V

Methods for the Analysis of CCRC Data

by Bruce L. Jones

Abstract

This paper presents an approach to analyzing continuing care retirement community (CCRC) data and demonstrates the methodology by using data from a CCRC. It is assumed that residents make "transitions" among a number of "states" that represent the levels of care required by residents. There is randomness associated with both the transition times and the states entered at these times. The model is conveniently characterized in terms of "transition intensity functions," which represent the instantaneous rates of transition between pairs of states. Statistical methods for estimating these functions are discussed, and estimates are obtained from the dataset. A simulation approach for determining probabilities and other interesting quantities based on the estimated intensity functions is also described and illustrated.

1. Introduction

1.1 Background on CCRCs

Continuing care retirement communities (CCRCs) offer housing and a wide range of services to elderly individuals. These services typically include daily meals, housekeeping, flat linen, maintenance of apartment and grounds, emergency nursing, security, scheduled transportation, and activities. In addition, CCRCs usually provide two or three levels of long-term care.

CCRCs generally charge a rather substantial entry fee as well as periodic fees paid throughout an individual's duration of residence. Additional fees may also be charged for some services. Many CCRC contracts provide for the refund of some portion of the entry fee in the event of death or withdrawal. A key feature of most CCRC contracts is that some or all of the cost of long-term care is covered by the entry and periodic fees. Such contracts therefore provide a long-term-care insurance benefit.

Discussions of the characteristics of CCRCs and CCRC contracts, as well as actuarial issues relating to CCRCs, are given by the Actuarial Standards Board (1994), Brace (1994), Moorhead and Fischer (1995), and Winklevoss and Powell (1984).

1.2 Actuarial Models for CCRCs

CCRCs offer a unique challenge for actuaries. Most communities provide two or more levels of care, and residents may transfer temporarily or permanently to the care units. Actuarial models must therefore permit "transitions" among a large number of "states," usually six or more. To calculate actuarial present values, the actuary should be able to estimate the probability that a resident is in any given state at any future time as well as the probability that a resident will move between any two states during any time interval. To perform cash-flow and population projections, the actuary should be able to estimate the expected number of residents in each state at any future time and the expected number of transitions between any two states during any future time interval. It is also important to quantify the variation about these expected values.

Cumming and Bluhm (1992) describe a CCRC population and financial model that uses a multiple decrement approach. Expected results can be calculated directly and random variation estimated by simulation. Jones (1995, 1996b, 1997) explores continuous-time multistate stochastic models for analyzing CCRCs with emphasis on parsimonious models for which direct calculation is possible for many important probabilities, expected population values, and actuarial present values.

1.3 CCRC Data

The data source upon which model parameters are to be based should be chosen carefully. The characteristics of CCRC residents can differ greatly among CCRCs. Some communities are like expensive resorts, affordable only to the wealthy; others are much more modest with fees that reflect this. We expect to see differences in health care utilization between residents from different socioeconomic classes. In addition, differing management philosophies on resident transfers can affect CCRC experience.

Ideally, parameter values used in modeling a given community should be based on that community's experience. Unfortunately, many CCRCs have not maintained appropriate records for this purpose or are too small to have accumulated substantial recent experience. It is important, though, that as much information as possible be extracted from the available data. Modern statistical techniques can help in doing this.

At present, little CCRC industry data are publicly available. However, an ongoing study conducted by Actuarial Forecasting and Research, funded by the National Institute on Aging, and endorsed by the Society of Actuaries and the American Association of Homes and Services for the Aging will provide a good source of CCRC industry mortality and morbidity data.

Estimated rates of transition between model states should appropriately reflect the effect of various factors, including aspects of a resident's health history since entering the CCRC as well as other information such as gender, marital status, fees paid, contract type, and so on. Data that provide this information are therefore required.

1.4 Outline of Paper

The purpose of this paper is to present an approach to analyzing CCRC data and to demonstrate the methods by using data collected from a CCRC. Section 2 provides a description and some preliminary observations of the dataset used. This illustrates the nature of CCRC data and gives an appreciation of the challenge presented by such data. Statistical methods for analyzing CCRC data are described in Section 3. The state occupied by a resident is modeled as a continuous-time stochastic process characterized by transition intensity functions. I discuss a nonparametric approach to estimating these functions and the Cox regression model for quantifying the effect of important variables on these functions. The methods were used to obtain estimates based on the data introduced in Section 2, and the results are summarized in Section 4. Because these estimates are based on limited data from one CCRC, they are illustrative only and should not be used in actuarial analyses. In Section 5, I explain how probabilities and other quantities of interest can be obtained by simulation, and I use the estimated transition intensity functions to illustrate the approach. Finally, some conclusions are discussed in Section 6.

2. The Pilot Study Data

2.1 Background

In 1991, the Society of Actuaries and the American Association of Homes and Services for the Aging cosponsored a pilot study that involved the collection of data from a CCRC in Florida. The purpose of the pilot study was to gain insight into collecting CCRC data with a view to future large-scale data collection and analysis projects. Results of the pilot study were presented in a report prepared by H. Barney (SOA and AAHA 1991).

The CCRC under study provides three types of independent housing (single-family, garden apartment, and high-rise units) and two levels of health care (assisted living and skilled-care beds). Access to health care is guaranteed with an increased charge to the resident.

The data comprise information on all individuals who resided in the CCRC during the three-year period from April 1, 1988 to March 31, 1991, the "study period." Information was also coded for those who resided in the facility before this period with a spouse who remained in the CCRC through some or all of the study period. A total of 803 residents were included in the study. They spent a total of 1,605 life-years in the community during the study period. Some residents entered the CCRC during the study period, and some left during the study period.

Information recorded for each resident includes an identification number, name, birth date, sex, couple status, apartment type at entry, apartment type at beginning of study period (or entry for those who entered during the study period), entry fee, service fee, health status (at later of entry or beginning of study period) as indicated by the level of care provided and whether or not recovery was expected, roommate identification number (if any), entry date, contract type, and refund provision. In addition, for each change of health status that occurred during the study period, the new status, date of change, and cause of change (if known) were recorded.

2.2 Preliminary Examination of Data

Of the 803 residents in the study, most were typical CCRC residents receiving residential services and possibly meals. Others, such as those admitted directly into assisted living or those residing in assisted-living units or skilled-care beds on a per-diem basis, were removed from the dataset. This left 722 residents who spent 1,518 years in the CCRC during the study period.

Because the number of individuals in the study was fairly small, the three types of independent-living units were combined for estimating transition intensity functions. This reduced the number of functions to be estimated. In large-scale studies conducted in the future, it will be appropriate to test whether the type of independent-living unit affects the intensity function.

While in the community, individuals transfer among the following "states":

1. *Independent.* Residents in this state are capable of living alone or with a roommate without 24-hour supervision.
2. *Assisted Living.* Residents in this state require some ongoing, long-term supportive services in order to function. While some medical or nursing services may be provided, the emphasis is on personal care services (for example, help in walking, bathing, dressing, eating, and the like).
3. *Skilled Care (Temporary).* Residents in this state require continuous or ongoing nursing or medical care services provided by a licensed practical nurse, a registered nurse, or a physician. These residents are expected to recover and return to either the independent or the assisted living state.
4. *Skilled Care (Permanent).* This state is the same as state 3 except that residents in this state are not expected to recover.*

Departure from the CCRC during the study period occurs by either withdrawal or death. Figure 1 illustrates the setup. The boxes represent states that may be occupied by an individual, and the arrows indicate the possible transitions. The total number of years spent in each state during the study period is shown in the appropriate box. Near the head of each arrow is the number of transitions of the indicated type during the study period. Figure 1 shows that certain transitions occur with much greater frequency than others. For example, there were 371 transitions from state 1 to state 3, but only 2 transitions from state 1 to state 4. Thus, we should be able to say much more about the $1 \rightarrow 3$ transition intensity.

Certain transitions should, in theory, not occur. There should be no recoveries from the assisted-living state to the independent state. Although the word *permanent* has been omitted in describing the assisted-living state, all visits to this state were coded as permanent. Also, there should be no recoveries from the skilled care (permanent) state to the independent, assisted-living, or skilled care (temporary) states. Therefore, the numbers of $2 \rightarrow 1$, $4 \rightarrow 1$, $4 \rightarrow 2$, and $4 \rightarrow 3$ transitions should be zero. In practice, assessments of future health status cannot be performed with 100% accuracy. Figure 1 illustrates that some of these transitions did occur.

