Are At-Risk Measures Useful Measures of Risk at the Corporate Level?

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Abstract

Value-at-risk (VaR) is a widely used risk measure among financial institutions. Cash-flow-at-risk (CFaR) is an attempt to transfer the same ideas to the setting of a non-financial firm. In this paper I argue that at-risk measures may fall short of being the best guides for risk management decisions in the non-financial firm since they are not easily interpreted in terms of existing theories about risk management, which emphasize costly financial distress and underinvestment due to financing constraints. Assessing such outcomes requires information on the firm’s presumed access to external sources of funding. What is called for is thus a framework that qualifies cash-flow-based measures of risk on the state of the firm’s debt capacity. The group of risk measures presented in this paper incorporates this information and thus renders hedgeable magnitudes that can be given interpretations consistent with risk management theory. They can inform risk management strategies by indicating if a hedge is likely to act as an effective substitute to equity capital, and whether these benefits are large enough to outweigh the costs of implementing a hedge.
1. **Introduction**

As far as risk measures go, value-at-risk\(^1\) (VaR) can be considered “successful.” After it was first developed at JP Morgan in 1993, it has spread rapidly throughout the financial world. A survey published in 2007 by Deloitte indicates that over 60 percent of global financial institutions use VaR in managing their risk exposures. The concepts behind VaR have also been transferred to the setting of a non-financial firm, usually with cash flow (CFaR) or earnings (EaR) replacing value as the target variable. This was done in response to a “growing interest in the business community about the methodology behind value-at-risk.”\(^2\)

However, VaR has also been criticized. Culp, Merton and Neves (1998) claim that VaR would have failed to detect and thereby avoid any of the corporate scandals in the 1990s involving risk management gone awry. Culp et al. then discuss some inherent limitations of the at-risk framework. They argue that an at-risk measure comes attached with an arbitrary probability, but there is rarely a compelling reason to choose one confidence level over another. According to Culp et al., another weakness of at-risk is that it fails to penalize large shortfalls more than small ones, whereas a realistic risk measure should recognize that larger ones are more undesired and should be penalized more heavily.

In this paper I will submit another argument why an at-risk measure may be inadequate as a tool for guiding corporate level decisions on the firm’s risk profile. Framing a firm’s risk tolerance in terms of an amount that is “at risk” is potentially misguided because this magnitude may correspond poorly with outcomes that are truly costly to the firm’s shareholders. It is unclear whether a corporate manager that hedges to reduce his firm’s CFaR really has shifted the firm’s profile in a way that is of benefit to the company’s owners. Put differently, at-risk measures are not easily reconciled with existing theories for how risk management creates value.\(^3\) For example, how does the CFaR measure relate to the risk of default? Or how does it relate to the risk of under-investing relative to an optimal level?

Knowing more about whether the hedge is likely to mitigate a relevant risk is desirable given that a hedging program entails certain costs. If there were no costs to executing a hedge, we wouldn’t have to worry. But there are transaction costs, including a bid-ask spread, and costs to administer the hedge (Smith, 1996). It may also involve lost upside, and a loss to business reputation if a large-scale hedge loses money (Brown, 2001). Interim cash settlements of certain types of

\(^1\) According to RiskMetrics, a leading authority on VaR, VaR is defined as “the predicted worst-case loss with a specific confidence level (for example, 95%) over a period of time (for example, 1 day).”

\(^2\) Corporate Metrics (1999). Other approaches to CFaR are found in Stein et al. (2001) and Andrén et al. (2005).

\(^3\) These include costly bankruptcy (Smith and Stulz, 1985); progressive tax rate (ibid.); reduced product-market competitiveness (Shapiro and Titman, 1986); underinvestment due to external financing costs (Froot, Stein and Scharfstein, 1993).
financial contracts may result in sub-optimal liquidity management (Mello and Parson, 2000). For the financially constrained firm, using up scarce liquidity to purchase insurance may come at a cost in terms of exacerbated underinvestment (Adam, 2002). Furthermore, a hedge may have undesired effects in the financial statements due to the hedge being marked to market (as is required under IFRS rules). Consequently, to help in assessing whether as hedge is worthwhile despite such costs, a useful risk measure at the corporate level should convey information on outcomes that can plausibly be linked to costly consequences. If one accepts this as a fundamental property of a meaningful risk measure, then an at-risk measure is unlikely to go all the way.

