Guaranteed Minimum Withdrawal Benefit in Variable Annuities

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Variable Annuities

- Popular savings instruments

- Guaranteed Minimum Withdrawal Benefit (GMWB) is the most popular VA rider.
Guaranteed Minimum Withdrawal Benefit

- Downside income protection against investment risk

- Potential upside equity gain
  If the investment performance is strong, policyholders will benefit.
GMWB Example

- GMWB is elected at the issuance of the variable annuity.
- Single premium: $100,000
- Initial Guaranteed Withdrawal Balance (GWB): $100,000
- Maximum Annual Withdrawal Amount (MAWA): $7,000 = 7% of initial GWB
<table>
<thead>
<tr>
<th>Contract year</th>
<th>Investment return</th>
<th>Fund before withdrawal</th>
<th>Annual withdrawal</th>
<th>Fund after withdrawal</th>
<th>Remaining benefit</th>
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<tbody>
<tr>
<td>1</td>
<td>10%</td>
<td>110,000</td>
<td>7,000</td>
<td>103,000</td>
<td>93,000</td>
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<td>2</td>
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<td>r</td>
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</table>
GMWB charge

- The charge is deducted from the account value.
  Example:
  - 50 basis points of average daily account value
Financial view of GMWB

- GMWB is a put option attached to an equity-like insurance product.
- If the account value is always higher than withdrawal amount, there is no liability under the GMWB.
- If the account balance reaches zero, GMWB guarantees all remaining periodic payments.
- This put option has a random exercise time.
- In theory market value of the put option should equal market value of the premium contributions.
Pricing a Simple GMWB

- Initial investment: $100
- Contract term: 15 years
- Annual withdrawal amount: $6.67
- Fund volatility: 20%
- Risk-free rate: 5%
- Charge as a percentage $q$ of account value continuously
- All policyholder fully utilize GMWB from the first contract year.
- All policyholders have the same entry age.
- No lapses, no deaths, no dynamic behavior.
Figure 7: Plot of present value of contributions against value of put option. Lines cross when $q = 0.0048$ when put value = $4.40$
Policyholder Behavior

Policyholder behavior dramatically affects the cost of GMWB in the real world.

- Dynamic GMWB utilization
  Possible influencing factors:
  1. in-the-moneyness of GMWB
  2. contract duration
  3. age
  4. different benefit features
- Lapse
- Mortality
Model Policyholder Behavior using Three-state Model

State 1: Active

State 2: Utilize GMWB

State 3: Lapse

Figure 1: State Transition Diagram For Policyholders
Base Transition Intensities

- Utilization rate:
  \[ \bar{\mu}^{12} = 0.2 \]

- Lapse rate \( \bar{\mu}^{13}(t) \) changes over time because of decreasing surrender charges.

![Graph showing base lapse rates over the contract duration (Surrender charge ends at the end of the 7th year)]
Dynamic Transition Intensities

We use an exponential function to model the impact of in-the-moneyness on utilization and lapse intensity. Assume

\[ \mu_{12}(t) = \bar{\mu}_{12} \exp\{\lambda(ITM(t))\} \]

and

\[ \mu_{13}(t) = \bar{\mu}_{13} \exp\{-\nu(ITM(t))\} \]

where

\[ ITM(t) = \max\left(0, \left(\frac{GB(t)}{AV(t)} - 1\right)\right). \]

\( GB(t) \) is the remaining guaranteed benefit at time \( t \).

\( AV(t) \) is the current account value at time \( t \).
Sensitivity Analysis

Assume there are 1,000 policyholders elected GMWB at the same time. Using the previous GMWB example, we can see how the values of put benefit and contributions change with $\lambda$ and $\nu$.

![Diagram showing PV of put benefit and contribution under different $\lambda$ when $\nu=1$](image1)

![Diagram showing PV of put benefit and contribution under different $\nu$ when $\lambda=1$](image2)
Summary

1. The value of GMWB depends on policyholder behavior.
2. It’s hard to model, because no published data available.
3. My future research will explore pricing, hedging, and risk management of GMWB.