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Reinvestment Strategies for Life Insurance Products in a Changing Economic Environment

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Section 1: Introduction

1.1 MOTIVATION AND PURPOSE

Most life insurance annuities are in force for many years and contain embedded long-dated options that can last through economic periods of both rapid change and sustained extreme conditions. A current example of such a condition is the sustained low interest rate environment that began in 2009. In such situations, annuities' primary account values perform poorly throughout the period, while the value of their shadow accounts (which determine whether the guarantees are in effect) remains positive for a longer part of that time.

Since such embedded options are long dated and may result in liabilities rather than assets, it is prudent to examine the reinvestment strategy for these products on a frequent basis. The same approach of asset-liability management (ALM) modeling of the embedded value from the original investment strategy can be repeated with stochastic scenarios to develop new reinvestment strategies. To accomplish this, we can analyze two different sets of stochastic scenarios: one would use the most current corporate assumptions, while the other would assume that the mean reversion target of the interest rates should be identical to the starting yield assumption. When a sustained low interest rate environment exists, the first set of scenarios can represent the company outlook, and the second set can illustrate the impact of low interest rates continuing far into the future. A side benefit of using two sets of scenarios for this analysis is the value of comparing the results from the status quo environment to those based on embedded corporate philosophy.

Starting from these considerations, our research addresses how to model different reinvestment strategies to determine which ones maximize the economic results under both sets of scenarios in different economic conditions.

1.2 DESCRIPTION OF VARIABLE ANNUITIES WITH GUARANTEES

This paper focuses on insurance products with embedded options, particularly on interest rate guarantees and variable annuities (VAs).

Insurance policies in many markets are sold with minimum interest rate guarantees. The guarantees on a point-to-point basis are very popular. With this process, a certain minimum interest rate is credited to the insured's account annually, and the policyholder is guaranteed to receive that amount at maturity of the contract. The insurer can credit less to accounts in years with poor interest rates and more in years with higher interest rates as long as the minimum amount is met in the end. The insurer has considerable flexibility in crediting interest to specific accounts.

Recently, other flexible products have become attractive. Variable annuities were introduced in the United States in the 1970s. This market has increased considerably over the past decade, as bullish financial markets and low interest rates have tempted investors to look for higher returns. A VA, whose benefits are based on the performance of an underlying fund (known as "unit-linked" in Europe), are attractive because they provide participation in the stock market and partial protection against the downside movements of interest rates or the equity market. Since the 1990s, two kinds of embedded guarantees have been offered in such policies (Hanif et al 2007): the guaranteed minimum death benefit (GMDB) and the guaranteed minimum living benefit (GMLB). The GMDB is an increasing-strike put option with a stochastic maturity date (Haberman and Piscopo 2008), meaning that if the insured dies during the deferment period, the beneficiary obtains a death benefit equal, in the basic form of the product, to the maximum of the premium accumulated at a guaranteed rate and the account value linked to the fund. This guarantee is paid by the VA holder in the form of a perpetual fee that is deducted from the account value linked to the underlying asset.

Guaranteed living benefits fall into three basic categories. The two earliest forms, the guaranteed minimum accumulation benefit (GMAB) and the guaranteed minimum income benefit (GMIB), offer the policyholder a guaranteed minimum at the maturity T of the contract; however, with the GMIB, this guarantee only applies if the account value is annuitized at maturity. In 2002, a new type of GMLB was issued—the guaranteed minimum withdrawal benefit (GMWB), which allows the insured to withdraw a prespecified amount annually, even if the account value has fallen below this amount (Piscopo

2009; Piscopo 2010b; Piscopo and Haberman 2011). In 2004, each of the 15 largest VA providers in the United States offered this guarantee, and 69% of the variable annuities sold included a GMWB option (Lehman Brothers 2005); in 2007, the percentage had risen to 86% (Ledlie at al. 2007).

1.3 BACKGROUND

Our research has a twofold basis: the literature on the valuation of variable annuities and interest rate guarantees, and the contributions of asset-liability management in the life insurance setting.

1.3.1 LITERATURE ON VARIABLE ANNUITIES

Many practical and academic contributions have described VAs and embedded guarantees. Most of the earlier literature is constituted by empirical works dealing with product comparisons rather than pricing and hedging issues (Sloane 1970). It was not until recently that practitioners discussed some guarantees in relation to the growing opportunities to introduce VAs in new markets (J. P. Morgan 2004; Lehman Brothers 2005; Milliman 2007). Recently, the academic literature has also shown a fervent interest in the topic (Bauer et al. 2006; Chen et al. 2008; Coleman et al. 2006; Dai et al. 2008; Holz et al. 2006; Milevsky and Panyagometh 2001; Milevsky and Salisbury 2006; Nielsen and Sandmann 2003). Bauer et al. (2006) offered the first general framework on which any design of options and guarantees currently offered within variable annuities can be modeled. Besides valuing the contract on the assumption that the policyholder will follow a given strategy with respect to surrender and withdrawals, the framework allows contracts to be priced with different embedded options. Piscopo (2010b) studied the guaranteed withdrawal benefit option in VAs and found that the periodic withdrawals from the fund are similar to the benefits paid to pensioners during retirement.

