Modeling Capital Market with Financial Signal Processing

Jenher Jeng
Ph.D., Statistics, U.C. Berkeley
Founder & CTO
of
Harmonic Financial Engineering,
www.harmonicfinance.com
Outline

• Theory and Techniques
  – Theoretic Framework of Modeling Capital Markets: Index-Based Composition Methodology
  – Statistical Procedure of Model Construction and Extension: Wavelets-Based Financial Signal Processing Technique

• Implications and Applications (ex. S&P 500)
  – Measuring Market Uncertainty and Volatility
  – Formatting Dynamic Strategies into Strategic Curves: Adaptive Futures Leveraging & Efficient Options Pricing
  – Monitoring Impacts of Smart-Money Timing Strategies
  – Gauging Cyclic Structure & Forecasting Market Crises Converging Patterns towards Market Crashes and Bubbles
Theoretic Framework of Modeling Capital Markets

**Index-Based Composition Methodology**

\[
R_{i+1} = r_i + \Psi(S_i) + \Sigma(S_i) \cdot \epsilon_{i+1}, \quad \epsilon_i's \text{ i.i.d. } \sim \mathcal{N}(0,1).
\]

- **Strategic Index**
  \[
  S_i = \Gamma(R_{i-L+1}, ..., R_i)
  \]

- **Static Regression**
  \[
  R_{i+1} = r_i + \mu(R_{i-L+1}, ..., R_i) + \sigma(R_{i-L+1}, ..., R_i) \cdot \epsilon_{i+1},
  \]
  \[
  \mu = \Psi \circ \Gamma; \quad \sigma = \Sigma \circ \Gamma
  \]

- **Dynamic Auto-Regression**

- **Non-stationary Correlation**

- **Noise barrier**

- **Dimension Curse**

- **Note:**
  - \( R_{i+1} \): (i+1)-th periodical short-term market return rate - say, S&P 500 monthly;
  - \( r_i \): i-th periodical average short-term interest rate - say, FFR monthly average;
  - \( S_i \): i-th periodically updated strategic index value – say, STTB Index, shown next
Mission Impossible to De-noise through Dimension Curse

Piecewise (Monthly) Constant Geometric Brownian Motion

\[ \frac{dX_t}{X_t} = r_i + \mu_i + \sigma_i \cdot dW_t, \quad \text{for } t_{i-1} \leq t < t_i \]

Key of Long-Term Consistent Profitability: Low-Frequency Component of Market Fluctuation

Patterns about Interactions between \(\mu_i\)'s and \(\sigma_i\)'s on Time-Domain

Non-Stationarity & High-Frequency Noise-Barrier

Knowledge in \(\Psi\) and \(\Sigma\) on S-Domain
Strategic Index

Short-Term Trend Bias (STTB)

Γ (non-parametric statistic)

A Typical Series of Patterns for Illustrating the Point of STTB

<table>
<thead>
<tr>
<th>Month</th>
<th>STTB</th>
</tr>
</thead>
<tbody>
<tr>
<td>09/2002</td>
<td>2.9143</td>
</tr>
<tr>
<td>10/2002</td>
<td>2.5357</td>
</tr>
<tr>
<td>11/2002</td>
<td>2.2032</td>
</tr>
<tr>
<td>12/2002</td>
<td>2.0318</td>
</tr>
<tr>
<td>01/2003</td>
<td>1.8631</td>
</tr>
<tr>
<td>02/2003</td>
<td>1.9359</td>
</tr>
<tr>
<td>03/2003</td>
<td>1.3931</td>
</tr>
</tbody>
</table>
Distribution of STTB

Histogram of Monthly STTB from 01/51 to 02/03
Gauging the Structure of a Market Cycle

higher Red up, steeper Blue down; longer Red stays up, deeper Blue sinks down; vice versa
Red Series (MA_STTB) is leading Blue Series in turnaround in a smooth conclusive way.
The Fundamental Model – Quantitative Psychological Model

Parametric Model: Linear Heteroscedastic Parabolic Model

\[ R_{i+1} - r_i = \Psi(S_i) + \Sigma(S_i) \cdot \varepsilon_{i+1}, \]
\[ \Psi(S) = k \cdot (S-a)^2 + b; \quad \Sigma(S) = c \cdot S + d, \text{ for } 1 \leq S < 3. \]

- **a** – Maximum Uncertainty Level: MLE = 2.0092
- **b** – Uncertainty Aversion Rate: MLE = -0.0014
- **k** – Rational Confidence Coefficient: MLE = 0.0107
- **c** – Stability Coefficient: MLE = 0.0096
- **d** – Efficient Market Volatility: MLE = 0.0230

