespan looks like consisting of two lines – one for shorter-lived mothers (died before age 85) with very weak dependence of daughter’s lifespan on maternal lifespan and another – for longer-lived mothers (died after age 85) with extremely strong and steep dependence.
Daughters born to long-lived mothers may live 10 years longer on average if their mother reached age 100. This indicates that long-lived people are fundamentally different from other people in the sense that their children also live significantly longer lives. The breaking point at about age 85 for mothers indicates the age when deaths becomes much more selective in their timing and when population heterogeneity becomes an important issue.

Overall, the discussed paper contains interesting ideas and important findings on the oldest-old mortality, which correspond nicely with results obtained by other authors.

References