Other authors have studied the interest rate guarantees embedded in unit-linked products and profit-sharing products, focusing on analytical valuation and numerical methods (such as the Monte Carlo simulation and the lattice method). Persson and Knut (1997) developed an analytical solution for maturity guarantees on savings accounts in a Vasicek stochastic interest rate environment. Miltersen and Persson (1999) presented analytical solutions for maturity and multi-period guarantees on stock accounts in a general stochastic interest rate environment. Lindset (2001) created an analytical formula of multi-period guarantees, giving numerical examples for up to five periods. Schrager and Pelsser (2003) gave analytical solutions to the problem of the rate of return guarantees in regular premium, unit-linked products. Grosen and Jorgensen (2000) proposed a recursive binomial lattice method for valuing surrender options in profit-sharing contracts. Finkelstein et al (2003) used the Monte Carlo simulation to price interest rate guarantees embedded in both variable and participating products for single-premium and regular-premium payments.

1.3.2 CONSIDERATIONS ON ASSET-LIABILITY MANAGEMENT

The majority of the papers mentioned were based on the concept of risk neutral valuation, focusing essentially on valuing and pricing these products. In the risk neutral framework, the main aim is to describe how much policyholders should be charged for the guaranteed benefits. Little attention has been given to determining how much reserve and capital an insurer should hold to cover expected and unexpected losses. To answer to this question, the risk neutral framework has to be abandoned.

Hardy (2003) was among the first in the literature to address the question of exploiting risk management for equity-linked insurance. Indeed, a different point of view must be adopted when quantifying the real impact of the risk factors on an insurer’s balance sheet and the management of these risks. In this context, it is necessary to consider the real probability of the movements of the financial and demographic factors and their interaction. In recent years, interest rates have decreased considerably across the world, and the real probability measure—used to produce stochastic simulations of insurance quantities and for hedging purposes—has to take this evolution into account. Regarding hedge considerations, in this environment life insurers that typically used fixed income to hedge interest rate guarantees have often appeared only partially hedged. In the past, many life insurers used hedging strategies such as duration matching. These techniques generally work well when interest rates are stable but not when there is a large change in rates such as the sustained decrease that occurred after the 2008 financial crisis. Another important factor in evaluating interest rate risk in the real world is that life insurers can be exposed to risk through the behavior of policyholders, especially through products with guaranteed returns. Some insurance products offer policyholders the option of contributing additional funds at their discretion or of partially or totally surrendering the contract. When interest rates change, policyholders are more likely to act on these options. The interaction of guarantees and policyholder behavior can make hedging interest rate risk much more complex.

An important tool to manage this interaction is the implementation of an accurate asset-liability model. The widespread presence of guarantees and embedded options have to be taken into account in ALM analyses, which need to estimate the medium- and long-term development of all assets and liabilities, as well as the interactions between them and the different types of financial and demographic risks. This aim can be achieved by performing stochastic simulations of ALM models. In the actuarial literature, much effort has been given to developing such models using risk neutral probability measures (see Grosen and Jorgensen 2000). Most authors focus on the fair valuation and contract design of unit-linked and participating life insurance policies. Exceptions were offered by Kling et al. (2007), who analyzed the financial risks and returns of participating policies using real-world probability measures.

1.4 ACTUARIAL GUIDELINE 43

Actuarial Guideline 43 (AG43) offers standards for valuing the reserves for variable annuities and other contracts involving certain guaranteed benefits similar to those offered with VAs. The guideline addresses these issues by including an approach that applies principles of asset adequacy analysis directly to the risks associated with the products and guarantees.

The guideline requires that reserves for contracts falling within its scope be based on the greater of a minimum floor determined using a standard scenario—referred to as the Standard Scenario Amount (SSA)—or a reserve calculated using a projection of the assets and estimated liabilities that support these contracts over a broad range of stochastically generated projection scenarios with prudent best estimate assumptions—referred to as the Conditional Tail Expectation (CTE) Amount. Within each of these scenarios, the greatest present value of accumulated losses, irrespective of federal income tax, is determined. The reserve calculated using projections is based on a CTE measure of the results for each scenario.

The objective of the approach used to determine the CTE Amount is to quantify the amount of statutory reserves the insurer needs to meet contractual obligations in light of the risks to which the company is exposed.

The calculation of the CTE Amount is based on the results of an analysis of asset and liability cash flows produced by the application of a stochastic cash flow model to equity return and interest rate scenarios. For each scenario, the greatest present value of accumulated surplus deficiency is calculated. The analysis reflects prudent best estimate assumptions for deterministic variables and is performed in aggregate to allow the natural offset of risks within a given scenario. The methodology uses a projected total statutory balance sheet approach by including all projected income, benefit and expense.

1.4.1 STATUTORY RESERVE CALCULATION

According to AG43, the statutory reserve is the Working Reserve based on the cash surrender value.