Are there any alternatives to the at-risk framework? Culp et al. (1998) discuss measures of risk associated with a specified target level below which the variable in question must never fall—the below-target probability and the below-target risk. The target level in the case of a non-financial firm could be the funds needed to implement the firm’s strategic plan, or the funds needed to avoid bankruptcy. Such shortfall risk measures—which are special cases of Fishburn’s (1977) lower partial moments (LPM) framework—convey information about risk that is potentially more meaningful to a wide array of decision-makers. The basic idea behind the LPM is that a penalty is applied to shortfalls to reflect the risk tolerance of the decision-makers and the perceived costs of the shortfall. By adjusting these parameters, a great deal of flexibility, which is lacking in the case of an at-risk measure, is achieved in terms of describing the risk preferences of the decision-maker.

The challenge to measure the risk of a company, however, is made more difficult by the fact that costly “states of nature” are typically the function of both weak liquidity and a weak balance sheet. The normative risk management literature often emphasizes the endogenous relationship between risk management, understood as cash flow hedging, and the capital structure. Stulz (1996), for example, views risk management as a direct substitute to equity capital. Even more to the point, Mello and Parsons (1999) argue that information on the firm’s debt capacity is a key input to corporate level risk management. They state: “Determining the external financing capacity is critical. Many firms have significant reserves in the form of unused debt capacity, and a shortfall in the internal supply of funds will not disrupt the investment strategy.”

What the statement by Mello and Parsons suggests is that cash-flow-based risk measures need to be qualified with a reference to the firm’s debt capacity. For the managers of a corporation, with its largely fixed portfolio of fairly illiquid assets, the state of the balance sheet is an important piece of information as it will indicate the firm’s ability to deal with short- to medium-term fluctuations in cash flow. Taking the corporate perspective, this paper suggests a conditional lower
partial moments (CLPM) framework for measurement of corporate level risk. The CLPM \((a_i, t_i, t_2)\) framework is a generalization of Fishburn’s LPM \((a, t)\) framework (1977) to include the debt capacity-parameter \(t_2\). The debt capacity-parameter concerns the identification and description of a liability overhang in the firm’s balance sheet that could lead the firm to conduct its business suboptimally in some states of the world.\(^4\) The resulting class of risk measures can be given interpretations that are consistent with existing theories about risk management. One obtains hedgeable magnitudes that indicate if and by how much a hedge mitigates costly states of nature by acting as a substitute to equity.

An illustrative example is provided. The aluminium company Norsk Hydro ASA wanted to evaluate if a hedge should accompany a substantial increase in the investment budget. Using simulation methodology, the company could see that exposure to substantial funding needs would indeed increase under the evaluated strategy, but that its strong balance sheet would absorb the large majority of these potential funding needs. An analysis of risk measures that incorporates information on its debt capacity revealed that there were virtually no scenarios under which a hedge would act as a substitute to equity capital. The company concluded that a hedge would not be worthwhile even if the evaluated strategy were to be implemented.

The article is outlined as follows. The next section discusses the merits and demerits of the at-risk framework. The section after that looks at the LPM framework for creating risk measures. In the section thereafter I discuss how to obtain more relevant risk measures by conditioning cash-flow-based risk measures on the firm’s debt capacity. The section after that discusses various ways to approach the debt-capacity parameter, followed by an illustrative application. A final section concludes.