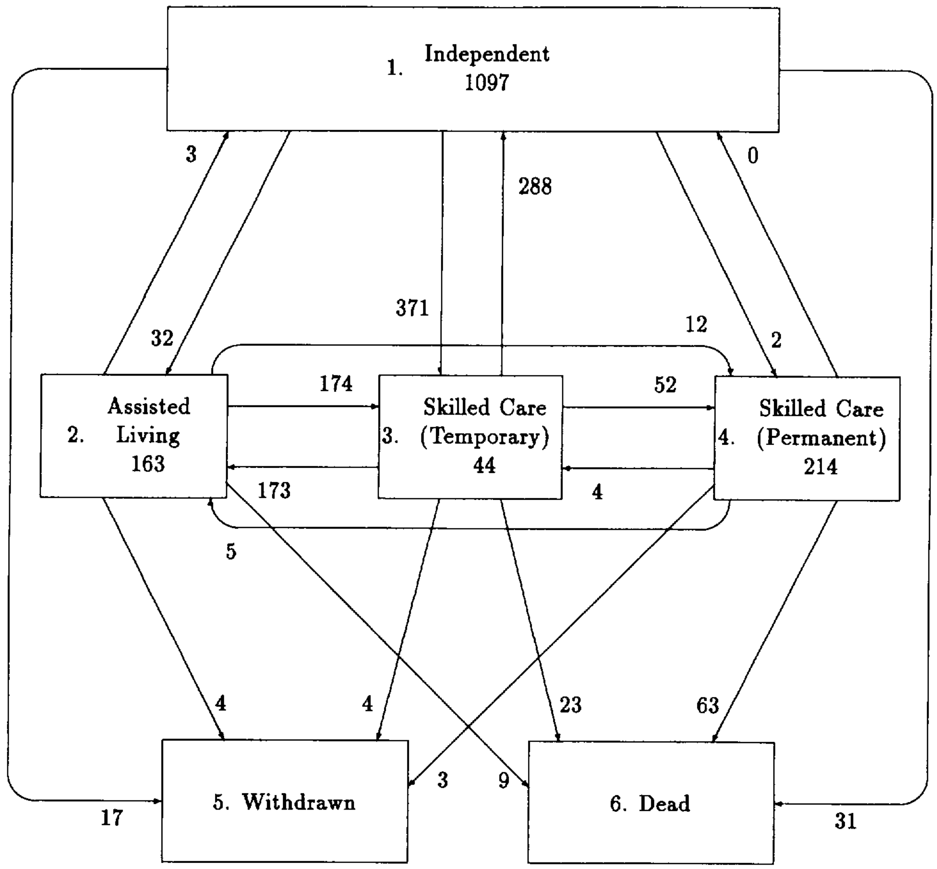
Females in the dataset outnumbered males. Of the 722 individuals included in the analysis, only 198 were males. Table 1 shows the total time spent in the CCRC during the study period by state and sex. Table 2 provides a breakdown of the number of transitions by transition type and sex.

One important variable in determining transition intensities is the age of the resident. Therefore, it is helpful to understand how the CCRC population is distributed by age. To this end, I prepared graphs showing the number of residents attaining each age during the study period (see Figure 2). Separate graphs are displayed for all residents and residents in each state. These graphs show the exposure by age in total and in each state. For each graph, the area under the curve between any two ages is the exposure for that age interval.

Another potentially relevant variable is duration since entry to the CCRC, due to the selection that occurs at the time of entry. Many CCRCs require residents to demonstrate that they are in good health before they are admitted. Figure 3 shows the number of residents attaining each duration during the study period. The figure illustrates how the population was distributed

*This is the traditional distinction between temporary and permanent. However, today it is common for residents to be classified as permanent only when the unit at the lower level of care is made available for another resident. If the individual was residing with a spouse, then this may not occur until the spouse vacates the unit. One must therefore recognize that the labeling of transfers to skilled care may differ between CCRCs.

FIGURE 1
STATE TRANSITION DIAGRAM FOR CCRC RESIDENTS



by duration (that is, the exposure by duration). The distribution is heavily skewed to the right. Roughly half of the total time that residents spent in the community during the study period was spent during the first five years since entry. However, some residents had been in the community for as long as 25 years.

3. Statistical Methods for Analyzing CCRC Data

As stated in Section 1, CCRCs present a challenge for actuaries because of the complexity of the possible outcomes for a given resident. A CCRC resident may transfer many times before leaving the community by death or withdrawal. Thus, it is easiest to think of the outcome as a realization of a stochastic process. I then attempt to find a model that reasonably describes the behavior of this process.

Suppose we have n residents in the study. For $j=1, 2, ..., n$, let $X_j(t)$ represent the state occupied by resident j at time t. Then $\{X_j(t), t \geq 0\}$ is a continuous-time stochastic process (see Ross 1983, p. 26) with state space $\{1, 2, ..., 6\}$. Often t will represent age. However, it will sometimes be convenient to let t measure the time since some event such as entry to the CCRC or entry to a given state. It is assumed that these processes are independent across residents. However, when resources are limited, decisions about resident transfers may well be influenced by the states occupied by other residents. If this is the case, the effect of this independence assumption should be examined.

We can characterize the above processes in terms of transition intensity functions. These functions are also

TABLE 1
TIME SPENT IN EACH STATE BY SEX

State	Females	Males	Total
Independent	807.7	288.8	1,096.5
Assisted Living	141.1	22.1	163.2
Skilled Care (Temporary)	37.4	6.9	44.3
Skilled Care (Permanent)	185.3	28.4	213.7
Total	1,171.4	346.2	1,517.6

TABLE 2
NUMBER OF TRANSITIONS BY TYPE AND SEX

Type	Females	Males	Total
1→2	29	3	32
1→3	308	63	371
1→4	2	0	2
1→5	11	6	17
1→6	21	10	31
2→1	3	0	3
2→3	156	18	174
2→4	10	2	12
2→5	3	1	4
2→6	8	1	9
3→1	244	44	288
3→2	155	18	173
3→4	42	10	52
3→5	4	0	4
3→6	16	7	23
4→1	0	0	0
4→2	3	2	5
4→3	4	0	4
4→5	2	1	3
4→6	56	7	63

referred to as forces of transition because they are analogous to the force of mortality. Let

$$\alpha_{hij}[t;\, \mathbf{Z}_j(t)] = \frac{\lim\limits_{u \to 0+} \Pr\,[X(t+u) = i \mid X(t) = h,\, \mathbf{Z}_j(t)]}{u}, \quad (1)$$

$$h,\ i = 1, 2, ..., 6,\ h \neq i,\ j = 1, 2, ..., n$$

be the transition intensity function for transitions from state h to state i by individual j. The term $\mathbf{Z}_j(t)$ is a vector of covariates containing relevant information about resident j that is available just prior to time t. Examples of possible components of $\mathbf{Z}_j(t)$ are the time since resident j entered state h and an indicator of the sex of resident j. The former depends on t and is referred to as a time-dependent covariate. I assume that the limits in (1) exist for all $t \geq 0$ and therefore that the probability of a transition at any fixed time t is zero. Initially, I consider the special case in which the transition intensity functions do not involve any covariates. I further assume that these functions are the same for all residents; that is,

$$\alpha_{hij}[t;\, \mathbf{Z}_j(t)] = \alpha_{hi}(t).$$

My objective is to estimate the transition intensity functions. I attack this problem by first finding estimators for the corresponding cumulative intensity functions,

$$A_{hi}(t) = \int_0^t \alpha_{hi}(s)\,ds.$$

Let $Y_{hj}(t) = I[X_j(t-) = h]$ and $Y_h(t) = \Sigma_{j=1}^{n}\, Y_{hj}(t)$, where $I(A)$ is the indicator random variable of the event A. The term $Y_h(t)$ can be thought of as the number of residents "at risk" just prior to time t of a transition from state h. Note that Figures 2 and 3 present graphs of $\Sigma_{h=1}^{4}\, Y_h(t)$ (and $Y_h(t)$ for $h = 1, 2, 3, 4$ in Figure 2), where t measures age and duration since entry, respectively. Also, let $N_{hij}(t)$ represent the number of observed $h \to i$ transitions made by resident j during [0, t], and let $N_{hi}(t) = \Sigma_{j=1}^{n}\, N_{hij}(t)$. Then $\{N_{hij}(t),\ t \geq 0\}$ and $\{N_{hi}(t),\ t \geq 0\}$ are counting processes. An elaborate theory has been developed for statistical models involving counting processes. This began with the work of Aalen (1975) and is well described in books by Andersen et al. (1993) and Fleming and Harrington (1991). The theory is based on the fact that the difference between a counting process and its integrated intensity process is a martingale. Variances of statistics that are stochastic integrals with respect to this martingale can be obtained, and asymptotic distributions can be found by using martingale central limit theory. The reader need not have an understanding of the theory of counting processes and martingales.

