Risk Management of a DB Underpin Pension Plan

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Outline

- Introduction and Background
- Risk Management
  - Hedging Strategies
  - Numerical Results
- Comments and Future Work
Introduction of Current Pension Systems

- Two basic kinds of pension plans
  Defined Benefit (DB) and Defined Contribution (DC)

- Pension Reforms
  Three-dimensional classification (Lindbeck, 2003):
  - Actuarial vs Non-Actuarial
  - Funded vs Unfunded
  - Defined Benefit vs Defined Contribution
Introduction and Background

- “Greater of” benefit a DB and DC hybrid plan
  - DC plus DB guaranteed minimum

- The payoff of the guarantee is same as an exchange option payoff.
  - Eg. McGill University, WLU offer a hybrid pension plan with minimum DB guarantee.
Description of Benefits (DB)

The DB benefits depend on the salary at exit age $x_r$:

\[ D_{xe,xr}(t) = \alpha \times S_{xr} \times n \times \bar{a}_{xe+t}^{(12)} \]

- $D_{xe,xr}(t)$: The actuarial value at age $xe+t$
- $\alpha$: The accrual rate
- $S_{xr}$: The salary at age $xr$
- $n$: The years of service
- $\bar{a}_{xe+t}^{(12)}$: The annuity factor
Description of Benefits (DC)

- The DC benefits depend on the monthly contribution:

\[ A_{xe,xr}(t + h) = A_{xe,xr}(t) \cdot (1 + hf_t) + c \cdot h \cdot S_{xe+t} \]

Where

- \( c \) : The contribution rate
- \( h \) : The time interval
Risk Management

- Our problem
  - Valuation and risk management of $\text{Max}(D-A, 0)$ at exit.

- Assume no exits
  - Given retirement time $T = x_r - x_e$, the value of guarantee at entry age $x_e$ is,

$$e^{-r(x_r-x_e)} \text{Max}(D_{x_e,x_r}(T) - A_{x_e,x_r}(T), 0)$$
Exchange Option

- Rates of return of two assets
  \[ dS_i = S_i [\alpha_i \, dt + \sigma_i \, dZ_i], \quad i = 1, 2 \]

- A European-type option with maturity time \( T \)
  - Payoff at maturity time \( T \)
    \[ w(S_1, S_2, T) = \text{Max}(S_1 - S_2, 0) \]
  - Price at any time \( t \)
    \[ w(S_1, S_2, t) = S_1 N(d_1) - S_2 N(d_2) \]
    \[ d_1 = \frac{\ln(S_1 / S_2) + 1/2 \cdot \sigma^2 (T - t)}{\sigma \sqrt{T - t}} \]
    \[ d_2 = d_1 - \sigma \sqrt{T - t} \]
    \[ \sigma^2 = \sigma_1^2 - 2\sigma_1 \sigma_2 \rho + \sigma_2^2 \]
Monte Carlo Hedging

- At any time $t$, change the value of DB and DC accounts
- \[ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \]
- the projected value of guarantee
- \[ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \]
- calculate delta
- \[ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \]
- rebalance at time $t+h$
Strategy 1

- Too Volatile!

EAN/Aggregate Cost Method
- Final Salary
- Future Service
- Future Contributions
Strategy 2

Projected Unit Credit (PUC) Cost Method

- Final Salary
- Past Service
- Past Contributions

Eg. Entry age 35, Current age 40, Normal Retirement Age 65

Projected value of DB: \[ \alpha \cdot 5^{25|} \bar{a}^{(12)}_{40} \cdot S_{40} e^{\int_{40}^{65} s(u)du} \]

Projected value of DC: \[ A_{40} e^{\int_{40}^{65} f(u)du} \]

Projected value of guarantee:

\[ \text{Max} \left( 0, \alpha \cdot 5^{25|} \bar{a}^{(12)}_{40} \cdot S_{40} e^{\int_{40}^{65} s(u)du} - A_{40} e^{\int_{40}^{65} f(u)du} \right) \]
Eg. Entry age 35, Current age 40, Normal Retirement Age 65
Projected value of DB: $\alpha \cdot 5 \cdot 25|\ddot{a}^{(12)}_{40} \cdot S_{40}$
Projected value of DC: $A_{40} e^{\int_{40}^{65} f(u) du}$
Projected value of guarantee:

$$Max(0, \alpha \cdot 5 \cdot 25|\ddot{a}^{(12)}_{40} \cdot S_{40} - A_{40} e^{\int_{40}^{65} f(u) du})$$
Hedging Costs

- At time $t$, assume delta hedging shares are $\Delta_{1,t}$ and $\Delta_{2,t}$.
  