The Working Reserve is the assumed reserve used in the projections of accumulated deficiencies supporting the calculation on a "greatest of present values" basis. At any point in the projections, including at the beginning, the Working Reserve equals the projected cash surrender value. For a variable payout annuity without a cash surrender value, the Working Reserve equals the present value at the valuation interest rate and the valuation mortality table specified for such a product by the Standard Valuation Law of future income payments projected using a return based on the valuation interest rate less appropriate asset-based charges.

For purposes of the guideline, the cash surrender value for a contract is the amount available to the contractholder upon surrender of the contract. Generally, it is equal to the account value less any applicable surrender charges, where the surrender charge reflects the availability of any free partial surrender options. For contracts where all or a portion of the amount available to the contractholder upon surrender is subject to a market value adjustment, however, the cash surrender value reflects the market value adjustment consistent with the required treatment of the underlying assets. For this, the cash surrender value must reflect any market value adjustments in which the underlying assets are reported at market value but not those reported at book value.

1.4.2 CONDITIONAL TAIL EXPECTATION

The reserve calculated using projections is based on a CTE measure of the results for each scenario. Conditional Tail Expectation is a statistical risk measure that provides enhanced information about the tail of a distribution beyond that provided by the traditional use of percentiles. Instead of identifying a value only at a particular percentile and thus ignoring the possibility of extremely large values in the tail, CTE recognizes a portion of the tail by providing the average overall values in the tail beyond the CTE percentile. According to AG43 (NAIC 2006), the CTE Amount is equal to the numerical average of the 30% largest values of the scenario. This is CTE(70).

1.4.3 ACCUMULATED DEFICIENCY AND DISTRIBUTABLE EARNINGS

Accumulated deficiency is an amount measured at the end of a projection year and equals the projected Working Reserve less the amount of projected assets from the valuation date to the end of the year. Accumulated deficiencies may be positive or negative. A negative accumulated deficiency means a positive surplus.

Distributable earnings equal the amount of a negative accumulated deficiency less the solvency capital margin. The solvency capital requirement is set at 6% of reserves.

Section 2: The Model

2.1 THE AIM

The aim of this paper is to define a rule for investment/reinvestment strategies in low interest environment for life insurance products with long duration and guarantees. In particular, if interest rates are not sufficient to cover the guarantees, this has a strong impact on insurers' balance sheets, and they have to resort to their own capital for reserves. Here, we evaluate the investment strategy at the beginning of each year in terms of the optimal portfolio weights by looking at the stochastic evolution of the prior year's cash flows. Despite the common approach to this kind of problem, we do not evaluate cash flows during the whole of the contract, thus avoiding the choice of a single discounted rate. In the context of stochastic evolution of interest rates, choosing a constant discounted rate deeply shows its weakness, because it does not allow for the interest rate sensitivity of these cash flows and the tail distribution of the results. Our research explicitly evaluates this sensitivity and tail distribution to inform the reinvestment strategy choice.

In light of these considerations, we propose a dynamic strategy by which we evaluate how to modify the investment strategy at the beginning of each year based on the results realized during the prior year. The traditional approach to investment and reinvestment strategies is to start with static asset class weights that remain constant for the entire projection with the duration matched at time zero. One way to improve this approach is to have the weights shift over time to continuously match duration. An even more advanced approach, which is the aim of this model, is to select the investment/reinvestment strategy dynamically, which involves continual matching to criteria other than just duration. To define the optimization problem, we need to select the objective function. In the following discussion, we assume that our aim is to maximize the expected value of distributable earnings by looking at their variance and CTE. We evaluate statutory reserve in terms of Working Reserve as required by AG43; the projected cash flows achieved on the asset side; and the difference in terms of accumulated deficiency and its mean and dispersion under different strategies.

2.2 MATHEMATICAL FORMULATION

Let $\pi_t = (\pi_t^1, \pi_t^2, \dots, \pi_t^j)$ be the weights of the portfolio composed by *j* asset classes with the following statutory bounds: $\pi_t^1 \leq b^i$. Let W_0 be the premium paid by the policyholder and invested in the investment portfolio after the initial expenses. Let A_t be the value of assets at the end of the year t, with $A_0 = W_0$, and let R_t be the statutory reserve at the end of the year t influenced by $g(t)$, the amount guaranteed at t. Starting from t = 0, we simulate the evolutionary path of assets and contractual obligations, considering the interaction of financial, demographic and behavioral factors. For each path at the end of each year, we calculate the statutory reserve needed to meet future contractual obligations, according to the statutory prescriptions, and evaluate the investment portfolio. Based on the investment returns achieved on the assets, we calculate the accumulated deficiency.