3.1 The Nelson-Aalen Estimator

A well-known nonparametric estimator of $A_{hi}(t) = \int_0^t \alpha_{hi}(s)\,ds$ is

$$\hat{A}_{hi}(t) = \int_0^t J_h(s)\,Y_h(s)^{-1}\,dN_{hi}(s), \quad (2)$$

where $J_h(s) = I[Y_h(s) > 0]$ and the integrand is defined to be zero when $Y_h(s) = 0$. Intuitively, this estimator makes

FIGURE 2
NUMBER OF RESIDENTS ATTAINING EACH AGE DURING THE STUDY PERIOD

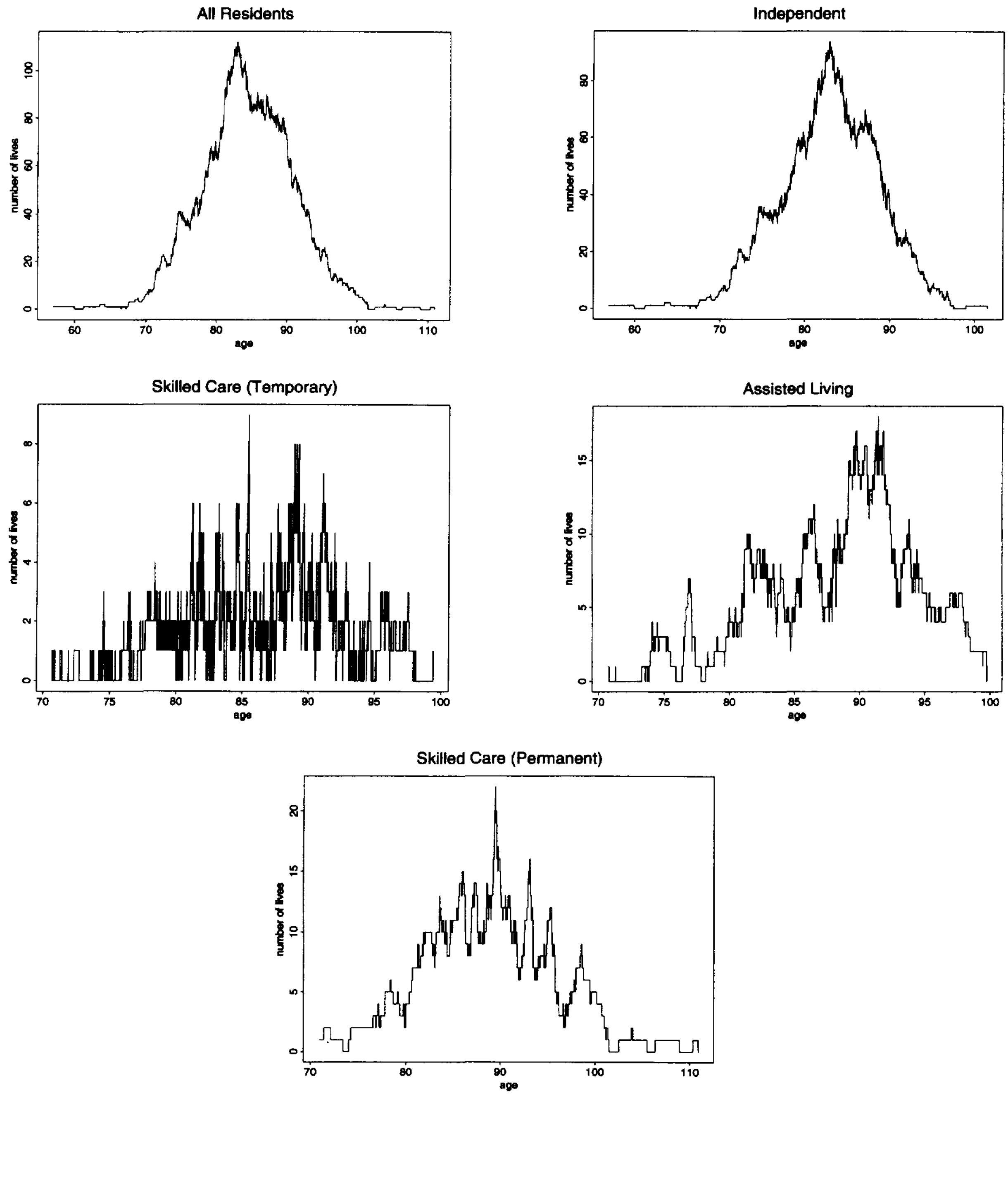
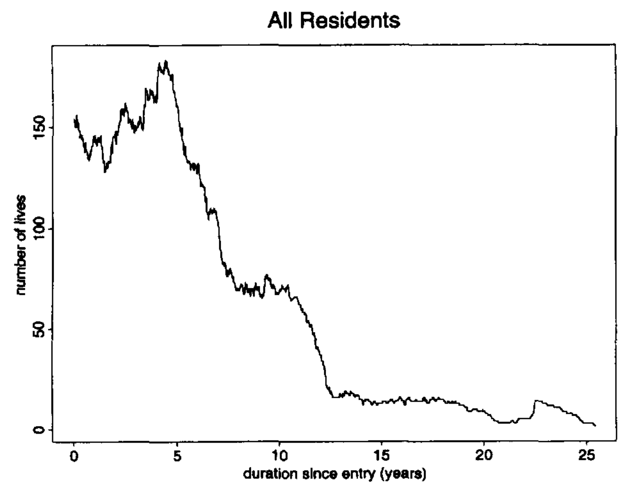


FIGURE 3
NUMBER OF RESIDENTS ATTAINING EACH DURATION DURING THE STUDY PERIOD



sense if we break up the interval $[0, t]$ into many subintervals of length ds. The probability that an individual in state h at time $s-ds$ moves to state i by time s is $\alpha_{hi}(s)ds$. A reasonable estimator of this probability is the number of observed $h \rightarrow i$ transitions during $(s-ds, s]$, which is $dN_{hi}(s)$, divided by the number of individuals in state h at time $s-ds$, which is $Y_h(s)$. Summing the actual probabilities over all subintervals in $[0, t]$ gives $A_{hi}(t)$, and summing the estimators gives the right-hand side of (2). If $T_{hi1}, T_{hi2}, \ldots$ are the observed times of the $h \rightarrow i$ transitions, then $\hat{A}_{hi}(t)$ can be expressed as a simple sum,

$$\hat{A}_{hi}(t) = \sum_{k:T_{hik} \le t} Y_h(T_{hik})^{-1}.$$

$\hat{A}_{hi}$ is the well-known Nelson-Aalen estimator, and it can be verified that the above expression is equivalent to Formula (7.90) of London (1988, p. 170). This estimator was introduced by Nelson (1969) in the context of estimating the hazard function of failure time distributions using censored data. Nelson explored how to use plots of the estimates to gain information about the distribution. Aalen (1978) discussed the estimator in a general counting process framework and considered exact and asymptotic properties of the estimator.

