Suboptimality of Asian Executive Indexed Options

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Outline

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2. Assumptions
3. Asian Executive Indexed Option
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Options Preliminaries

Sample Price Paths

- $\hat{S}_4 = \sqrt[4]{S_1 S_2 S_3 S_4} = 92.12$, $\hat{H}_4 = \sqrt[4]{H_1 H_2 H_3 H_4} = 99.19$
- European Call Option Payoff $= \max(S_4 - K, 0) = 0$
- Asian Option Payoff $= \max(\hat{S}_4 - K, 0) = 2.12$
- Asian Indexed Option Payoff $= \max(\hat{S}_4 - \hat{H}_4, 0) = 0$
Assumptions

1. Black-Scholes market:
   - Extension to Vasicek short rate
2. Stock $S_t$ and benchmark $H_t$ driven by Brownian motions
3. Existence of state-price process $\xi_t$
4. Agents preferences depend only on the terminal distribution of wealth
Asian Executive Indexed Option (AIO) proposed by Tian (2011):

- **Averaging**: Prevent stock price manipulation
- **Indexing**: Only reward out-performance
- More cost-effective than traditional stock options
- Provide stronger incentives to increase stock prices

Construct a better payoff:

- Same features as the AIO
- Strictly cheaper
- Use the concept of **cost-efficiency**
Cost-Efficiency

From Bernard, Boyle and Vanduffel (2011):

Definition (1)
The cost of a strategy with terminal payoff $X_T$ is given by

$$c(X_T) = E_P[\xi_T X_T]$$

where the expectation is taken under the physical measure $P$.

Intuition: $\xi_T$ represents the price of a particular state

Definition (2)
A payoff is cost-efficient (CE) if any other strategy that generates the same distribution costs at least as much.
Cost-Efficiency

Theorem (1)

Let $\xi_T$ be continuous. Define

$$ Y_T^* = F_{X_T}^{-1}(1 - F_{\xi_T}(\xi_T)) $$

as the cost-efficient counterpart (CEC) of the payoff $X_T$. Then, $Y_T^*$ is a CE payoff with the same distribution as $X_T$ and is almost surely unique.

Intuition: CEC is achieved by reshuffling the outcome of $X_T$ in each state in reverse order with $\xi_T$ while preserving the original distribution.
Constructing a Cheaper Payoff

1. Apply Theorem 1 to each term of the AIO

\[ \hat{A}_T = \max(\hat{S}_T - \hat{H}_T, 0) \]

to get

\[ A^*_T = \max \left( d_S S_T^{1/\sqrt{3}} - d_H H_T^{1/\sqrt{3}}, 0 \right) \]

2. It can be shown that:
   - \( \hat{A}_T \overset{d}{=} A^*_T \)
   - \( A^*_T \) costs strictly less than \( \hat{A}_T \)

\( A^*_T \) inherits the desired features of \( \hat{A}_T \), but comes at a cheaper price
**True Cost Efficient Counterpart**

True CEC

\[ A_T = F_{\hat{A}_T}^{-1}(1 - F_{\xi_T}(\xi_T)) \]

is estimated numerically

Examples:

1. Empirical cumulative distribution functions (CDFs) for each payoff in the base case \(^1\)
2. Reshuffling of \(\hat{A}_T\) to \(A^*_T\) and \(A_T\)
3. Order of \(\hat{A}_T\), \(A^*\) and \(A_T\) vs \(\xi_T\)
4. Price of each payoff and the efficiency loss

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\(^1\) \(K = 100, S_0 = 100, r = 6\%, \mu_S = 12\%, \mu_I = 10\%, \sigma_S = 30\%, \sigma_I = 20\%, \rho = 0.75, q_S = 2\%, q_I = 3\%, T = 1\)
Numerical Results

Figure: Comparison of the CDFs of $A_T$, $A^*_T$ and $\hat{A}_T$. 
Numerical Results

Figure: Reshuffling of outcomes of $\hat{A}_T$ to $A^*_T$
Numerical Results

Plot of $\hat{A}_T$ vs $A_T$

Figure: Reshuffling of outcomes of $\hat{A}_T$ to $A_T$
Numerical Results

**Figure:** Plot of outcomes of $\hat{A}_T$ vs $\xi_T$
Plot of $A^*_T \ vs \ \xi_T$

Figure: Plot of outcomes of $A^*_T \ vs \ \xi_T$
Numerical Results

Plot of $A_T$ vs $\xi_T$

Figure: Plot of outcomes of $A_T$ vs $\xi_T$
### Numerical Results

<table>
<thead>
<tr>
<th>Case</th>
<th>$A_T$</th>
<th>$A^*_T$</th>
<th>$\hat{A}_T$</th>
<th>$\hat{V}_T$</th>
<th>Eff Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_T$</td>
<td>$V^*_T$</td>
<td>Eff Loss</td>
<td>$\hat{V}_T$</td>
<td>Eff Loss</td>
</tr>
<tr>
<td>Base Case</td>
<td>3.26</td>
<td>4.34</td>
<td>33%</td>
<td>4.36</td>
<td>34%</td>
</tr>
<tr>
<td>$r = 4%$</td>
<td>2.96</td>
<td>4.37</td>
<td>48%</td>
<td>4.40</td>
<td>49%</td>
</tr>
<tr>
<td>$\mu_s = 8%$</td>
<td>3.97</td>
<td>4.35</td>
<td>10%</td>
<td>4.36</td>
<td>10%</td>
</tr>
<tr>
<td>$\mu_l = 13%$</td>
<td>3.26</td>
<td>4.34</td>
<td>33%</td>
<td>4.36</td>
<td>34%</td>
</tr>
<tr>
<td>$\sigma_s = 35%$</td>
<td>3.97</td>
<td>5.04</td>
<td>27%</td>
<td>5.07</td>
<td>28%</td>
</tr>
<tr>
<td>$\sigma_l = 15%$</td>
<td>3.27</td>
<td>4.34</td>
<td>33%</td>
<td>4.36</td>
<td>33%</td>
</tr>
<tr>
<td>$\rho = 0.9$</td>
<td>2.28</td>
<td>2.86</td>
<td>25%</td>
<td>2.87</td>
<td>26%</td>
</tr>
<tr>
<td>$q_s = 1.5%$</td>
<td>3.27</td>
<td>4.35</td>
<td>33%</td>
<td>4.37</td>
<td>34%</td>
</tr>
<tr>
<td>$q_l = 2%$</td>
<td>3.25</td>
<td>4.34</td>
<td>33%</td>
<td>4.36</td>
<td>34%</td>
</tr>
</tbody>
</table>

**Table:** Prices and efficiency loss of $A^*_T$ and $\hat{A}_T$ compared against $A_T$ across different parameters.
Stochastic Interest Rates

Extension to a market with Vasicek short rate:

1. State price process expressed as a function of market variables

2. Pricing formula for the AIO
Summary

- Reviewed the use of *averaging* and *indexing* in the context of executive compensation
- Constructed a strictly cheaper payoff with the same features as the AIO using *cost-efficiency*
- Numerical examples that illustrate reshuffling of payoffs and loss of efficiency
- Extension to the case of stochastic interest rates