  The value of hedging fund is
  $$\Delta_{1,t} \cdot S_{1,t} + \Delta_{2,t} \cdot S_{2,t}$$

- At time $(t+1)$, the value of hedging fund is
  $$\Delta_{1,t} \cdot S_{1,t+1} + \Delta_{2,t} \cdot S_{2,t+1}$$

- At time $(t+1)$, the value of hedging fund is
  $$\Delta_{1,t+1} \cdot S_{1,t+1} + \Delta_{2,t+1} \cdot S_{2,t+1}$$

- At time $(t+1)$, the hedging cash flow $CF_{t+1}$ is
  $$(\Delta_{1,t+1} - \Delta_{1,t}) \cdot S_{1,t+1} + (\Delta_{2,t+1} - \Delta_{2,t}) \cdot S_{2,t+1}$$
Two ways to calculate hedging costs

- Discount all hedging cash flows and amortize by the salary-related annuity

\[ \frac{\sum_j e^{-r_j CF_j}}{\sum_j e^{-r_j S_j}} \]

- At time t, calculate the proportion of the hedging cash flows of the salary at time t, then calculate the average

\[ \frac{1}{n} \sum_j CF_j / S_j \]
Parameters

- **Base Projections**
  - Mean of Salary Growth Rate: 0.04
  - Std of Salary Growth Rate: 0.02
  - Mean of Crediting Rate: 0.10
  - Std of Crediting Rate: 0.20
  - Contribution Rate: 0.125
  - Discount Rate: 0.05
Results Comparison (% of Salary)

Table 1: Discount Rate: 5%, Strategy 2

<table>
<thead>
<tr>
<th>Entry Age</th>
<th>Hedging Cost 1</th>
<th>Hedging Cost 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>4.69 (0.0086)</td>
<td>4.33 (0.0086)</td>
</tr>
<tr>
<td>25</td>
<td>4.55 (0.0082)</td>
<td>4.25 (0.0080)</td>
</tr>
<tr>
<td>30</td>
<td>4.38 (0.0078)</td>
<td>4.14 (0.0076)</td>
</tr>
<tr>
<td>35</td>
<td>4.20 (0.0074)</td>
<td>4.01 (0.0073)</td>
</tr>
<tr>
<td>40</td>
<td>4.03 (0.0069)</td>
<td>3.87 (0.0068)</td>
</tr>
<tr>
<td>45</td>
<td>3.80 (0.0063)</td>
<td>3.68 (0.0062)</td>
</tr>
<tr>
<td>50</td>
<td>3.56 (0.0056)</td>
<td>3.48 (0.0055)</td>
</tr>
<tr>
<td>55</td>
<td>3.36 (0.0049)</td>
<td>3.20 (0.0048)</td>
</tr>
</tbody>
</table>

Table 2: Hedging Cost 1, Strategy 2

<table>
<thead>
<tr>
<th>Entry Age</th>
<th>Discount Rate:5%</th>
<th>Discount Rate:3%</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>4.69 (0.0086)</td>
<td>4.01 (0.0094)</td>
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<tr>
<td>25</td>
<td>4.55 (0.0082)</td>
<td>3.98 (0.0091)</td>
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<td>30</td>
<td>4.38 (0.0078)</td>
<td>3.92 (0.0085)</td>
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<tr>
<td>35</td>
<td>4.20 (0.0074)</td>
<td>3.84 (0.0081)</td>
</tr>
<tr>
<td>40</td>
<td>4.03 (0.0069)</td>
<td>3.74 (0.0073)</td>
</tr>
<tr>
<td>45</td>
<td>3.80 (0.0063)</td>
<td>3.61 (0.0068)</td>
</tr>
<tr>
<td>50</td>
<td>3.56 (0.0056)</td>
<td>3.43 (0.0060)</td>
</tr>
<tr>
<td>55</td>
<td>3.36 (0.0049)</td>
<td>3.19 (0.0051)</td>
</tr>
</tbody>
</table>
Monthly Hedging Cost
Quantile
Scenario Tests
(Strategy 2, Hedging Cost 2)
Conclusions

- Hedging costs depend on many factors, but are not very sensitive to the entry age or the salary growth rate.

- Standard errors depend on the entry age and increase when the entry age increases.
Future Work

- Use inflation-linked bonds hedge salary and explore basis risk.
- Introduce stochastic interest and annuity.
Thank You!