The Nelson-Aalen estimator is not an unbiased estimator of A_{hi} but is biased downward. Let

$$A^*_{hi}(t) = \int_0^t \alpha_{hi}(s) J_h(s) ds.$$

The term $A^*_{hi}(t)$ is almost the same as $A_{hi}(t)$ when $\Pr[Y_h(s)=0]$ is small for all $s \le t$. It turns out that

$$E[\hat{A}_{hi}(t)] = E[A^*_{hi}(t)] = \int_0^t \alpha_{hi}(s) \Pr[Y_h(s) > 0] ds.$$

Hence, the bias in using $\hat{A}_{hi}(t)$ to estimate $A_{hi}(t)$ is

$$E[\hat{A}_{hi}(t)] - A_{hi}(t) = -\int_0^t \alpha_{hi}(s) \Pr[Y_h(s) = 0] ds.$$

The implications of this in estimating transition intensity functions using the CCRC pilot study data will be discussed shortly.

It is important to be able to quantify the variability of an estimator. The variance of the Nelson-Aalen estimator is

$$\begin{aligned} \operatorname{Var}[\hat{A}_{hi}(t)] &= E[\{\hat{A}_{hi}(t) - A^*_{hi}(t)\}^2] \\ &= \int_0^t E[J_h(s) Y_h(s)^{-1}]\, dA_{hi}(s). \end{aligned}$$

An unbiased estimator of the variance is

$$\begin{aligned} \operatorname{Var}[\hat{A}_{hi}(t)] &= \int_0^t J_h(s) Y_h(s)^{-1} d\hat{A}_{hi}(s) \\ &= \int_0^t J_h(s) Y_h(s)^{-2} dN_{hi}(s). \end{aligned} \tag{3}$$

As with Equation (2), we can express the right-hand side of (3) as a sum,

$$\operatorname{Var}[\hat{A}_{hi}(t)] = \sum_{k:T_{hik} \le t} Y_h(T_{hik})^{-2}.$$

This variance estimator can be used to obtain approximate pointwise confidence limits for the cumulative intensity functions. In doing so, we use the fact that the asymptotic distribution of $\hat{A}_{hi}(t)$ is normal. Because the distribution may depart significantly from the normal distribution when $Y_h(t)$ is small, confidence limits obtained using the normal distribution assumption are not reliable in this case.

To illustrate the ideas discussed in this subsection, I now examine the use of the Nelson-Aalen estimator in analyzing one transition type by using the CCRC pilot study data. I consider transitions from state 1 to state 6, that is, deaths from the independent state. Because only 31 such transitions occurred, we can clearly see how the Nelson-Aalen estimator works.

Table 3 shows, for both males and females, the ages at which each death from the independent state occurred, as well as the number of residents at risk of dying while in the independent state at each of those ages. The corresponding Nelson-Aalen estimates of the cumulative intensity functions for females and males

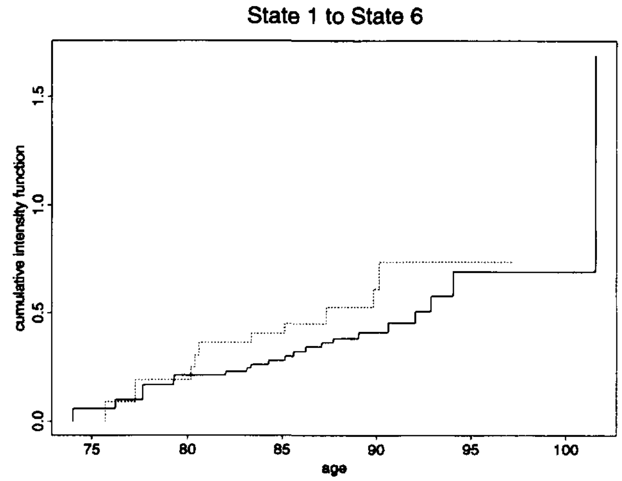
TABLE 3
DEATHS FROM THE INDEPENDENT STATE

Females		Males	
Age	At Risk	Age	At Risk
74.00958	17	75.69884	11
76.21903	24	77.27584	10
77.67830	30	80.18891	17
77.67830	30	80.38877	18
79.29363	45	80.60780	17
79.29911	44	83.38946	24
82.02053	61	85.13621	22
83.14305	62	87.34565	13
83.36482	62	89.84531	12
84.27926	53	90.14648	8
85.17180	46		
85.60986	46		
86.25873	46		
87.09651	53		
87.72621	53		
89.06776	35		
90.62834	23		
92.06571	19		
92.90075	14		
94.06160	9		
101.54689	1		

are shown in Figure 4. The estimated cumulative intensity functions are step functions with jumps at each of the transition (death) ages. The size of each jump equals the number of transitions that occurred at that age divided by the number of residents at risk of making the transition at that age. Perhaps the most appealing aspect of using Nelson-Aalen estimates is the ability to plot the estimates and observe the general shape of the estimated cumulative intensity function. A cumulative intensity function that appears to increase linearly suggests a constant intensity function (because the cumulative intensity function is the integral of the intensity function). A cumulative intensity function that is convex (concave) suggests an increasing (decreasing) intensity function. Keep in mind that estimates based on a small number of transitions, as in this example, are limited in how much information they can convey. Figure 4 seems to indicate that the female intensity function is increasing with age, and the male intensity function may be constant, though there are only ten male transitions.

To interpret the estimated cumulative intensity function appropriately, we should also examine confidence intervals associated with the estimates. This will help

FIGURE 4
ESTIMATED CUMULATIVE INTENSITY FUNCTIONS FOR FEMALES AND MALES (DOTTED)



in distinguishing whether a flat portion of an estimated cumulative intensity function arises due to a small transition intensity or a small number of lives exposed.

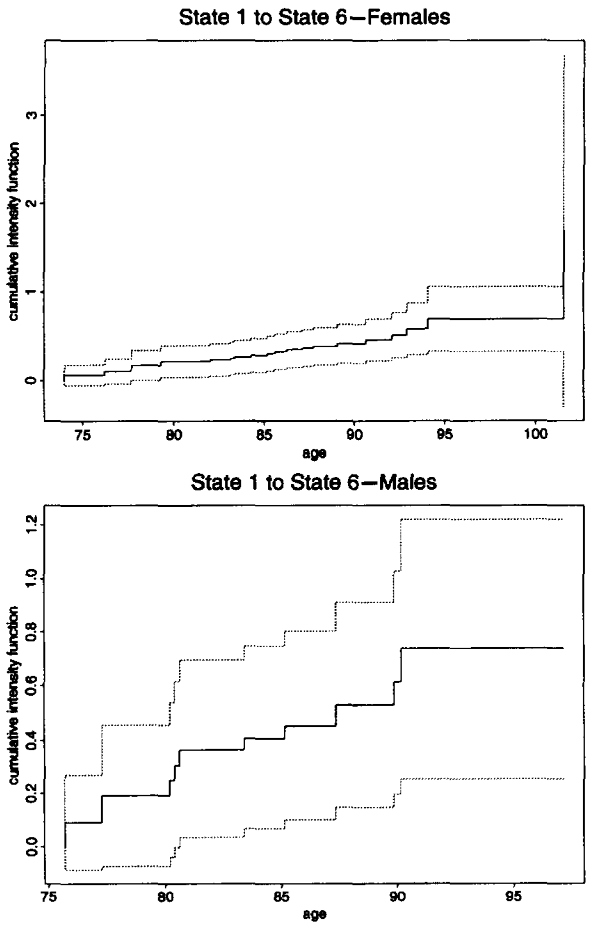
As stated above, pointwise confidence limits can be obtained by assuming that estimators have a normal distribution. For example, an approximate 95% confidence interval for $A^*_{hi}(t)$ is given by

$$\hat{A}_{hi}(t) \pm 1.96\sqrt{\mathrm{Var}[\hat{A}_{hi}(t)]}.$$

Figure 5 shows the estimated cumulative intensity functions along with these 95% confidence limits. Since, for males, the number at risk at each age is rather small, the confidence limits should not be trusted.

I mentioned earlier that $\hat{A}_{hi}(t)$ is an unbiased estimator of $E[A^*_{hi}(t)]$ and a biased estimator of $A_{hi}(t)$. The terms A^*_{hi} and A_{hi} are quite different in the above example because there are no residents at the younger ages. In fact, $Y_h(s)=0$ for all $s < 55$. Fortunately, we are less interested in estimating the function A_{hi} than we are in estimating $\alpha_{hi}(t)$ for values of t in the age range of the CCRC residents. Now $\alpha_{hi}(t)$ is the slope of A_{hi} at age t. For an age interval with $Y_h(t) > 0$, the slopes of A_{hi} and A^*_{hi} are the same. Thus, we can estimate $\alpha_{hi}(t)$ by estimating the rate of increase of A^*_{hi} at

FIGURE 5
ESTIMATED CUMULATIVE INTENSITY FUNCTIONS WITH APPROXIMATE 95% POINTWISE CONFIDENCE LIMITS (DOTTED)



time t. This can be done by averaging the jumps in $\hat{A}_{hi}$ at ages near t, which is considered next.

