MR. ERIC STALLARD: In terms of the mixed-Weibull model, there are two interpretations that can be given to a mixture of four groups. One is that, if you in fact have four groups, then everybody in the population gets slotted into one of the four groups, so the population is treated as a finite-population mixture of four distinct groups. The estimation problem is to figure out what are the proportions in each group. There may be some variable that you could measure for each individual, such as hair color, gender, or something else that would tell you the group to which each individual belonged. A second interpretation is that the mixture is nothing more than a meaningless convenience and that there really are not four distinct groups.

I’ve done some work with mixed-Weibull models. I used that type of model to represent differences in susceptibility to stomach cancer, assuming that there were two distinct groups. One group was composed of persons who were susceptible to stomach cancer; the other group was composed of persons who were not susceptible to stomach cancer. We had two mutually exclusive groups: each person was in one group or the other. In looking at the application in the paper presented today with the mixed-Weibull model, I can not tell whether you intend the mixtures to be truly four separate population groups or whether the mixture is just a convenience to allow you to represent the different bends that the Heligman-Pollard formula can represent. I did not look at the paper, so I apologize if you commented on that, but it is a fundamental interpretation because, if the groups are truly separate, then you are saying that there is one subpopulation that is subjected only to childhood mortality and to nothing else.
They can never escape that childhood mortality function. That sounds like it would be a strong assumption, if that's what you intend. I think the question is clear.

**MR. MASAKAZU OZEKI:** That's the composition of mixed-Weibull models. I don't intend to distinguish population into four categories. It is more a convenient way to understand or connect the mixed-Weibull model to the Heligman-Pollard model. The model does not distinguish the population of the four categories.

**DR. LEONID GAVRILOV:** When evolutionary theory was mentioned, there were five evolutionary theories. One, which claims that aging is programmed, and now it is considered to be not true. It's historicalism. There is mutation accumulation theory, and the reason it makes some models for mortality. The third model is antagonistic pleiotropy theory, suggested by George Williams. The next evolutionary theory was the extension by Ronald Lee, and recently moved from demographic studies to evolutionary theorist. He included intergenerational transfers, and so for humans he makes an extension of evolutionary theory. When we say "evolutionary theory," we need to be more specific and say which particular theory we mean and why. Maybe this is not important for this audience.

**MODERATOR:** Do we have any more questions or comments from the audience?

**MR. JEAN-CLAUDE MENARD:** Mr. Li, I would like to commend you for your excellent paper. Although I somewhat agree with the results of your work about the predicted distribution of the highest attainable age, I am much more concerned by what you have written on bottom of slide 40, and I don't want to be rude but I take the opportunity to talk to you about this because Jean Marie Robine is in the room too and you said, "This agrees with the maximum age observed in the international data longevity database, which has increased by roughly 20 years during the past 10 years."

This statement has been used a lot of times in the past three or five years, maybe just three years, and I was asked to be a provider of this international database. What I don’t like with this statement is that in my view, it is much more correct to say that the maximum age has increased by roughly 10 years during the 20th century because there was a documented case in 1900 of a Danish person who died at 109. Of course, this person is not in the IDL because this base did not exist at that time, but people could be misled by that statement and say, "We are on the verge of breaking the barriers," so I don’t know if you have a view on this statement or if anyone in the audience has a view on this statement.

**Mr. SIU-HANG LI:** Thanks very much for the question, and I would like to apologize if this statement has caused you any discomfort. In fact, when I attempt to predict the highest
attainable age, I really want to have the real data for the higher or very advanced ages rather than using some extrapolative methods that make the data more artificial rather than real. I attempted to access the international database of longevity, but I failed to get the real data because they provide only the introduction, but I can't access the real data. When I wrote this statement, I think I just quote this statement or I observe this pattern in some previous paper by Mr. Robine, and that's because I don't have the whole picture of the international database of longevity at that moment, so I do apologize for any unhappiness or something else caused by me, and I'm looking forward to the publicizing of the international database of longevity, and thanks again for your question.

MR. JEAN-MARIE ROBINE: I think it looks like typo because effectively it has increased by 10 years in 20 years, and not the opposite. You can find that easily in a special issue of the North American Journal of Actuaries from three years ago.

MS. ANNA RAPPAPORT: I'd like to make a follow-up suggestion in terms of where we might go from here. This question of what people here think about maximum ages and about different things has come up several times. In 1997 or 1998, I believe, the SOA and some other groups did a symposium on Social Security mortality, and we followed it up with a questionnaire, which wasn't scientific, but it was a way to get some information back from the participants who joined. It was quite interesting, so maybe a thing that we'd like to do after this is go back to the participants with 10 or 12 questions in terms of what their views are on some of these questions, which, when we've asked them, a couple of people have spoke, but not a lot. I'd like to suggest that this would be a good follow-up activity, and then it could be published in The Actuary and maybe even extended as a little post log or something to the online monograph.