2. The At-Risk Framework: Contributions and Limitations

Value-at-risk was developed by JP Morgan in 1993 in response to a desire to know more about the aggregated, firm-level risk that resulted from the firm’s trading positions. Up to that point, risk analysis was largely about establishing sensitivities on each trading position separately. VaR, on the other hand, enabled senior managers to know how risky JP Morgan’s total portfolio of trading positions was by aggregating all trading desks, taking into account the correlation between different asset classes.

\(^4\) This constraint can be framed, for example, in terms of a critical level of a firm’s debt-to-assets ratio; a covenant restricting financial policy; an existing credit facility; a target level of a financial ratio deemed crucial by credit rating agencies (see Section 5 for a discussion).
VaR’s rise to stardom can be attributed to its ability to aggregate a large number of exposures into a single summary of overall risk, which greatly facilitates risk monitoring and communication. VaR makes the risk of different types of financial instruments directly comparable. Furthermore, by viewing only shortfalls relative target (“downside”) as risk, VaR achieves great intuitive appeal. This is not the case with the standard deviation, the dominant proxy for risk in financial theory. In fact, the standard deviation penalizes positive deviations as much as negative ones, whereas people in general do not tend to think of performing better than a benchmark as “risk” (see, for example, Kahneman and Tversky, 1979).

Due to a growing interest among non-financial firms about the at-risk methodology, the same principles were eventually applied in other contexts. Cash-flow-at-risk (CFaR) is thus the cash flow equivalent of the VaR measure. As such, it shares many of its advantages. Compared with the traditional “silo” approach to risk management, it sums up the firm’s various risk exposures into a single measure of risk. Approaches to CFaR modelling can be found in Risk Metrics (1999), Stein et al. (2001) and Andrén et al. (2005). Such modelling efforts can be helpful in managing the firm’s operating cash flow and provide a sense of the firm’s overall liquidity risk over a certain time period.

While these are important improvements, at-risk measures may have limits in terms of their scope of applications. Culp et al. (1998) discuss at length some of the pitfalls in using VaR. Some of these pitfalls may be common to any risk measurement framework that tries to assign probabilities where the true distributions can’t be known with any real certainty. But even at a conceptual level, the at-risk framework seems to have certain limitations. One such limitation is that an at-risk measure cannot really distinguish successfully between shortfalls that come at a significant cost to the firm from those that have little or no consequence. In consequence, it systematically fails to incorporate the cost of risk. Also, the at-risk measure comes attached with a certain probability, but there is little guidance as to why any confidence level should be chosen over another.

3. Moving Beyond Cash Flow-at-Risk

As an alternative to an at-risk type of measure, Culp et al. suggest measuring risk relative to some critical target level. This “doomsday level” is a level of performance that would have truly dramatic consequences to the organization if not met. For example, it could be the level at which bankruptcy occurs, or at which some covenant is breached. Such “below target” measures have the advantage over an at-risk measure in that the reference point for measuring risk springs from the organization’s own context. Management can ask itself “at
what level of performance do we really feel the pain?” and use this level as reference point when constructing risk measures.

The measures proposed by Culp are actually part of a more general framework concerned with downside risk. The general framework is the LPM \((a, t)\) model (Fishburn, 1977), in which risk is measured relative to a benchmark level.\(^5\) A penalty is applied to outcomes that are worse than the benchmark, but not those in which the target level is met or outperformed. Risk tolerance is described by selecting a value for \(a\), the penalty applied to shortfalls below target, and \(t\), the threshold level of performance. The general principle is that the higher the value of \(a\), the higher the penalty placed on a given shortfall (and by extension the higher the risk aversion of the decision-maker).