3.2 *Kernel Function Estimators*

Smooth estimates of α_{hi} can be obtained by using a kernel function estimator. This approach is discussed by Ramlau-Hansen (1983a, 1983b), Andersen et al. (1993), and Gavin et al. (1993). The estimator is defined as

$$\hat{\alpha}_{hi}(t) = \frac{1}{b}\int_{-\infty}^{\infty} K\left(\frac{t-s}{b}\right) d\hat{A}_{hi}(s), \qquad (4)$$

where $\int_{-\infty}^{\infty} K(x)dx=1$ and $K(x)=0$ for $|x| > 1$. The function K is called the kernel function, and b is called the bandwidth, or window size. Viewing the real line as many small intervals of length ds, we see that $\hat{\alpha}_{hi}(t)$ is a weighted average of the jumps in the Nelson-Aalen estimator that occur in the interval $[t-b, t+b]$. The smoothness of the estimates increases as the value of b increases. Again letting T_{hi1}, T_{hi2}, ... be the observed $h \rightarrow i$ transition times, (4) can be written as a sum,

$$\hat{\alpha}_{hi}(t) = \frac{1}{b}\sum_k K\left(\frac{t-T_{hik}}{b}\right) Y_h(T_{hik})^{-1}.$$

The kernel function estimator is a consistent, though not unbiased, estimator of $\alpha_{hi}(t)$.

A variance estimator of the kernel function estimator is given by

$$\text{Var}[\hat{\alpha}_{hi}(t)] = \frac{1}{b^2}\int_{-\infty}^{\infty} K^2\left(\frac{t-s}{b}\right) J_h(s)Y_h(s)^{-2}dN_{hi}(s),$$

which can also be written as

$$\text{Var}[\hat{\alpha}_{hi}(t)] = \frac{1}{b^2}\sum_k K^2\left(\frac{t-T_{hik}}{b}\right) Y_h(T_{hik})^{-2}.$$

A popular choice of kernel function is the Epanechnikov kernel function (see Epanechnikov 1969),

$$K(x) = 0.75(1 - x^2), \ |x| \leq 1.$$

This kernel function minimizes the mean square error asymptotically. I use the Epanechnikov kernel function to obtain smooth transition intensity functions. Other kernel functions are discussed by Ramlau-Hansen (1983b). Bandwidth selection is reviewed by Jones et al. (1996).

Note that it is appropriate to use (4) only if $Y_h(s) > 0$ for all $s \in [t-b, t+b]$. Otherwise, a substantial downward bias could result since the absence of exposure in a given time range will produce a low-transition-intensity estimate. The actual transition intensity might be quite large, but no residents were at risk of making the transition. I stated earlier that Nelson-Aalen estimates are informative only if calculated for a time interval $[t_1, t_2]$, where $Y_h(t) > 0$ for all $t \in [t_1, t_2]$. If $Y_h(t)=0$ for t outside this interval, then we should restrict use of (4) to obtaining estimates of $\alpha_{hi}(t)$ for $t \in [t_1+b, t_2-b]$.

Smoothed transition intensities for deaths from the independent state were calculated by using the CCRC pilot study data; they are shown in Figure 6 along with approximate 95% confidence limits. A window size of

6 was used for both females and males. In this example, we have too few observations to make conclusions about the transition intensity functions.

3.3 Regression Models

The transition intensity functions defined in (1) are resident-specific and depend on $\mathbf{Z}_j(t)$, a vector of covariates that provide relevant information about resident j that is available just prior to time t. One approach to reflecting the effect of covariates is by using a multiplicative hazards model. This was introduced by Cox (1972) in the context of analyzing censored survival data and is often referred to as the Cox regression model. Andersen and Gill (1982) extended the ideas to general counting processes. Let

$$\alpha_{hij}[t; \mathbf{Z}_j(t)] = \alpha_{hi0}(t) \exp[\boldsymbol{\beta}^T_{hi}\mathbf{Z}_j(t)], \qquad (5)$$

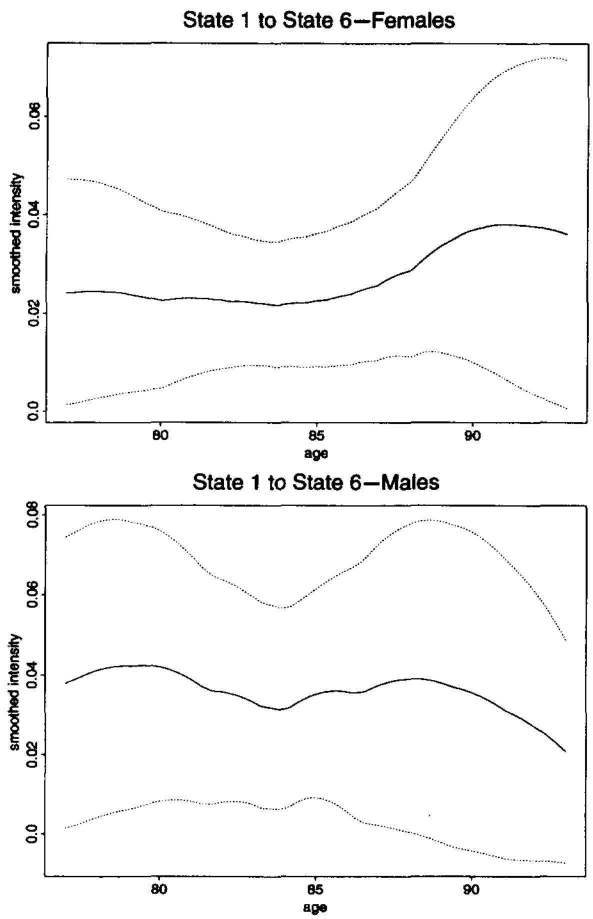
where $\boldsymbol{\beta}_{hi}$ is a parameter vector, and α_{hi0} is the "baseline" intensity function. Under this model, the transition intensity functions for different values of a fixed covariate are proportional. For example, if $Z^1_j(t)$, the first component of the covariate vector for resident j, is 0 if the individual is a female and 1 if a male, then the model assumes that the male transition intensity function is $\exp(\beta^1_{hi})$ times the female transition intensity function, where β^1_{hi} is the first component of $\boldsymbol{\beta}_{hi}$.

To fit a model of this type, we must estimate the components of $\boldsymbol{\beta}_{hi}$ and the baseline intensity function. It can be argued (see Cox 1972) that little information about $\boldsymbol{\beta}_{hi}$ is provided by the transition times since the baseline transition intensity function could be very small, except near the transition times where it could be very large. Hence, most of the information about $\boldsymbol{\beta}_{hi}$ is provided by the knowledge of which individuals made transitions given the transition times and the individuals at risk of transitions just prior to these times. This is the motivation for the partial likelihood,

$$L(\boldsymbol{\beta}_{hi}) = \prod_k \prod_{j=1}^{n} \frac{\exp[\boldsymbol{\beta}^T_{hi}\ \mathbf{Z}_j(T_{hijk})]}{\Sigma^n_{l=1} \exp[\boldsymbol{\beta}^T_{hi}\ \mathbf{Z}_l(T_{hijk})]Y_{hl}(T_{hijk})},$$

where T_{hij1}, T_{hij2}, ... are the $h \rightarrow i$ transition times for individual j. The components of $\boldsymbol{\beta}_{hi}$ can be estimated by maximizing $L(\boldsymbol{\beta}_{hi})$. Inferences can then be made about $\boldsymbol{\beta}_{hi}$ as in the usual maximum likelihood setting (see Hogg and Craig 1995). An advantage of using the partial likelihood is that the effect of covariates on the transition intensity can be estimated without specifying

FIGURE 6
SMOOTHED INTENSITY FUNCTIONS WITH APPROXIMATE 95% POINTWISE CONFIDENCE LIMITS (DOTTED)



a functional form for the baseline intensity function. This differs from the approach discussed by London (1988, p. 208) in which the baseline intensity is assumed to be a simple (constant) function, and parameters are estimated by maximizing the full likelihood.