We define the optimization problem as follows:

$$\begin{cases} max_{\pi_t} \quad (E[A_{t^+} - R_{t^+}] | SD[A_{t^+} - R_{t^+}]) \text{ for each } t \\ \pi_t^i \leq b_i \\ \pi_t^1 + \pi_t^2 + \cdots + \pi_t^j = 1 \end{cases}$$

where the decision of a target allocation is made at the beginning of each year t based on the simulated value of assets and reserve at the end of the same period t^+. In this formulation, the model produces different results depending on how the *standard deviation (SD)* is constrained—for example, fixing a given level of risk the insurer is willing to assume. The problem can be standardized using the following formula:

$$\begin{cases} max_{\pi_t} \quad E[A_{t^+} - R_{t^+}]/SD[A_{t^+} - R_{t^+}] \text{for each } t \\ \pi_t^i \leq b_i \\ \pi_t^1 + \pi_t^2 + \cdots + \pi_t^j = 1 \end{cases}$$

There is an alternative optimization formula as well:

$$\begin{cases} max_{\pi_t}(EA_{t^+} - R_{t^+} - \mathrm{CTE}(70)_{t^+}) \; \forall t \\ \pi_t^i \leq b_i \\ \pi_t^1 + \pi_t^2 + \cdots + \pi_t^j = 1 \end{cases}$$

where $\mathrm{CTE}_{(70)}$ is the CTE of the simulated distribution of the accumulated deficiency.

This model is flexible and can be modified to meet specific needs. The central idea is that the strategy is dynamic based on the results obtained during the year for whatever formula one decides to maximize.

2.2.1 CASH FLOW ANALYSIS IN AN ASSET-LIABILITY MANAGEMENT FRAMEWORK

The cash flows associated with A_t and R_t are simulated across different scenarios.

The Society of Actuaries defines ALM as "the ongoing process of formulating, implementing, monitoring and revising strategies related to assets and liabilities to achieve an organization's financial objectives, given the organization's risk tolerances and other constraints."

The aim of ALM is to manage the investment portfolio in a risk efficient manner to meet estimated insurance obligation cash flows and taking into account guarantees, embedded options, policyholder behavior and mortality factors. From this perspective, the available capital has to be invested profitably using asset management strategies and the obligations assumed must be met.

Following the ALM models, we project asset and liability cash flows based on financial and demographic assumptions at the valuation date. The model takes into account the interaction of the following variables:

1. Financial variables: interest rate and return on other investments
2. Demographic variables: lapse and death
3. Investment choices by management

2.2.2 RISK MEASURE

The function of the maximization problem can be defined according to the strategic asset allocation goals. Our model deals with the maximization of the expected value of distributable earnings, taking into account the dispersion of the simulated distribution. Thus we try to maximize the expected value for a given standard deviation. Another way to allow for the dispersion could be to maximize the expected value minus the expected loss in the tail (the CTE).

2.3 FINANCIAL SCENARIO GENERATOR

The financial variables in this model are simulated using the Financial Scenario Generator Version 7.1.201805 developed by the American Academy of Actuaries and available on the SOA website (https://www.soa.org/tables-calcs-tools/research-scenario). The generator produces scenarios for the future paths of interest rates for U.S. Treasury securities and several kinds of investment portfolios, including equity and fixed-income portfolios. Interest rate shocks are independent of equity returns, while bond index returns are modeled as a function of interest rate.

The U.S. Treasury yields are generated using the C-3 Phase I interest rate model designed by the American Academy of Actuaries. The model simulates Treasury bond yields according to a stochastic variance process with mean reversion under the real-world probability measure. This report refers to "Appendix III: Technical Aspects of the Scenario Generator and the Scenario Selection Process" in the Academy's Phase I report (American Academy of Actuaries 1999).

The Academy's interest rate generator is based on a stochastic process that defines interest rates for 20-year maturities and 1-year maturities. An additional process must be defined to derive the other eight points on the yield curve included in the generator output. Version 7 of the generator uses a process based on a Nelson-Siegel formula that uses four parameters: the long-term rate, the excess of the instantaneous short rate over the long-term rate (normally negative), the size of the hump in the yield curve and the curvature and location of the hump. The Nelson-Siegel parameters are fitted to the yield curve.

The mean reversion point for the 20-year Treasury bond rate is calculated dynamically, based on historical interest rates as they emerge. The formula for the dynamic mean reversion point has been defined by the National Association of Insurance Commissioners (NAIC; American Academy of Actuaries 2010).

The equity return scenarios are generated from a monthly stochastic local volatility (SLV) model in which the natural logarithm of the annualized volatility follows a strong mean-reverting stochastic process, and the annualized drift is a deterministic quadratic function of volatility. This model is able to capture many of the dynamics observed in the equity market data: the negative skewness and positive kurtosis ("fat tails") over short holding periods; the time-varying volatility and volatility clustering; and the increased volatility in bear markets.

The monthly SLV model is governed by the following equations:

$$\tilde{v}(t) = Min\left[v^{+}, (1-\phi)\times v(t-1) + \phi \times \ln(\tau)\right] + \sigma_v \times {}_vZ_t$$

$$v(t) = Max\left\{v^{-}, Min\left[v^{*}, \tilde{v}(t)\right]\right\}$$

$$\mu(t) = A + B\times\sigma(t) + C\times\sigma^2(t)$$

$$\ln\left[\frac{S(t)}{S(t-1)}\right] = \frac{\mu(t)}{12} + \frac{\sigma(t)}{\sqrt{12}} \times {}_sZ_t$$

$$S(t) = \text{stock index level at time } t$$

$$v(t) = \text{natural logarithm of annualized volatility in month } t$$

$$\sigma(t) = \text{annualized volatility of stock return process in month } t = \exp\left[v(t)\right]$$

$$\mu(t) = \text{mean annualized log return ("drift") in month } t$$

$$v^{-} = \text{lower bound for log volatility} = \ln\sigma^{-}$$

$$v^{+} = \text{upper bound for log volatility (before random component)} = \ln\sigma^{+}$$

$$v^{*} = \text{absolute upper bound for log volatility} = \ln\sigma^{*}$$

where vZ_t and sZ_t are correlated normal processes. (To deepen the construction of the financial model, see American Academy of Actuaries 2006.)