MR. STALLARD: I have another other observation, and I'm not sure exactly what to say because it's not a specific critique. In terms of the estimates in the Li and Chan paper, the modal estimate of the maximum age increases about three years every decade, which is a fairly sizable increase. Oeppen and Vaupel's analysis (Science, Vol. 296, 10 May 2002, p. 1029–1031) of life expectancy at birth over 160 years has an increase of 2.5 years per decade. To have an increase of the maximum age at death that's larger than the increase in the mean age at death, which is what life expectancy represents, the distribution of age at death has to be spreading out.

Virtually the entire body of literature on mortality describes a compression of mortality or a "rectangularization" of the survival curve wherein the most extreme age could increase, or is allowed to increase, but the most extreme age is generally assumed to increase at a rate that is substantially less than the rate of increase in the mean age at death or, equivalently, in the average life expectancy value. The upper bound of the increase of the mean age at death is likely to be less than the 2.5 year rate of increase reported by Oeppen and Vaupel, because this is the rate of improvement at the world record pace, as estimated by plotting the highest life expectancy in the world at each year for 160 years, yielding a line with a slope of 2.5 years per
decade. With respect to the estimate that the maximum age at death is increasing by three years per decade, I have trouble reconciling that number with the limits imposed by the compression of mortality hypothesis. I’m thinking of 2.5 years as an upper bound of that type of estimate and something maybe on the order of one year per decade or even less as being a number that I’d be more comfortable with.

In essence, my comment is that your number is about three times my comfort level, but I am just one person in a room of experts. I do not know whether others feel the way I do, but I think that 2.5 years is an upper limit and that some number below that would be more plausible. I would like to get some insight as to how to make estimates like that. Any comment? You can defend your estimate or whatever.

MR. LI: I also am not very comfortable with the increase of three years for a 10-year period, but I should clarify that the results of the prediction of the highest attainable age relies on two assumptions. The first assumption is the age pattern of mortality at very advanced ages. In this study we assumed that it follows the relational mortality models suggested by Himes. The other critical factor is as we have looked into the mortality improvement, the results of the prediction of the highest attainable age are heavily dependent on the Lee-Carter forecast. Your question paraphrased is whether we should believe in the Lee-Carter forecast, or should we believe in the extension of the mortality using the duration mortality models suggested by Himes?

We are not prophets. Nobody can justify whether the Lee-Carter forecast is good or not or whether the relation of the mortality model is good or not unless they’re realized. My work here is a very first step on the study of the highest attainable age by relying on these two assumptions. Later, we may try some more studies on the highest attainable age by using other forms of mortality projection models. If we can have the real data of the very high ages from the international database of longevity and can discard the relational model, we may attempt this question again and probably the answer will not be three years for a 10-year period. It may be slightly less than three years for a 10-year period, I don’t know. We relied heavily on these two assumptions. These two assumptions may be correct and may be wrong, but it’s hard to tell whether the increase of three years in a 10-year period can be justified or not. That’s my comment on your question.

MR. JAY SIEGEL: On Wednesday when I spoke when I was pressed to finish my comments, I said, “These are my dying words.” We can now take that more literally because I’m not in good shape as several of you know for the moment, but I’ll risk it and there are enough of you here to pick me up if I fall. I’m inclined to comment on the commenters. What Eric said is right as far as I can tell, and I made those remarks and covered that in my talk. I believe that Oeppen and Vaupel while setting 100 as a terminal figure was just life expectancy, not life span, remember, and that was the upper limit of a series, I believe. It was not simply the only estimate they used in the projections that Vaupel made with Alberg several years ago.
I may be missing something, but some of these models here do not deal with age, such as the Lee-Carter. They’re not models of age variation or change; rather they’re simply projections of a time series, whereas the Heligman-Pollard model is an age model as is the mixed-Weibull model.

Now I’m agreeing with the comments here of Dr. Gavrilov when you talk of the background for the setting age limits. It’s an odd summary to write down evolutionary theory and wear and tear, but wear and tear really is a part of evolutionary theory otherwise described as Dr. Gavrilov said as cumulative mutation and in the other part or the extension of the theory it’s a negative pleiotropy, and now there’s something added to that called positive pleiotropy, and I won’t go into that now. That’s it.

**MR. OZEKI:** It is possible to estimate a highest attainable age and also incorporate the _event of mortality improvement with the Heligman-Pollard model. We may use the Heligman-Pollard model to historical life tables and then do the projections, and we can get another set of results. I haven’t done that. It may be the same, it may be similar, it may be totally different, but yes your theory is totally possible.