In the LPM risk is defined by the following function.

\[
F_a(t) = \int_{-\infty}^{t} (t-x)^a dF(x) \quad a > 0
\]

While a non-financial firm typically is concerned with many steering parameters, this paper will emphasize liquidity. Assuming that the variable of interest \(x\) is the firm’s cash holdings at some future point in time we obtain the following group of risk measures.\(^6\)

\[
\text{LPM}_0 = P(x < t) \quad (B)
\]
\[
\text{LPM}_1 = E\{(x), \text{for all } x < t\} \quad (C)
\]
\[
\text{LPM}_2 = E\{(x)^2, \text{for all } x < t\} \quad (D)
\]

\(\text{LPM}_0\) is the probability that cash falls below a certain target level.\(^7\) It can be interpreted as the probability of a funding need. \(\text{LPM}_1\) weighs each of these potential funding needs with its respective probability of occurrence. Compared to \(\text{LPM}_0\), this measure takes into account the magnitude of shortfalls below the target. It can therefore be interpreted as the expected, or probability-weighted, funding need (not to be confused with the most likely funding need). \(\text{LPM}_2\) applies a heavier

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\(^5\) The LPM framework has been applied to the portfolio optimization problem by many authors; see for example Harlow (1991).

\(^6\) Note that these three measures with risk aversion coefficients of 0, 1 and 2 are not the only ones possible. In fact, the coefficient can take any value depending on risk preferences.

\(^7\) This could be zero, but also some positive number that reflects the fact that a firm may want a minimum buffer of cash at all times for working capital needs.
penalty to potential funding needs, which reflects that these funding needs may be more problematic and expensive the larger they get.\textsuperscript{8}

But while possibly informative in their own right, these risk measures suffer from not having a clear-cut interpretation. Do such funding needs pose a problem or not? Do they mean that the firm’s strategic plan could be derailed, or whether the firm risks having insufficient funds to meet debt obligations? It is hard to tell without additional information. Mello and Parsons (1999) suggest that we can obtain more informative risk measures by qualifying the firm’s cash flow risk using information on its spare debt capacity. For example, a firm whose conservative financial policy has landed its debt-to-equity ratio at 0.3 may find the prospect of a funding need to be less than frightening. An otherwise similar firm whose aggressive use of debt has put its debt-to-equity ratio at 2 may not be as keen to exposure to refinancing risk.

\textbf{4. Conditioning Liquidity Risk on the Firm’s Debt Capacity}

This paper has argued that a truly useful summary risk statistic at the corporate level should make reference to any existing debt capacity. This can be accomplished if risk measures are derived from the interaction between the firm’s cash balance and its debt capacity. More precisely, we make liquidity risk conditional on the firm’s debt capacity. The general conditional lower partial moments (CLPM) framework is written CLPM \((a, t_1, t_2)\). In the CLPM risk measures are defined by:

\[ F_a(t_2) = \int_{-\infty}^{t_2} (t_1 - x)^a dF(x) \quad (t_1 - x) = 0 \text{ for all } (y \leq t_2) \quad a > 0 \]  \hspace{1cm} \text{(E)}

Again, the unit of analysis \(x\) is the firm’s cash holdings. Again, \(t_1\) refers to the critical, or threshold, level for the firm’s cash balance. \(t_2\), in turn, refers to the critical, or threshold level for the proxy for the firm’s debt capacity, which is denoted \(y\) (more on this later). \(a\), finally, is the risk coefficient as applied to the shortfalls below the target level for cash that are \textit{not covered within the firm’s remaining debt capacity}. The difference between the LPM and CLPM resides in the debt capacity parameter \(t_2\).

In the CLPM a risk event can thus be thought of as an indicator variable that takes the value one if both the cash balance and the proxy for debt capacity breach

\textsuperscript{8} This assumption is common in the risk management literature. See, for example, Froot, Scharfstein and Stein (1993).
their respective target levels simultaneously. We obtain the following set of risk measures.

\[
\text{CLPM}_0 = P(x < t_1 \mid y > t_2)
\]  

(F)

\[
\text{CLPM}_1 = E\{(x) \text{ for all } x < t_1 \mid y > t_2\}
\]  

(G)

\[
\text{CLPM}_2 = E\{(x)^2 \text{ for all } x < t_1 \mid y > t_2\}
\]  

(H)

How can we interpret these measures? CLPM\(_0\) is the probability that a funding need occurs at the same time that a firm’s debt capacity is presumed to be exhausted. It can be interpreted as the probability of the firm having to cut back its investment spending, or some other planned cash outlay, such as the dividend. Depending on the firm’s initial financial status, it might also be interpreted as the probability of default. CLPM\(_1\) can be interpreted as the expected, or probability-weighted, underinvestment. For example, if LPM\(_1\) is estimated to 10bn and CLPM\(_1\) to 9bn, then the firm “expects” to underinvest by 1bn due to weaknesses in the balance sheet. Put another way, there are plausible outcomes of the firm’s financial situation such that it could end up having to scale back on its planned capital expenditure.