2.4 DEMOGRAPHIC SCENARIO

Every year an insurer has to consider whether a contract is still in force or the policyholder has died or surrendered. For mortality, taking a deterministic approach is justified by the fact that mortality risk can be diversified in a large portfolio; it is prudent to use appropriate projected mortality tables.

With regard to the choice of lapse rate, the risk is not fully diversifiable. Both academics and practitioners have tried to explain and model policyholder behavior and the factors that drive the choice of whether to lapse or not. From a financial point of view, during a period of decreasing markets, the value of the underlying fund will decrease and the economic value of the guarantee will rise, giving the policyholder incentive not to exercise the surrender option. In reality, insurance companies usually do not assume such behavior for all policyholders, because some of them may not be rational or well informed, they may not know about the economic value of the guarantee, or exogenous factors such as a need for liquidity may drive their actions. Wrong hypotheses on lapse rate could have a strong impact on the sustainability of the VA market and insurers' financial stability. For example, White Mountain Life Re experienced a loss in 2010 due to a reduction in the surrender assumptions used to calculate its variable annuity guarantee liability. The change in the surrender assumptions increased the company's liability by $48 million.

In the literature, some authors assume that fees are collected from some policyholders who surrender their policies prior to maturity without reason. Swiss Re (2003) noted that lapse rate is higher and more volatile for unit-linked life insurance contracts than for other traditional insurance products.

Let $l(t)$ denote the probability of the policyholder not having lapsed at anniversary t. This probability may reflect the historical experience of the insurer or the market average. A survey conducted by the Society of Actuaries (2011) showed that "company experience studies continue to be the most popular source of lapse assumptions."

Section 3: Numerical Application

3.1 PRODUCT DESCRIPTION

In recent years, insurers have introduced variable annuity products with guaranteed options. They can be broken down into fixed annuity plus variable annuity. Part of the premium is invested in a fixed account, and the excess goes into a separate variable account. In our numerical application, we take into account a variable annuity with a Guaranteed Minimum Death Benefit with the following features summarized in Table 1:

Table 1: Hypothesis on the Variable Annuity

Policyholder	U.S. male, 50 years old	
Projection period	30 years? 40 years?	
Premium	10,000	
Percentage of investment in fixed account	20%	
Fund chosen for the variable account	Balanced fund	
Separate account fund expense	0.01	
Fixed account credited rate formula	Min (New Money Rate – Base Expense Margin + Inversion Adjustment; 0%)	
Base expense margin	0.024	
Inversion adjustment	0.25% when the 10-year Treasury is less than the 2-year Treasury and the 10-year Treasury is less than 5.00%; otherwise, 0.00%	
Guaranteed rate	0.01	
Administrative fee (on VA)	0.0015	
Mortality and expense (on VA)	0.0125	
Fixed account credited rate	Set to the maximum of the rate determined by the fixed account credited rate formula and the guaranteed interest rate; reset on each policy anniversary	
GMDB	0.04	
Surrender charges	1 year	7%
	2 years	6%
	3 years	5%
	4 years	4%
	5 years	3%
	6 years	2%
	7 years	1%
	8 years+	0
Annuitization	No annuitization assumed	
Partial withdrawal	No partial withdrawal assumed	

LAPSE RATE				1 year	2 years		
				0.007	0.0142		
3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
0.0214	0.0286	0.0358	0.043	0.05	0.22	0.15	0.05
	6% of the statutory reserve						

statutory capital requirement	

The lapse assumption is given by the median base lapse rate for GMDB according to the Society of Actuaries' (2016) survey of assumptions of policyholder behavior.

3.2 DYNAMIC REINVESTMENT STRATEGIES

3.2.1 ASSET UNIVERSE DESCRIPTION

The variables generated by the Financial Scenario Generator are summarized in Table 2, while Table 3 provides the composition of each asset class. Parameters are calculated by fitting the models to historical data.

