MULTIVARIATE MODELING OF ASSET RETURNS FOR INVESTMENT GUARANTEES VALUATION

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Introduction

• Canadian insurance industry has widely accepted the regime-switching model for valuation purposes

• However, different funds are offered and dependence between them must be accounted for

• Objectives
  – Present multivariate models that can be used for valuation purposes
  – Compare the CTE provisions for each model
  – Analyze the impacts of a multivariate estimation on the CTE provision
Outline

1. Introduction
2. Models
3. Data
4. Estimation
5. Results
6. Monte Carlo experiment
Models – Multivariate modeling

• Families of models considered
  – Multivariate regime-switching
  – Multivariate GARCH

• Desired features
  – Time-varying volatility and dependence
  – Manageable number of parameters
  – PSD covariance matrix

• Need to specify the dependence structure
  – Copulas or multivariate error distribution
Models – Regime-switching

• Types of regimes
  – Global regimes
    • Each market is influenced by a common process
    • Each market is simultaneously in the same regime
  – Local regimes
    • Each market has its own set of regimes
    • Captures specific market conditions
    • Transitions are independent between markets
  – Local-global regimes
    • Similar to local regimes
    • Transitions are dependent between markets
Models – GARCH

• VECH model (or diagonal VECH)
  – **Idea**: use a univariate GARCH(1,1) for each element of the covariance matrix
  – Covariance matrix:
    • Not PSD unless we impose PSD parameter matrices
    • PSD if we redefine the PSD parameter matrices
      ➔ Model known as PSD VECH

• BEKK model
  – Matrix quadratic form: PSD covariance matrix
  – Features volatility and dependence diffusion
Models – GARCH

• Dynamic Conditional Correlation (DCC)
  – Idea: modeling the dynamics of the conditional correlation of returns is equivalent to modeling conditional covariance of standardized returns.
  – 2 steps modeling/estimation
    (1) Volatility
    (2) Correlation
  – Can use many models for the correlation dynamics
  – PSD covariance matrix if correlation matrix is PSD
  – Particular case: constant correlation model (CCORR or CCC)
Data

- Monthly data from January 1956 to September 2005
- 4 different markets
  - Canada: S&P TSX total return index
  - U.S.: S&P 500 total return index
  - U.K.: monthly historical return of
    - Actuaries Investment Index (01/1956 – 04/1962)
    - FTSE All Shares Index (05/1962 – 09/2005)
  - Japan: monthly historical return of TOPIX
Estimation – Regime-switching

• Different pairs of markets considered
  – Canada – U.S. (high correlation)
  – U.S. – Japan (low correlation)

• Global regime is the most parsimonious
  – Even for low correlated markets
  – Global-local approach has too much parameters to be chosen
  – Global regime approach only will be considered
Estimation – Regime-switching

• Different global regime models considered
  – Gaussian or Student copula
  – 2 or 3 regimes
  – 1 or 2 correlation matrices
  – Compared with constrained univariate model used in practice

• Constrained univariate model – 3 steps
  (1) Transition probabilities: univariate estimation on some arbitrary market or portfolio
  (2) Means and variances: given (1), perform constrained univariate estimation on each market
  (3) Correlation matrix: estimated empirically
Estimation

• Ranked parsimonious models (SBC)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Model</th>
<th>Param.</th>
<th>Log-like.</th>
<th>SBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CCORR GARCH</td>
<td>22</td>
<td>4424.38</td>
<td>4354.08</td>
</tr>
<tr>
<td>2</td>
<td>RS2LN w/ 1 corr. matrix</td>
<td>24</td>
<td>4423.72</td>
<td>4347.04</td>
</tr>
<tr>
<td>3</td>
<td>Same as (2) w/ Student copula</td>
<td>25</td>
<td>4426.50</td>
<td>4346.62</td>
</tr>
<tr>
<td>Last</td>
<td>Constrained Univ. RS2LN</td>
<td>24</td>
<td>4321.89</td>
<td>4245.20</td>
</tr>
</tbody>
</table>

• Constant correlation GARCH model is the most parsimonious
• Consistent with the regime-switching model with one correlation matrix
Results - Assumptions

- CTE provision with 3% annual MER (continuously compounded)
- Different CTE levels (60%, 80%, 95%)
- Initial ratio of MV/GV = 100%
- We consider:
  - CTE computation of an equally weighted portfolio in the 4 markets
  - Comparison with univariate models: Canadian market
## Results – 4 markets