The difference between LPM\(_1\) and CLPM\(_1\) is a hedgeable magnitude with a meaningful interpretation; it indicates the size of the underinvestment problem caused by a weak balance sheet. If positive, it indicates that a hedge stands a chance of acting as a substitute for equity capital. A hedge can then be evaluated according to its ability to reduce this magnitude.

The CLPM\(_2\) doesn’t have an equally straightforward interpretation. However, it appears to make sense to apply some penalty larger than one to uncovered shortfalls, because they intuitively should get more costly the larger they get. To appreciate this point, consider the following sequence of events. A firm’s first response to insufficient liquidity might be to cut down the investment spending with the lowest expected return. It may then choose to cancel its dividend payment, sending clear distress signals to the investor community. For even larger uncovered shortfalls, the firm may have to scale back its strategic investments where the expected profitability is the highest. If a firm’s interest payments are threatened, a variety of financial-distress-related costs might set in, as stakeholders demand a higher risk premium for doing business with the firm. That is, the first set of cash commitments that the firm would scale back on can be assumed to be less important than the next set. Table 1 summarizes the main approaches to corporate risk measurement.
Table 1
Main Approaches to Corporate Risk Measurement

<table>
<thead>
<tr>
<th>Risk framework</th>
<th>Concept</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>Measures the degree of dispersion around the mean</td>
<td>• Symmetric perception of risk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Relies on normal distribution</td>
</tr>
<tr>
<td>Cash Flow at Risk</td>
<td>Measures the maximum loss associated with a certain statistical confidence level</td>
<td>• Asymmetric, i.e., treats losses different than gains</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Based on operating cash flow</td>
</tr>
<tr>
<td>Lower Partial Moments</td>
<td>Measures risk as the deviations below a target level penalized by a risk aversion coefficient a</td>
<td>• Adopts easily to varying levels of risk aversion</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Makes no reference to debt capacity</td>
</tr>
<tr>
<td>Conditional Lower Partial Moments</td>
<td>Makes reference to a second probability distribution to separate risky from non-risky shortfalls</td>
<td>• Incorporates information on debt capacity</td>
</tr>
</tbody>
</table>

5. Know thy Constraints

The debt capacity parameter \( t_2 \) concerns the identification of a liability overhang in the firm’s balance sheet that could lead it to conduct its business suboptimally in some states of the world. It represents the point at which the firm’s balance sheet becomes so weak that external financing can be considered unattainable or prohibitively expensive. How could this variable be obtained? The literature on capital structure has long recognized that contracting problems in the financial markets increase in the level of debt (see, for example, Myers, 1977). The firm’s debt-to-equity-ratio would therefore appear like a natural candidate to proxy for the firm’s capital market access (or debt-to-total assets, depending on which is deemed most closely correlated with the firm’s capital markets access). As a firm’s leverage increases, so does the potential for information and agency
problems with respect to the investor community. For risk calculations, one would therefore have to identify a critical level for the debt-to-equity ratio at which the firm’s balance sheet constraints become operative. Mello and Parsons (1999) suggest that the industry average could be a useful indicator.