Table 2: Asset Classes in the Academy's Interest Rate Generator

Asset Class	Short Name	Scenario File
3-month U.S. Treasury yields	UST_3m	UST_3m.csv
6-month U.S. Treasury yields	UST_6m	UST_6m.csv
1-year U.S. Treasury yields	UST_1y	UST_1y.csv
2-year U.S. Treasury yields	UST_2y	UST_2y.csv
3-year U.S. Treasury yields	UST_3y	UST_3y.csv
5-year U.S. Treasury yields	UST_5y	UST_5y.csv
7-year U.S. Treasury yields	UST_7y	UST_7y.csv
10-year U.S. Treasury yields	UST_10y	UST_10y.csv
20-year U.S. Treasury yields	UST_20y	UST_20y.csv
30-year U.S. Treasury yields	UST_30y	UST_30y.csv
Money Market / Short-Term	MONEY	MONEY.csv
U.S. Intermediate Term Government Bonds	U.S. ITGVT	ITGVT.csv
U.S. Long Term Corporate Bonds	U.S. LTCORP	LTCORP.csv
Diversified Fixed Income	FIXED	FIXED.csv
Diversified Balanced Allocation	BALANCED	BALANCED.csv
Diversified Large Capitalized U.S. Equity	US	US.csv
Diversified International Equity	INTL	INTL.csv
Intermediate Risk Equity	SMALL	SMALL.csv
Aggressive or Specialized Equity	AGGR	AGGR.csv

Table 3: Composition and Historical Window of Each Asset Class

Asset Class	Market Proxies	Historic Period
Money Market	3 Month Treasury returns	1955.12 – 2003.12
U.S. ITGVT	U.S. Intermediate Term Government Bonds	1955.12 – 2003.12
U.S. LTCORP	U.S. Long Term Corporate Bonds	1955.12 – 2003.12
Fixed Income	65% ITGVT + 35% LTCORP	n/a
Balanced Allocation	60% Diversified Equity + 40% Fixed Income	n/a
Diversified Large Cap U.S. Equity	S&P500 Total Return Index	1955.12 – 2003.12
Diversified International Equity	MSCI-EAFE $USD Total Return Index	1969.12 – 2003.12
Intermediate Risk Equity	U.S. Small Capitalization Index	1955.12 – 2003.12
Aggressive Equity	Emerging Markets, NASDAQ, Hang Seng	1984.12 – 2003.12

3.2.2 THE FINANCIAL PARAMETERS

Using the ALM framework, the maximization model is implemented through two different financial sets of stochastic scenarios. In the first, the mean reversion target of the interest rates is derived from the December 2016 yield curve, while in the second, the reference point for the interest rate curve is December 2000 before the financial crisis. In a sustained low interest rate environment, the first set of scenarios might represent the company's prudential outlook based on continuation of the current climate, while the second might consider the impact of interest rates rising again in the near future. Table 4 gives the Financial Scenario Generator's parameters for the two different sets of scenarios.

Table 4: The Two Financial Sets of Stochastic Scenarios for the Interest Rate

Starting date:	201612	Starting date:	200012
Yield curve on starting date:		Yield curve on starting date:	
3 Month	0,51%	3 Month	5,89%
6 Month	0,62%	6 Month	5,70%
1 Years	0,85%	1 Years	5,32%
2 Years	1,20%	2 Years	5,11%
3 Years	1,47%	3 Years	5,06%
5 Years	1,93%	5 Years	4,99%
7 Years	2,25%	7 Years	5,16%
10 Years	2,45%	10 Years	5,12%
20 Years	2,79%	20 Years	5,59%
30 Years	3,06%	30 Years	5,46%
Mean reversion to:	3,75%	Mean reversion to:	6,50%

The U.S. Treasury yields are generated using the C-3 Phase I interest rate model designed by the American Academy of Actuaries. The model simulates Treasury bond yields according to a stochastic variance process with mean reversion under the real-world probability measure. (For full details, see American Academy of Actuaries 1999). Interest rate movements are not correlated with other model factors. The interest rate model is designed for cash flow projections only: it is not arbitrage-free and may give inappropriate values if used to price options and other derivatives as part of an asset-liability management strategy.

The parameters of the SLV model for each equity market are given in Table 5. The estimated correlation matrix (derived from American Academy of Actuaries 2006) is given in Table 6.

Table 5: Estimated Parameters of the SLV Process for Equity Markets

	Description	U.S.	INTL	SMALL	AGGR
τ	Long-run target volatility (annualized)	0.12515	0.14506	0.16341	0.20201
ϕ	Strength of mean reversion	0.35229	0.41676	0.3632	0.35277
σ_v	Standard deviation of the log volatility process (monthly)	0.32645	0.32634	0.35789	0.34302
ρ	Correlation co-efficient between ${}_vZ_t$, ${}_sZ_t$	−0.2488	−0.1572	−0.2756	−0.2843
A	Drift of stock return process as $\sigma(t) \to 0$ (i.e., intercept)	0.055	0.055	0.055	0.055
B	Co-efficient of quadratic function for $\mu(t)$	0.56	0.466	0.67	0.715
C	Co-efficient of quadratic function for $\mu(t)$	−0.9	−0.9	−0.95	−1
$\sigma(0)$	Starting volatility (annualized)	0.1476	0.1688	0.2049	0.2496
σ^-	Minimum volatility (annualized)	0.0305	0.0354	0.0403	0.0492
σ^+	Maximum volatility (annualized), before random component	0.3	0.3	0.4	0.55
σ^*	Maximum volatility (annualized), after random component	0.7988	0.4519	0.9463	1.1387