Once the estimates $\hat{\boldsymbol{\beta}}_{hi}$ have been determined, the estimated baseline cumulative intensity function is

$$\hat{A}_{hi0}(t, \hat{\boldsymbol{\beta}}_{hi}) = \sum_{k: T_{hik} \le t} \left\{ \sum_{j=1}^{n} \exp[\hat{\boldsymbol{\beta}}^T_{hi}\ \mathbf{Z}_j(T_{hik})]Y_{hj}(T_{hik}) \right\}^{-1}. \qquad (6)$$

This generalizes the Nelson-Aalen estimator and is often referred to as the Breslow estimator.

The traditional actuarial approach to handling data with covariates is to group the data into homogeneous cells. All data in a given cell would have the same (or approximately the same) values of the important covariates. Separate estimates are then obtained for each cell by using only data from that cell. Some smoothing across cells can then be done.

The Cox regression approach offers some advantages. Statistical tests can be performed to determine which covariates are important. Accuracy is improved since transition intensity functions are estimated by using all the data, not just the data from a given cell. The only cost is that we must be willing to assume that the transition intensity functions have the multiplicative form given in (5). This assumption should, of course, be tested. Methods of performing such a test are discussed by Andersen et al. (1993).

The statistical package S-PLUS can be used to fit a Cox regression model.

4. Estimation of Transition Intensity Functions Using the Pilot Study Data

This section illustrates the techniques of Section 3 by using the pilot study data described in Section 2. The goals are to demonstrate the methods of estimation and to identify variables that may be significant in modeling transition intensity functions. The results of this analysis are used to determine probabilities in Section 5.

Note that the pilot study data are not sufficient to estimate all the intensity functions with reasonable accuracy. In practice, we should consult other sources of information when faced with this situation. Since my objective is to demonstrate the methods described earlier and to find estimates to be used later in the paper, I am content with estimates based solely on the pilot study data. In addition, in using the Cox regression model, I do not perform a thorough regression analysis. My goal is simply to gain an understanding of what covariate information may influence the transition intensity functions. I use the regression coefficients that result to illustrate the nature of the intensity estimates that might be obtained. Tests of the multiplicative intensity assumption and analyses of residuals, which are beyond the scope of this paper, should be undertaken if this approach is used in practice (see Andersen et al. 1993, sec. VII.3).

To begin, note that the intensity functions corresponding to certain transitions shown in Figure 1 should be zero. In particular, the $2 \to 1$, $4 \to 1$, $4 \to 2$, and $4 \to 3$ intensity functions should be zero in light of the permanent nature of states 2 and 4. In addition, the $1 \to 4$ transition intensity function will be set to zero. Only two such transitions occurred. Given the total time spent in state 1 by all residents, this suggests a very small intensity for this CCRC.

4.1 Mortality

Only 126 deaths occurred during the study period. However, we have considerable prior knowledge of mortality patterns. We expect the intensity (force of mortality) to be greater for males than females, and we expect the intensity to increase with age and the level of care provided. Rather than model the four different mortality transitions separately as described in Section 3.3 and suggested by (5), we can model them together by assuming that the intensity functions are proportional. This leads to the model

$$\alpha_{\cdot 6j}[t; \mathbf{Z}_j(t)] = \alpha_{\cdot 60}(t) \exp[\boldsymbol{\beta}_{\cdot 6}^{T} \mathbf{Z}_j(t)],$$

where $\alpha_{\cdot 60}$, the baseline intensity function for deaths from any state, is a function of age. We can assume that the last three components of the covariate vector, $\mathbf{Z}_j(t)$, are variables that indicate (1 if yes, 0 if no) whether resident j was in state 2, 3, and 4 just prior to time t. If the resident was in state 1, then all three variables are zero. Thus, if $Z_j^k(t)$, the k-th component of $\mathbf{Z}_j(t)$, is the indicator for state 2, then the intensity for deaths from state 2 is $\exp(\beta_{\cdot 6}^k)$ times the intensity for deaths from state 1, where $\beta_{\cdot 6}^k$ is the k-th component of $\boldsymbol{\beta}_{\cdot 6}$.

Table 4 shows the results of a Cox regression run performed using S-PLUS. Four covariates were included in the model. The first was an indicator of whether the resident was a male. The remaining three covariates were the indicators corresponding to states 2 (assisted living), 3 (skilled care temporary), and 4 (skilled care permanent), as described above. For each covariate, the table provides the estimate of the coefficient, $\beta_{\cdot 5}^k$, the corresponding proportionality factor, $\exp(\beta_{\cdot 6}^k)$, the standard error of the coefficient, the p value for a two-tailed test of the hypothesis that the coefficient equals zero, and upper and lower 95% confidence limits for the proportionality factor. At the bottom of the table are the likelihood ratio and efficient score statistics, which can be used for an overall test of whether the variables in the model are related to the transition intensity.

Table 4
Cox Regression Results for Mortality

Covariate	Coefficient	Exp(Coef)	Standard Error	p Value	Lower 95%	Upper 95%
Male	0.239	1.27	0.240	0.320	0.793	2.03
Assisted	0.448	1.57	0.394	0.256	0.723	3.39
Skilled (Temporary)	2.773	16.01	0.292	0	9.042	28.35
Skilled (Permanent)	2.082	8.02	0.245	0	4.964	12.95

Likelihood ratio statistic = 123 on 4 df, $p=0$
Efficient score statistic = 166 on 4 df, $p=0$

The table indicates that only the two skilled-care variables are significant. The p values for the male indicator and the assisted living indicator are 0.320 and 0.256, respectively, suggesting no evidence against the hypothesis that the corresponding two coefficients are zero. However, recognizing that this may be due to insufficient data, I retained all four variables. I therefore estimate the intensity functions in terms of the estimated baseline intensity function, $\hat{\alpha}_{\cdot 60}(t)$, as follows:

$$\begin{aligned}
\hat{\alpha}_{16j}[t;\ \mathbf{Z}_j(t)] &= \hat{\alpha}_{\cdot 60}(t),\\
\hat{\alpha}_{26j}[t;\ \mathbf{Z}_j(t)] &= 1.57\hat{\alpha}_{\cdot 60}(t),\\
\hat{\alpha}_{36j}[t;\ \mathbf{Z}_j(t)] &= 16.01\hat{\alpha}_{\cdot 60}(t),\\
\hat{\alpha}_{46j}[t;\ \mathbf{Z}_j(t)] &= 8.02\hat{\alpha}_{\cdot 60}(t),
\end{aligned}$$

if resident j is a female. Each function should be multiplied by 1.27 if resident j is a male.

Figure 7 shows the Breslow estimates of the baseline cumulative intensity function as well as kernel function estimates of the baseline intensity function obtained by using a bandwidth of 6. The estimated cumulative intensity function has a convex shape through the 80s and early 90s, suggesting an increasing intensity function. The smooth intensity function estimates exhibit this increasing behavior. To obtain a simple mathematical expression for the intensity function and to improve the smoothness, I assumed that the intensity function is of the form $\alpha_{\cdot 60}(t)=a+bc^t$ (Makeham's law). The corresponding cumulative intensity function is