A leverage ratio, however, does not inform us whether a particular debt covenant is actually likely to restrain the firm’s activities (Matsunaga, Shevlin and Shores, 1992). A debt covenant is a provision included in debt contracts that restricts the firm’s activities after the bond is sold, the purpose of which is to mitigate the bondholder-stockholder conflict (Smith and Warner, 1979). A covenant targeting subsequent financial policy may stipulate that the firm is restricted from issuing additional debt, for example by subjecting it to aggregate dollar limitations (ibid.). Covenants in existing debt contracts may constitute a very real constraint on a firm’s borrowing, so it might sometimes be possible to rely on clearly formulated debt covenants that make subjectivity less necessary when estimating debt capacity.

Another ratio-based proxy for the firm’s balance sheet constraints might come in the form of a particular ratio on which the firm’s credit rating is perceived to hinge. This presumes that it is highly likely that the firm would be downgraded if it fails to meet the rating agencies’ target level, and that this event would materially impair its capital market access. A somewhat more tangible constraint may be derived from the size of a firm’s existing credit facility. It may be reasonable to assume that a firm can expect to draw credits under existing arrangements but fail to attract new funding beyond that.

6. An Illustrative Example

Norsk Hydro ASA is an integrated aluminium company headquartered in Oslo, Norway. At one point its managers posed the question: “Is a hedge desirable to accompany a strategy involving a substantially increased investment budget”? To help answer such questions, the company operates a risk model that simulates the product prices, exchange rates and interest rates the company is exposed to. The model simulates a complete set of interrelated financial statements five years into the future. Based on these statements, the company is able to evaluate its liquidity and financial strength under a large number of scenarios. The firm defines and communicates its financial capacity in terms of two key financial ratios: the net interest-bearing debt-to-equity and the funds-from-operations to net interest-bearing debt. The maximum amount of borrowing, in each simulated scenario, is imputed by the position of these ratios relative the company’s targets.
To enlighten the issue at hand, CFaR was found insufficient. This measure could not inform sufficiently about the interplay between the firm’s operating cash flow, its planned investment strategy and the strength of the firm’s balance sheet. Running 10,000 simulations assuming the new investment strategy, both LPM₀ and LPM₁ rose substantially. From this it was clear that exposure to significant borrowing needs would increase compared to the base case if the new plan were implemented. However, CLPM₀ and CLPM₁ remained close to zero. This was an indication that the balance sheet would be able to absorb practically all funding needs that might arise. Since the difference between the LPM and CLPM measures was extremely small, there was no sign that a hedge would work as an effective substitute to equity capital. The main reason was that the company, following a de-merger of one of its major operating units, had lower-than-usual debt. Spare debt capacity was correspondingly high. The company concluded that, at the time being, a hedge was not worthwhile even if the firm were to implement a high growth strategy.

7. Summary

This paper has outlined a risk measurement framework in which risk measures are derived by qualifying cash-flow-based risk measures on the state of the firm’s debt capacity. This is based on the intuitive idea that a liquidity shortfall entails some truly negative consequence only to the extent the firm also has a weak balance sheet. In so doing, one obtains measures of risk that can be given interpretations consistent with existing theories of how risk management can create value (i.e., costly financial distress, underinvestment). The risk measures in the framework presented will indicate if a hedge is likely to mitigate costly consequences of volatility and thus be worthwhile. Given costs to implementing a risk management program and executing a strategic hedge, this is a desirable property of a risk measure at the corporate level. Furthermore, the risk measures have the attractive property that they easily allow for varying degrees of risk tolerance.

This risk framework overcomes some of the weaknesses of the popular at-risk framework. At-risk, being the maximum loss associated with an arbitrary probability, lacks interpretations that are firmly grounded in theories of corporate risk management. Also, by its very construction, it does not systematically incorporate the cost of risk.

The approach is illustrated with the example of the Norwegian aluminium company Norsk Hydro. Its risk simulations indicated that exposure to liquidity risk would increase following an evaluated expansive investment plan. However, measures of risk incorporating information on debt capacity showed that these
potential funding needs could well be handled within pre-defined limits. They indicated that there were very few scenarios in which a strategic hedge would act as an effective substitute to equity capital. The company concluded that the benefits did not appear to outweigh the costs of implementing such a hedge.
References