*** MLE results are based on S&P 500 monthly data from 01/1951 to 02/2003 ***

Market Volatility = Dynamic (Low-Freq.: Uncertainty) + Stochastic (High-Freq.: Stability)
Basic Structure of Shaping Dynamic Investment Strategies over Domain of Strategic Index

**Strategic Curve**

- **Action Parameter (for Investment Decision-Making)**
- **Shape of Strategic Curve**
- **Goal of the Strategy**
- **Knowledge about the Market**

**Strategic Index S (e.g. STTB)**

**Elementary Examples**

- **Future Leveraging Strategy** –
  maximizing cumulative return of a simple portfolio combining **S&P 500 Stock Index Future and Cash** (leverage-multiple of the total invested capital)

  \[ \Theta(S) = \frac{\Psi(S)}{\Sigma^2(S)} \]

- **Option Pricing Strategy** –
  fairly pricing the value of a **One-Month At-The-Money Call** contract (as a fraction of the current value of the underlying asset)

  \[ \Theta(S) = \Phi[(r/\Sigma^2 + \Psi/\Sigma^2 + 1/2) \cdot \Sigma] - e^{(r+\Psi)} \cdot \Phi[(r/\Sigma^2 + \Psi/\Sigma^2 - 1/2) \cdot \Sigma] \]
Efficient Options Pricing

Black-Scholes Model

\[ R_{i+1} = r_i + \mu + \sigma \cdot \epsilon_{i+1} \]

L.H.P. Composite Model

\[ R_{i+1} = r_i + \psi(S_i) + \sum(S_i) \cdot \epsilon_{i+1} \]

Expected Return = \( r + \mu \)?; Volatility = \( \sigma \)?

Predict Expected Return & Volatility by Interest Rate and STTB
Adaptive Leveraging Strategy for S&P 500 Future
in comparison with one via model-free simulation

maximizing cumulative return without risk control

The Mystery of The Missing Bump?
The Complex
extending knowledge beyond the psychological factor

Complex Additive Model

\[ R_{i+1} = [\Psi(S_i) + \Delta(S_i) + \Omega(S_i)] + \Sigma(S_i) \cdot \epsilon_{i+1}, \quad \epsilon_i \text{'s i.i.d. } \sim \mathcal{N}(0,1). \]

• Psychological Factor
  Rationality-Oriented, such as Uncertainty, Momentum
  \( \Psi, \text{ Smooth Curve} \)

• Strategical Factor
  Discipline-Oriented, such as Contrarian, Hedge Fund Arbitrage
  \( \Delta = \Delta_0 + \Delta_1, \Delta_0, \text{ Concentrated} \rightarrow \text{Missing Bump} \)

• Economical Factor
  Policy-Oriented, such as Short-Term Interest Rate (Feds Fund Rate)
  \( \Omega(S_i) = r_i + \delta(S_i), \quad \delta, \text{ asymmetrically distributed} \)
Nonparametric Decomposition to realize Model-Free Simulation
distinguishing and recognizing factors moving the market

Advanced Financial Signal Processing via Wavelet Technique

Raw Financial Signal: \( R_{i+1} - r_i = [\Psi(S_i) + \Delta(S_i) + \delta(S_i)] + \Sigma(S_i) \cdot \epsilon_{i+1}, i=1, ..., n \)

decomposition levels (\( \log_2 n \)) with higher resolution are ignored –
almost nothing living there except for the components of the heteroscedastic white noise
Two-Peak Phenomenon

Histogram of STTB in 1980's
A Clue to the Remarkable Story of the Great Crash

Max Uncertainty Line

value of parameter a

12-Month STTB Moving Average
12-Month S&P 500 Return M.A. (%)

Feb 1986 Oct 1987
Nov 1981 Jul 1993
Similar Sign before Another Crash in Another Market

Nikkei 225 12-Month Moving Averages of Return Rate and STTB

10/97 Asia Crisis  08/98 Russia Crisis & LTCM Fallout

12/1985  03/2003
Striking Coincidence

Histogram of Nikkei Monthly STTB from 01/1991 to 12/1998
from the above two pre-crash patterns, it is intuitive to perceive the following principle:

When the market’s behavior eventually evolves into a rapid oscillation around the maximum uncertainty level rather than taking a typical cyclic course, the chance for the market to crash and the crash extent will increase day-by-day until that happens.
Bubble Phenomena:  
a dynamic picture for the principle of cyclic hazard

as the cycles keep converging to, thus oscillating around, the maximum uncertainty level,
accumulating fear of uncertainty builds up to end up with a market crisis
Underlying Mechanism for Principle of Cyclic Hazard
Ping-Pong Hazard - a physical illustration with a pendulum