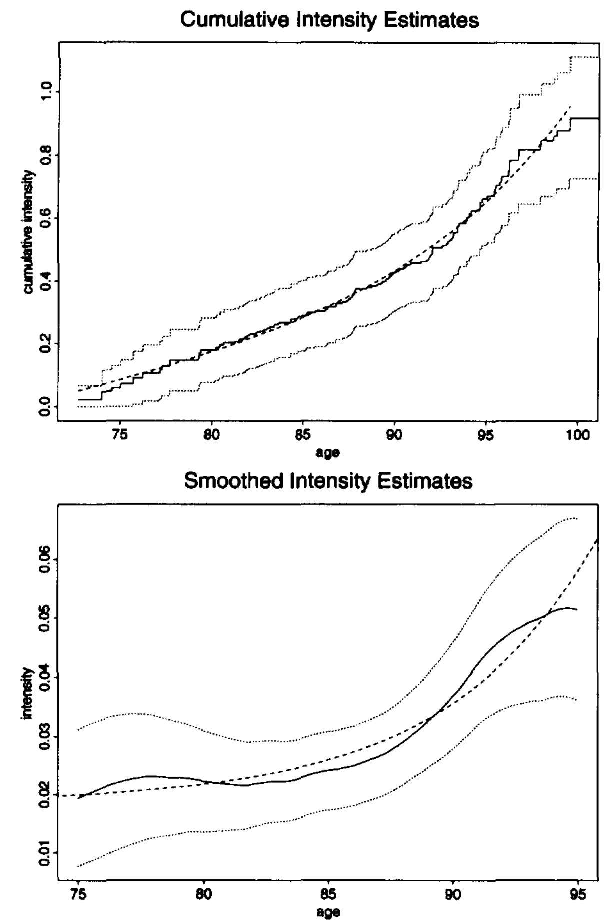
$$A_{\cdot 60}(t) = \frac{at + b(c^t - 1)}{\log c},$$

and I let

$$A^{*}_{\cdot 60}(t) = \frac{d + at + b(c^t - 1)}{\log c}.$$

It is necessary to distinguish between $A_{\cdot 60}$ and $A^{*}_{\cdot 60}$ because $\hat{A}_{\cdot 60}$ is greatly influenced by the fact that there are no residents at the younger ages. The latter function was fitted to the Breslow estimates by least squares.

Figure 7
Cumulative and Smoothed Baseline Mortality Intensity Estimates



The resulting parameter values are $a=0.01200684$, $b=7.075078\times10^{-7}$, $c=1.122718$, and $d=-0.8505983$, and the function is represented by the dashed line that appears along with the baseline cumulative intensity estimates in Figure 7. The corresponding intensity function is also plotted along with the smoothed intensity estimates.

4.2 Other Transitions

A similar approach was used to obtain transition intensity function estimates for the remaining transitions. Variables found to be significant in modeling these functions were age, sex, duration since entry to the current state, duration since entry to the CCRC, and the previous state.

For transitions from states 1 and 2, the baseline intensity functions were defined as functions of age. For transitions from states 3 and 4, the baseline intensity functions were defined as functions of time since entry to the current state. This was particularly necessary for transitions from state 3 because, for a number of age intervals, there were no lives exposed (see Figure 2). Since stays in states 3 and 4 were rather short, nearly all observed stays began during the study period. For transitions from states 3 and 4, the significance of age was examined by using a covariate. Similarly, for transitions from states 1 and 2, the impact of duration since entry to the current state was analyzed using a covariate.

See Jones (1996) for details on the estimated transition intensity functions.

5. Determining Probabilities and Other Quantities

As stated in Section 1.2, to calculate actuarial present values, we should be able to estimate the probability that a resident is in any given state at any future time, as well as the probability that a resident will move between any two states during any time interval. Depending on the complexity of the transition intensity functions, these probabilities may be difficult to calculate directly. In this section, I present an approach to determining probabilities and other quantities using simulation. The method can be used for very general forms of the transition intensity functions. The approach is described in Section 5.1, and numerical results obtained using the intensity functions estimated from the pilot study data are discussed in Section 5.2.

5.1 Simulation Approach

Consider the general setup in which the transition intensity functions are given by $\alpha_{hi}[t;\, \mathbf{Z}_j(t)]$. I assume that the components of $\mathbf{Z}_j(s)$ are either constant over time or depend only on the history of $\{X_j(t)\}$ up to time $s-$. Thus, if we know $X_j(t)$ for $0 \le t \le s$, then we also know $\alpha_{hi}[t;\, \mathbf{Z}_j(t)]$ for values of t up to time s. Furthermore, if we know $X_j(t)$ up to time s, and we assume that no transitions occur during the next w years, then we know $\alpha_{hi}[t;\, \mathbf{Z}_j(t)]$ up to time $s+w$.

For example, suppose that $\alpha_{hi}[t;\, \mathbf{Z}_j(t)]=\alpha_{hi}(t,\, u)$, where t is the age of the individual and u is the time since the individual entered state h. Now if, at age s, the individual has been in state h for v years, then assuming no transitions occur during the next w years, the $h \rightarrow i$ transition intensity at age $s+w$ is $\alpha_{hi}(s+w,\, v+w)$.

Assume that resident j is in state h at time s and we wish to determine the probability that this individual will be in state k at time $r > s$. We can do this by simulating the time and state entered at each transition time up to time r. Then if we repeat this a large number of times, the proportion of times that the individual is in state k at time r approximates the desired probability.

I use a method known as *thinning* (see Ross 1990, p. 73). As stated above, if no transitions occur by time $s+w$, then the values of $\alpha_{hi}[t;\, \mathbf{Z}_j(t)]$ are known for $s \le t \le s+w$ and $i=1, 2, \ldots, 6$, $i \ne h$. Let

$$\alpha = \max_{s \le t \le r} \left\{ \sum_{i:i \ne h} \alpha_{hi}[t;\, \mathbf{Z}_j(t)] \right\},$$

assuming that no transitions occur before time r. Now

$$\sum\nolimits_{i:i \ne h} \alpha_{hi}[t;\, \mathbf{Z}_j(t)]$$

is the intensity of transition out of state h at time t, and α is no less than this intensity for $t \in [s, r]$. To determine the first transition time after time s, successively generate the event times, T_1, T_2, ..., of a Poisson process with intensity α, and accept T_l with a probability equal to

$$\sum_{i:i \ne h} \alpha_{hi} \frac{s + T_l;\, \mathbf{Z}_j(s + T_l)}{\alpha}.$$

The event times of a Poisson process with intensity α are easily generated since the times between successive events are exponentially distributed with mean $1/\alpha$. An event time can be accepted with a given probability by generating a random number that is uniform on (0, 1) and accepting the event time if the number is no greater than this probability. Let T_1^* the first accepted time, be the time until the next transition. The state entered at time $s+T_1^*$ can then be generated based on its conditional distribution. For $i^* \neq h$, the probability that i^* is the state entered at time $s+T_1^*$equals

$$\alpha_{hi^*} \frac{s + T_1^*; \mathbf{Z}_j(s + T_1^*)}{\sum_{i:i\neq h} \alpha_{hi}[(s + T_1^*; \mathbf{Z}_j(s + T_1^*)]}.$$

Once the time of and state entered upon the next transition are found, we determine the intensity functions that will apply after this transition, recalculate α, and repeat the procedure. We continue until we have a transition that occurs after time r, or until death or withdrawal. We then know the value of $X_j(t)$ for $s \leq t \leq r$.

By repeating the procedure a large number of times, not only can we estimate various probabilities, but we can also estimate other interesting quantities. For example, for each simulation outcome we could compute the present value of the fee income that would result. Then the average of these present values provides an estimate of the actuarial (expected) present value of the fee income.

5.2 Numerical Illustration

By using the method described in Section 5.1 and the intensity function estimates obtained from the pilot study data, 10,000 simulations were performed for a 75-year-old female and for a 75-year-old male, both having just entered the CCRC. Table 5 summarizes the results of these simulations. The columns of the table provide the probability that the resident is in each of the six states at the end of each of the next 20 years. Since 10,000 simulations were performed, for each probability estimate, the standard deviation is at most $\sqrt{(0.5)^2/10000}=0.005$. Hence, the estimates should be within 0.01 of the true value (that is, true according to our estimated intensity functions) with probability at least 0.95.

One interesting observation from Table 5 is that the probability that the resident is in state 6 (dead) is higher for females than for males. This anomaly suggests that revisions to the transition intensity function estimates are required. I remind the reader that these estimates were obtained by using a small amount of data and are shown for illustration only.

6. Conclusions

This paper has described and demonstrated an approach to analyzing CCRC data. I conclude with some observations that are relevant to those wishing to conduct such an analysis.

Given the number of transitions that can be made by a given resident and the frequency with which these transitions occur, it is natural to use a continuous-time multistate stochastic model to capture fully the randomness in resident transitions. It is convenient to characterize such a model in terms of the transition intensity functions. The methods described in Section 3 can then be used to obtain estimates for these functions.