Table 6: The Estimated Correlation Matrix

	US LogVol	US LogRet	INTL LogVol	INTL LogRet	SMALL LogVol	SMALL LogRet	AGGR LogVol	AGGR LogRet	MONEY Return	ITGVT Return	LTCORP Return
US LogVol	1	−0.249	0.318	−0.082	0.625	−0.169	0.309	−0.183	0.023	0.075	0.080
US LogRet	−0.249	1	−0.046	0.630	−0.123	0.829	−0.136	0.665	−0.120	0.192	0.393
INTL LogVol	0.318	−0.046	1	−0.157	0.259	−0.050	0.236	−0.074	−0.066	0.034	0.044
INTL LogRet	−0.082	0.630	−0.157	1	−0.063	0.515	−0.098	0.558	−0.105	0.130	0.234
SMALL LogVol	0.625	−0.123	0.259	−0.063	1	−0.276	0.377	−0.180	0.034	0.028	0.054
SMALL LogRet	−0.169	0.829	−0.050	0.515	−0.276	1	−0.142	0.649	−0.106	0.067	0.267
AGGR LogVol	0.309	−0.136	0.236	−0.098	0.377	−0.142	1	−0.284	0.026	0.006	0.045
AGGR LogRet	−0.183	0.665	−0.074	0.558	−0.180	0.649	−0.284	1	0.034	−0.091	−0.002
MONEY Return	0.023	−0.120	−0.066	−0.105	0.034	−0.106	0.026	0.034	1	0.047	−0.028
ITGVT Return	0.075	0.192	0.034	0.130	0.028	0.067	0.006	−0.091	0.047	1	0.697
LTCORP Return	0.080	0.393	0.044	0.234	0.054	0.267	0.045	−0.002	-0.028	0.697	1

3.2.3 INVESTMENT CONSTRAINTS AND STRATEGIES

The dynamic asset allocation can be formulated in terms of types of investment strategies or in terms of parameters, taking regulatory constraints into account.

At t = 0, we evaluate different strategies in terms of proportion of investment in bond and equity and choose to follow a more or less aggressive strategy. Once the strategy has been chosen, the assets and liabilities are simulated, and the values of distributable earnings at the end of the first year are collected for each path. The simulations are repeated for both sets of financial scenarios. At the end of the year, a reinvestment strategy is defined. We assume that when distributable earnings are positive, they are not actually distributed but are invested in additional assets according to the following reinvestment strategies.

(A) Base strategy:

Assets are only bought/sold when there are net positive/negative distributable earnings at the end of the year, mantaining the same proportion of assets as the initial strategy. For example, if at the end of the year, a net positive cash flow occurs, it is invested in the initial portfolio so the investment weight of each asset does not change but the total amount invested in each asset increases.

A number of alternative investment strategies were also considered:

(B) Base strategy + shift to aggressive equity fund when distributable earnings are positive:

When distributable earnings are positive, assets are bought and the equity investment profile becomes aggressive; when distributable earnings are negative, assets are sold to maintain the same proportion of assets as in the initial strategy.

(C) Base strategy + shift to balanced equity fund when distributable earnings are negative:

When distributable earnings are negative, assets are sold and the equity investment profile becomes balanced; when distributable earnings are positive, assets are bought to maintain the same proportion of assets as in the initial strategy.

(D) Combination of strategies B and C:

When distributable earnings are positive, assets are bought and the equity investment profile becomes aggressive, while when distributable earnings are negative, assets are sold and the equity investment profile becomes balanced.

(E) Base strategy + interest rate swap:

The company hedges the U.S. 10-year Treasury assets, paying the U.S. 10-year swap rate (historical value at December 2016), and receives the variable U.S. 10-year interest rate.

The costs of rebalancing are ignored. In practice, other more sophisticated investment strategies could be implemented, but we have limited this example to these five for illustrative reasons. The model can be implemented iteratively, starting with a range of investment strategies and then varying the parameters of these strategies to determine the combination that gives the optimal result.

3.3 RESULTS

The first step of the numerical application is to generate 10,000 paths of the asset-liability cash flows for both scenarios considered. At t = 0, the insurer chooses the first allocation and calculates the distributable earnings for each of the selected scenarios at the end of the first year. In the second step, at the end of the first year, the insurer uses the results obtained for each path to simulate the distributable earnings at the end of the following year and chooses the reinvestment strategy (A through E) that will produce the best results. The reserves are modeled according to the investment choice made by the policyholder, while the assets are modeled according to the allocation selected by the insurer.

On the liability side, the insurer offers the policyholder an opportunity to choose the risk/return investment profile of the separate account, and we assume that the policyholder decides to link the separate account to a diversified, balanced allocation portfolio. We simulate the liability cash flows of both fixed and variable accounts, taking into account the features of the product described in Table 1 as well as the guarantees, the investment choice of the policyholder, the stochastic occurence of mortality and the lapse.

On the asset side, we assume that at the inception of the contract, four different explanatory investment strategies vary the proportion of investment between bonds and equity according to the parameters shown in Table 7. When compared to U.S. Treasury bonds with 2- and 10-year maturities, the equity asset class is the riskiest. At $t = 0$, the insurer chooses the first allocation between the following strategies I -IV.

Table 7 Initial explanatory possible strategies

	STR I	STR II	STR III	STR IV
Cash	5%	5%	5%	5%
U.S. Treasure (2 y)	30%	30%	30%	30%
U.S. Treasury (10 y)	60%	50%	40%	30%
INT.R EQUITY	5%	15%	25%	35%

Starting from each of the described strategy, the expected value and deviations of the simulated distributable earnings at the end of the first year are summarized in Table 8.

Table 8: Expected Value and Standard Deviations of Simulated Distributable Earnings for Each Strategy I-IV

	Scenario 2000				Scenario 2016			
	STR I	STR II	STR III	STR IV	STR I	STR II	STR III	STR IV
Mean	570.6482	613.2906	655.933	698.5754	323.294	392.6313	461.9686	531.3059
SD	812.9938	854.2247	932.6598	1039.915	784.647	827.3418	908.1488	1018.032
Mean/SD	0.70191	0.71795	0.703293	0.671762	0.412025	0.47457	0.508693	0.521895

Under the hypothesis of scenario 2016, according to which the interest rates will remain low in the future, starting strategy IV has the greatest mean/SD, while under the hypothesis of scenario 2000, strategy II dominates the others. Remember that scenario 2000 means higher interest rates and scenario 2016 means low interest rates.

Considering that the second strategy is more sensitive to the changes in interest rates that represent the focus of this research and also is more realistic given the real constraints to asset allocation, we decide to implement the second strategy at the inception of the contract. Following this initial strategy, we follow the evolutionary paths of the assets and liabilities under both scenarios and evaluate the distributable earnings at the end of the first year. For each path of each scenario, starting with the results obtained from the first year, we implement reinvestment strategies A through E, and evaluate the distributable earnings at the end of the second year. The results are shown in Table 9.

Table 9: Distributable earning at the end of the second year for each reinvestment strategy A-E

	Scenario 2000					Scenario 2016				
	A	B	C	D	E	A	B	C	D	E
Mean	1362.284	1385.658	1356.734	1380.109	1585.539	704.3639	724.0884	691.257	710.9815	742.61
SD	2008.662	2037.038	2011.46	2039.86	2026.325	1907.988	1934.946	1915.90	1942.883	1911.5
Mean/SD	0.678204	0.680232	0.674502	0.67657	0.78247	0.369166	0.374216	0.3608	0.365942	0.3884
CTE	–915.355	–930.932	–927.268	–941.064	–707.907	–1456.91	–1465.31	–1493.2	–1500.89	–1426.4
E-|CTE|	446.9291	454.7257	429.4663	439.0444	877.6319	–752.541	–741.222	–802.00	–789.912	–683.8

Under both the scenarios, strategy E produces the best results in terms of the expected mean of distributable earnings per unit of risk measured by the standard deviation and the expected mean of distributable earnings minus the expected loss in the tail according to the requirements of AG43 (CTE).

Section 4: Conclusions

In a changing economic environment, life insurers are facing new challenges in asset-liability management. The long-term duration of liabilities and new products with guarantees and variable features have a strong impact on reserve calculations and solvency capital assessments. Insurers need to reformulate their strategic asset allocation to account for the increasing interest risk and its interaction with other financial and demographic risks.

Taking into account the statutory requirements for products with variable accounts and guarantees, this paper has proposed a dynamic strategy for maximizing distributable earnings year by year. The risk measure introduced conforms to the statutory requirements. We have analyzed the mean of distributable earnings and the CTE for two complementary metrics using the maximization formula; however, the model appears flexible, and other risk measures can easily be introduced. The stochastic simulation of the statutory reserve and investment portfolio permits us to consider the complex interaction of assets and liabilities, taking into account the relationship between financial, demographic and behavioral factors.

The results given in this report are for explicative purposes only. The model is simplified: no transition costs are included, the solvency capital requirement is set to 6% of the statutory reserve, and and the cash flows are generated based on just one policy issued and not a diversified insurance portfolio. The examples shown illustrate the techniques rather than providing results for a specific situation. More realistic strategies might be considered and different constraints introduced; other investment strategies may be optimal under different scenarios. Moreover, the introduction of a dynamic lapse formula offers inspiration for further research. The point appears interesting and debatable at the same time: both academics and practitioners are trying to explain and model policyholder behavior and the factors motivating the choice to let a policy lapse. From a financial perspective, during a period of rising markets, the value of the underlying fund will increase, and the economic value of the guarantee will fall, giving the policyholder a potential incentive to exercise the surrender option. In reality, insurance companies typically do not assume this behavior for all policyholders because some may not be rational or informed. They may not fully understand the economic value of the guarantee, or they may be motivated by exogenous factors such as a need for liquidity. A policyholder may exhibit suboptimal behavior as described by Piscopo and Ruede (2016).

The model described in this paper can offer interesting ideas for a company’s asset-liability management, providing a way to value and solve the optimization problems associated with choosing investment strategies. It can help to inform investment management decisions to identify the best investment strategies for the company on a case-by-case basis.

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