Forward transition rates in a multi-state model

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In the last decade the idea of forward rates was transferred from the interest world to the mortality framework. Forward mortality rates are mostly used in the context of securitization of mortality and longevity risk. In the course of the introduction of Solvency II the valuation of insurance products by means of forward rates becomes more and more important, since forward rates are a helpful tool for calculating market values. While there exists a lot of literature on forward mortality rates, forward transition rates in multi-state models are hardly discussed. We start our investigation by discussing the definition of forward rates in general and give a generally valid definition of forward rates. Thereby, we take into account that the forward rates depend on the considered product, so that this definition is even valid in cases where forward rates seem to be contradictory. Our paper continues the work of Norberg (Insurance: Mathematics and Economics, 2010, 47(2), 105-112) who made the first attempt to define general forward transition rates. While he calculates forward rates only for products that are traded on a financial market, we show (under certain circumstances) how market prices can be decomposed to single risks and then reassembled in order to price other products that are not traded on the market. More precisely, we discuss cycle-free multi-state models in more detail. By assuming independence between transition rates that go to different states and a certain dependency between transition rates that go to the same state, we are able to define forward rates for single risks and apply them to calculate the prices of complex products. Other dependency structures can also be incorporated. We demonstrate our results for joint life insurance and disability insurance. Furthermore, we discuss how forward rates can be defined via Kolmogorov forward equations.