The analysis summarized in Section 4 suggests that a model that incorporates the important sources of variation in resident outcomes will be rather complicated. Fitting such a model requires good data and careful use of statistical methods. I have introduced some useful methods in this paper.

I pointed out in Section 4 that the pilot study data are not sufficient to estimate accurately all the transition intensity functions. We should therefore consider how much data are required. The accuracy with which we can estimate a transition intensity function depends greatly on the number of observed transitions. Unfortunately, for some of the transition types, such as those from assisted living to skilled care (permanent), there were very few observed transitions, and I have little confidence in the estimates. Based on my analysis with this dataset, I believe that, with five to ten times as much data, all the transition intensity functions could reasonably be estimated.

Another important issue in deciding how much data are required is the completeness of the data. Unfortunately, for residents in the pilot study dataset, information was available only for transitions that occurred during a three-year study period. It would be ideal to have complete health status histories for all residents involved in the study. In estimating certain transition intensity functions, more data on a given group of residents may be better than data on more residents.

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Table 5
Simulation Results for 75-Year-Old New Residents

Age	Probability of Being in					
	State 1	State 2	State 3	State 4	State 5	State 6
			Females			
75	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
76	0.7492	0.0407	0.0759	0.0704	0.0284	0.0354
77	0.5474	0.0802	0.0728	0.1606	0.0566	0.0824
78	0.3906	0.1050	0.0561	0.2305	0.0804	0.1374
79	0.2734	0.1095	0.0442	0.2732	0.1002	0.1995
80	0.1912	0.1087	0.0311	0.2913	0.1167	0.2610
81	0.1339	0.0954	0.0219	0.2942	0.1315	0.3231
82	0.0885	0.0833	0.0178	0.2839	0.1434	0.3831
83	0.0554	0.0691	0.0151	0.2632	0.1528	0.4444
84	0.0363	0.0577	0.0088	0.2393	0.1609	0.4970
85	0.0236	0.0443	0.0049	0.2120	0.1668	0.5484
86	0.0154	0.0349	0.0035	0.1821	0.1718	0.5923
87	0.0097	0.0261	0.0037	0.1515	0.1754	0.6336
88	0.0051	0.0203	0.0020	0.1246	0.1771	0.6709
89	0.0024	0.0154	0.0015	0.1018	0.1790	0.6999
90	0.0012	0.0107	0.0011	0.0803	0.1802	0.7265
91	0.0006	0.0069	0.0007	0.0616	0.1811	0.7491
92	0.0003	0.0052	0.0005	0.0441	0.1817	0.7682
93	0.0002	0.0032	0.0008	0.0328	0.1819	0.7811
94	0.0001	0.0020	0.0000	0.0232	0.1819	0.7928
95	0.0000	0.0013	0.0001	0.0160	0.1819	0.8007
			Males			
75	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
76	0.8198	0.0256	0.0461	0.0417	0.0337	0.0331
77	0.6650	0.0567	0.0424	0.0949	0.0641	0.0769
78	0.5311	0.0764	0.0416	0.1327	0.0887	0.1295
79	0.4195	0.0931	0.0345	0.1551	0.1135	0.1843
80	0.3283	0.1039	0.0281	0.1639	0.1313	0.2445
81	0.2500	0.1092	0.0223	0.1746	0.1476	0.2963
82	0.1908	0.1079	0.0175	0.1691	0.1601	0.3546
83	0.1443	0.1054	0.0137	0.1585	0.1729	0.4052
84	0.1022	0.1021	0.0112	0.1465	0.1809	0.4571
85	0.0762	0.0936	0.0074	0.1328	0.1876	0.5024
86	0.0572	0.0822	0.0054	0.1162	0.1948	0.5442
87	0.0403	0.0742	0.0061	0.0985	0.1985	0.5824
88	0.0289	0.0650	0.0036	0.0841	0.2022	0.6162
89	0.0189	0.0549	0.0032	0.0691	0.2042	0.6497
90	0.0121	0.0454	0.0024	0.0582	0.2057	0.6762
91	0.0084	0.0391	0.0013	0.0438	0.2074	0.7000
92	0.0058	0.0308	0.0013	0.0319	0.2080	0.7222
93	0.0029	0.0264	0.0013	0.0236	0.2082	0.7376
94	0.0019	0.0202	0.0009	0.0176	0.2089	0.7505
95	0.0013	0.0165	0.0002	0.0119	0.2090	0.7611

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REFERENCES

Aalen, O.O. 1975. "Statistical Inference for a Family of Counting Processes." Ph.D. thesis, University of California, Berkeley.

Aalen, O.O. 1978. "Nonparametric Inference for a Family of Counting Process," *Annals of Statistics* 6:701–26.

Actuarial Standards Board. 1994. "Practices Relating to Continuing Care Retirement Communities," *Actuarial Standard of Practice No. 3*, rev. ed. Washington, D.C.: ASB, July.

Andersen, P.K., Borgan, Ø., Gill, R.D., and Keiding, N. 1993. *Statistical Models Based on Counting Processes*. New York: Springer-Verlag.

Andersen, P.K., and Gill, R.D. 1982. "Cox's Regression Model for Counting Processes: A Large Sample Study," *Annals of Statistics* 10:1100–20.

Brace, G.L. 1994. "Continuing Care Retirement Communities: A Primer," *Health Section News* no. 28 (February):4–6.

Cox, D.R. 1972. "Regression Models and Life-Tables" (with discussion), *Journal of the Royal Statistical Society B* 34:187–220.

Cumming, R.B., and Bluhm, W.F. 1992. "CCRC Population and Financial Model," *The Actuary* 26, no. 3 (March):3–5.

Epanechnikov, V.A. 1969. "Nonparametric Estimation of a Multidimensional Probability Density," *Theory of Probability and Its Applications* 14:153–58.

Fleming, T.R., and Harrington, D.P. 1991. *Counting Processes and Survival Analysis*. New York: Wiley.

Gavin, J., Haberman, S., and Verrall, R. 1995. "Graduation by Kernel and Adaptive Kernel Methods with Boundary Correction," *Transactions of the Society of Actuaries* XLVII:173–210.

Hogg, R.V., and Craig, A.T. 1995. *Introduction to Mathematical Statistics*. Englewood Cliffs, N.J.: Prentice Hall.

Jones, B.L. 1995. "A Stochastic Population Model for High Demand CCRCs," *Insurance: Mathematics and Economics* 16:69–77.

Jones, B.L. 1996a. "Analysis of CCRC Data," *ARCH* no. 1:19–82.

Jones, B.L. 1996b. "Transient Results for a High Demand CCRC Model," *Scandinavian Actuarial Journal* no. 2:165–82.

Jones, B.L. 1997. "Stochastic Models for Continuing Care Retirement Communities," *North American Actuarial Journal* 1, no. 1 (January):50–73.

Jones, M.C., Marron, J.S., and Sheather, S.J. 1996. "A Brief Survey of Bandwidth Selection for Density Estimation," *Journal of the American Statistical Association* 91, no. 433:401–7.

London, D. 1988. *Survival Models and Their Estimation*. Winsted, Conn.: Actex.

Moorhead, E.J., and Fischer, N.H. 1995. "Testing Financial Stability of Continuing Care Retirement Communities," *Transactions of the Society of Actuaries* XLVII:307–58.

Nelson, W. 1969. "Hazard Plotting for Incomplete Failure Data," *Journal of Quality Technology* 1: 27–52.

Ramlau-Hansen, H. 1983a. "Smoothing Counting Process Intensities by Means of Kernel Functions," *Annals of Statistics* 11:453–66.

Ramlau-Hansen, H. 1983b. "The Choice of a Kernel Function in the Graduation of Counting Process Intensities," *Scandinavian Actuarial Journal* 1983: 165–82.

Ross, S.M. 1983. *Stochastic Processes*. New York: Wiley.

Ross, S.M. 1990. *A Course in Simulation*. New York: Macmillan.

Society of Actuaries and the American Association of Homes for the Aging. 1991. "Pilot Study of a Continuing Care Retirement Community Mortality and Morbidity Data Collection Process." Schaumburg, Ill.: Society of Actuaries.

Winklevoss, H.E., and Powell, A.V. 1984. *Continuing Care Retirement Communities: An Empirical, Financial, and Legal Analysis*. Homewood, Ill.: Richard D. Irwin.