<table>
<thead>
<tr>
<th>Model</th>
<th>60%-CTE</th>
<th>80%-CTE</th>
<th>95%-CTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCORR GARCH</td>
<td>3.3%</td>
<td>6.6%</td>
<td>22.8%</td>
</tr>
<tr>
<td>RS2LN w/ 1 corr. matrix</td>
<td>5.6%</td>
<td>11.2%</td>
<td>33.8%</td>
</tr>
<tr>
<td>Same as (2) w/ Student copula</td>
<td>4.5%</td>
<td>8.9%</td>
<td>29.1%</td>
</tr>
<tr>
<td>Constrained Univ. RS2LN</td>
<td>6.5%</td>
<td>13.0%</td>
<td>38.1%</td>
</tr>
</tbody>
</table>

- Significant variation between RS models
- Less variation between GARCH models (not shown)
- Largest CTE provisions with constrained univariate RS model (for this portfolio) but it is not always the case
Results – 1 market

<table>
<thead>
<tr>
<th>Model</th>
<th>60%-CTE</th>
<th>80%-CTE</th>
<th>95%-CTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS2LN</td>
<td>8.9%</td>
<td>17.8%</td>
<td>47.0%</td>
</tr>
<tr>
<td>MRS2LN</td>
<td>8.2%</td>
<td>16.5%</td>
<td>43.9%</td>
</tr>
<tr>
<td>GARCH</td>
<td>2.7%</td>
<td>5.3%</td>
<td>20.9%</td>
</tr>
<tr>
<td>CCORR</td>
<td>4.8%</td>
<td>9.7%</td>
<td>30.6%</td>
</tr>
</tbody>
</table>

- **Univariate vs multivariate**
  - RS: not clear whether there is an increase in CTE
  - GARCH: significant increase in CTE

- **Regime-switching vs GARCH**
  - RS: CTE provisions are greater than with GARCH
  - True for all other portfolios
Monte Carlo experiment

- Market is more complex than GARCH or regime-switching
- Misspecification (and small samples) have impacts on CTE
  - Bias, non-normality
- **Purpose**: obtain the real distribution of the CTE given a complex market model
- Market model: bivariate stochastic volatility model
Monte Carlo experiment

• Market model specifications:
  – Log-volatilities: correlated VAR(1)
  – Errors: constant correlation bivariate normal
  – Parameters: usual range for stochastic volatility

• Assumptions:
  – CTE: 120 months, 95%, 3% MER, MV/GV=1
  – Portfolio allocation: 60%-40%
  – True CTE: approximated empirically using 100 000 paths of 120 months → 100 000 accumulation factors
  – CTE distribution: 1000 samples of 500 observations

• Approximating models used: RS2LN w/ 1 corr. matrix and CCORR GARCH
Monte Carlo experiment

- CTE distribution (smoothed histogram)

- Greater CTE underestimation risk with regime-switching model
Monte Carlo experiment

- Bias

<table>
<thead>
<tr>
<th></th>
<th>GARCH</th>
<th>Regime-switching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.634167</td>
<td>0.622031</td>
</tr>
<tr>
<td>True value</td>
<td>0.645354</td>
<td></td>
</tr>
<tr>
<td>Std dev.</td>
<td>0.137012</td>
<td>0.163142</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.137468</td>
<td>0.164801</td>
</tr>
</tbody>
</table>

- Path by path inconsistency
  - Only 3 of the 10 smallest (largest) CTEs are common to both models
  - Correlation of 78% between pairs of CTEs
Conclusion

• With the specific dataset presented:
  – CCORR GARCH model is the most parsimonious model (SBC)
  – Current practitioner’s approach provides the worst fit
  – Significant variation in CTEs among models
• Monte Carlo experiment was intended to assess the costs of misspecifying the market model
Conclusion

• Model selection alternative:
  – Choose a model with the lowest CTE RMSE since the target output is the CTE
  – CCORR GARCH has lowest CTE RMSE
• Even if the regime-switching model is widely accepted in a univariate framework
  – Should be reconsidered in a multivariate setting
• Final model selection and CTE computation
  – Depends on the context
  – Should reflect the actuary’s judgment
**Bibliography**

Main paper (does not include the Monte Carlo experiment results):


DCC:


VECH:

Bibliography

BEKK:

Regime-switching:

Stochastic volatility: